The Anderson-Higgs Mechanism in Superconductors

Asle Sudbø

Department of Physics,
Norwegian University of Science and Technology

Summer School "Symmetries and Phase Transitions from Crystals and Superconductors to the Higgs particle and the Cosmos" Dresden 2016

References

P. W. Anderson, Phys. Rev. **130**, 439 (1962)

P. Higgs, Phys. Rev. Lett. **13**, 508 (1964). Nobel Prize 2013.

F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964). Nobel Prize 2013.

G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964)

H. Kleinert, *Gauge fields in Condensed Matter Physics*, (World Scientific, Singapore (1989)

A. K. Nguyen and A. Sudbø, Phys. Rev **B 60**, 15307 (1999)

J. Hove and A. Sudbø, Phys. Rev. Lett. **84**, 3426 (2000)

E. Babaev, A. Sudbø, and N. W. Ashcroft, Nature 431, 666 (2004)

J. Smiseth, E. Smørgrav, E. Babaev, and A. Sudbø, Phys. Rev. B **71**, 214509 (2005)

A. Sudbø, *The Anderson Higgs Mechanism for the Meissner-effect in superconductors*, in K. Fossheim: *Superconductivity: Discovery and Discoverers. Ten Physics Nobel Laureates Tell Their Stories*, Springer (2013)

F. S. Nogueira, A. Sudbø, and I. Eremin, Phys. Rev. B **92**, 224507 (2015)

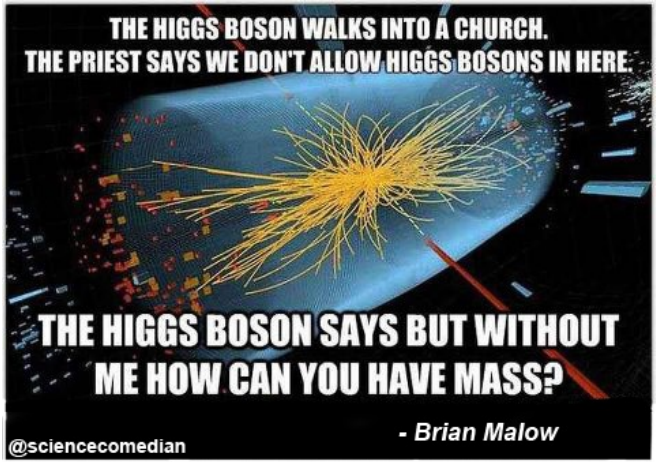
P. N. Galteland and A. Sudbø, Phys. Rev. B **94**, 054518 (2016)

Outline

Overview

- What is the Higgs mechanism?
- Definition of the superconducting state
- Theories of superconductors
- Higgs Mechanism I: Mean field theory physics
- Higgs Mechanism II: Fluctuation physics

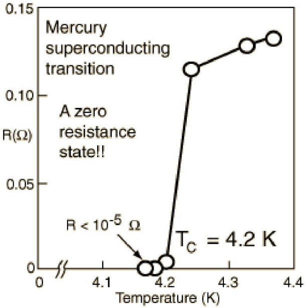
What is the Higgs mechanism?



What is the Higgs mechanism?

- The Higgs mechanism is the dynamical generation of a mass of a gauge-boson (mediating interactions) resulting from the coupling to some matter field.
- In superconductors, it is the generation of a mass of the photon.
- Superconductors have radically different electrodynamics compared to non-superconductors.
- The Higgs mass therefore distinguishes a metallic state from a superconducting state.

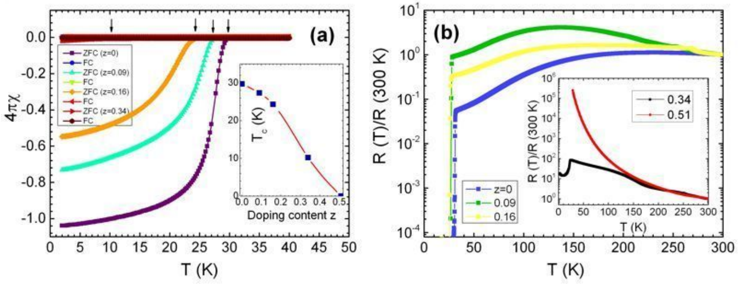
Defining properties of a superconductor



H. Kammerling-Onnes (1911). Nobel Prize 1913.

But: SC is much more than just perfect conductivity!

Defining properties of a superconductor

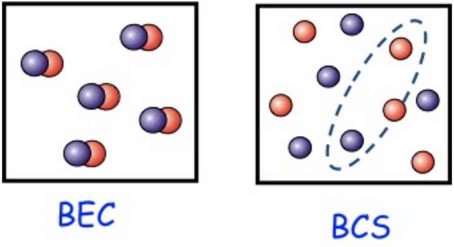


SC: Perfect diamagnet! Completely different from a perfect conductor!

W. Meissner, R. Oechsenfeld (1933)

Cooper-pairing

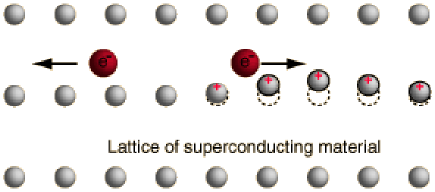
All theories of superconductivity somehow involve the concept of Cooper-pairing: Two electrons experience an *effective* attractive (!) interaction and form pairs.



NB!! No more than pairs! (L. Cooper (1956); J. Bardeen, L. Cooper, R. J. Schrieffer (1957)).

Cooper-pairing

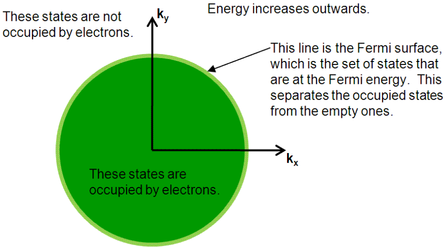
All theories of superconductivity somehow involve the concept of Cooper-pairing: Two electrons experience an *effective* attractive (!) interaction and form pairs.



NB!! No more than pairs! (L. Cooper (1956); J. Bardeen, L. Cooper, R. J. Schrieffer (1957)).

Cooper-pairing

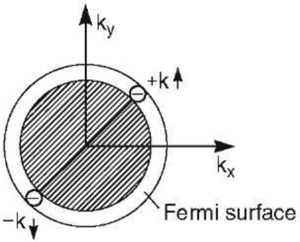
All theories of superconductivity somehow involve the concept of Cooper-pairing: Two electrons experience an *effective* attractive (!) interaction and form pairs.



NB!! No more than pairs! (L. Cooper (1956); J. Bardeen, L. Cooper, R. J. Schrieffer (1957)).

Cooper-pairing

All theories of superconductivity somehow involve the concept of Cooper-pairing: Two electrons experience an *effective* attractive (!) interaction and form pairs.



NB!! No more than pairs! (L. Cooper (1956); J. Bardeen, L. Cooper, R. J. Schrieffer (1957)).

Generalized BCS theory

Effective BCS Hamiltonian

$$
\begin{aligned}
H &= \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} \\
&+ \sum_{\mathbf{k},\mathbf{k}',\sigma} \underbrace{V_{\mathbf{k},\mathbf{k}'}}_{Effective \quad interaction} c^{\dagger}_{\mathbf{k},\sigma} c^{\dagger}_{-\mathbf{k},-\sigma} c_{-\mathbf{k}',-\sigma} c_{\mathbf{k}',\sigma} \\
\Delta_{\mathbf{k}} &\equiv \sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \langle c_{-\mathbf{k}',-\sigma} c_{\mathbf{k}',\sigma} \rangle
\end{aligned}
$$

- H is invariant if $c_{\mathbf{k},\sigma} \rightarrow c_{\mathbf{k},\sigma}\ e^{i\theta}$. Theory is $U(1)$-symmetric. Note: Global U(1)-invariance!
- $\Delta_{\mathbf{k}} \neq 0 \rightarrow U(1)$-symmetry is broken!
- If $V_{\mathbf{k},\mathbf{k}'} = V\ g_{\mathbf{k}}\ g_{\mathbf{k}'}$, then $\Delta_{\mathbf{k}} = \Delta_0(T) g_{\mathbf{k}}$

Normal metal in zero electric field

No net motion of electrons
Electrons (almost) do not see each other

Normal metal in electric field

Forced net motion of electrons
Electrons (almost) do not see each other

Superconductors in zero electric field

Unforced net motion of electrons
Electrons states are co-operatively highly organized

Outline

Building a theory of Superconductivity

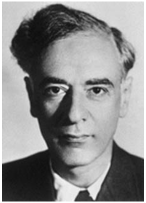
- A theory of wave-function of Cooper-pairs

$$\Psi(r) = |\Psi(r)|\, e^{i\theta(r)}$$

- This theory needs to be able to somehow distinguish (by an order parameter OP) the high-temperature normal metallic state from the low-temperature superconducting state.
- An OP is the expectation value of some operator that effectively distinguishes an ordered state from a disordered state, separated by a *phase transition*.
- Two states separated by a phase transition cannot be analytically continued from one to the other.
- What is the order parameter of a superconductor?

Conventional phase transitions

Conventional phase transitions are well described by the Landau-Ginzburg-Wilson (LGW) paradigm



L. D. Landau

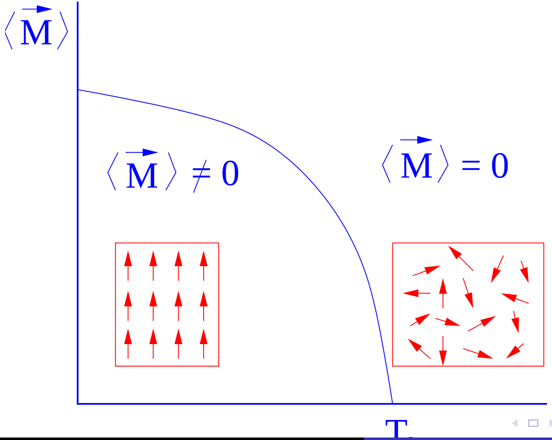
V. Ginzburg

K. G. Wilson

$$\begin{aligned} \mathcal{F} &= \frac{1}{2}(\nabla\vec{M})^2 + \frac{r}{2}\vec{M}^2 + \frac{u}{8}(\vec{M}^2)^2 \\ r &\propto T - T_c \\ r < 0 &\Longrightarrow \langle\vec{M}\rangle \neq 0 \end{aligned}$$

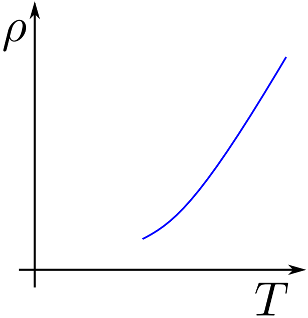
Conventional phase transitions

Conventional phase transitions are well described by the Landau-Ginzburg-Wilson (LGW) paradigm

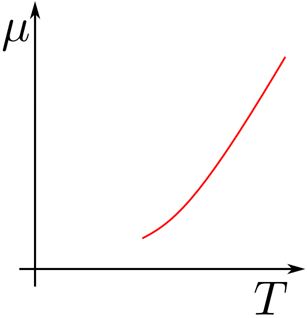


Superfluids and superconductors

Superconductor

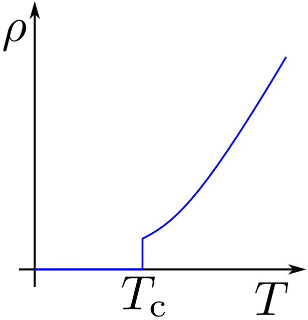


Superfluid



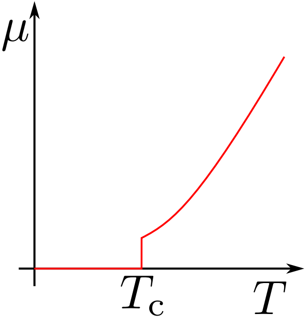
Superfluids and superconductors

Superconductor



Zero resistivity

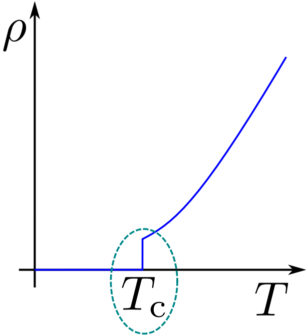
Superfluid



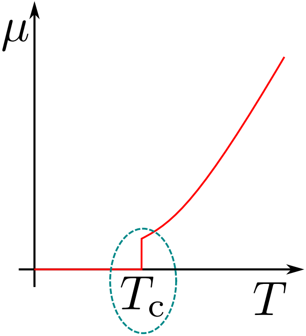
Zero viscosity

Superfluids and superconductors

Superconductor



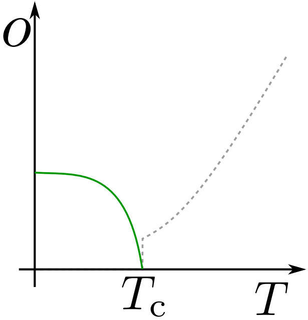
Superfluid



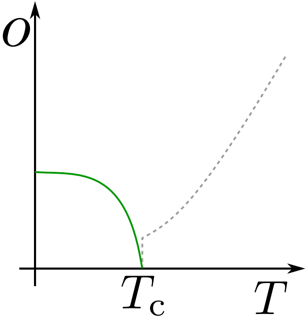
Phase transition

Superfluids and superconductors

Superconductor



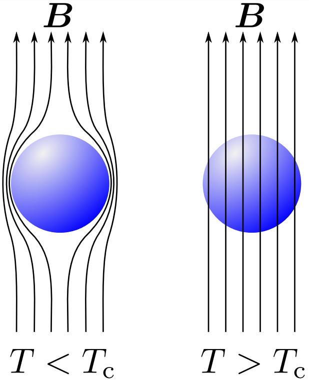
Superfluid



Order parameter

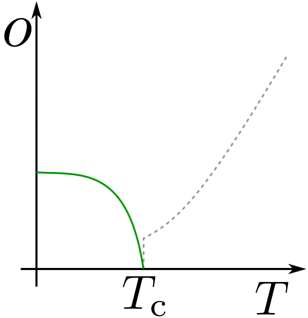
Superfluids and superconductors

Superconductor



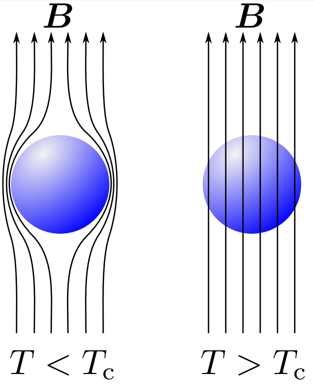
Meissner effect

Superfluid



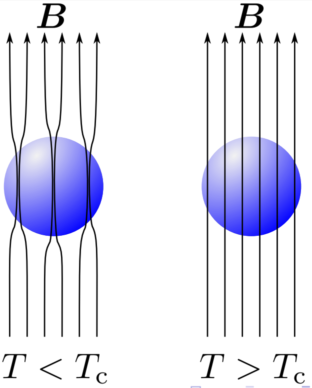
Superfluids and superconductors

Type I superconductor



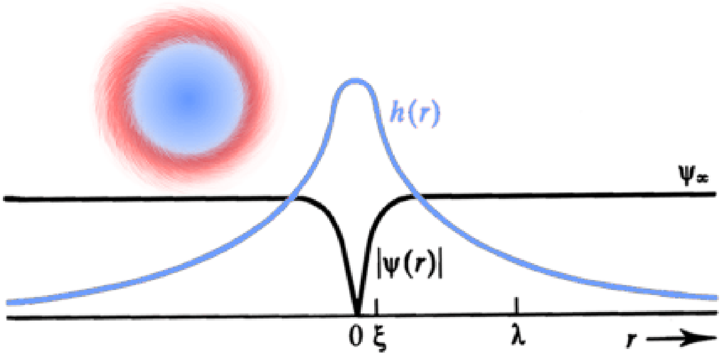
Meissner effect

Type II superconductor

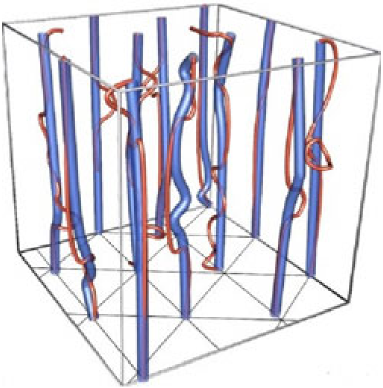


Mixed phase

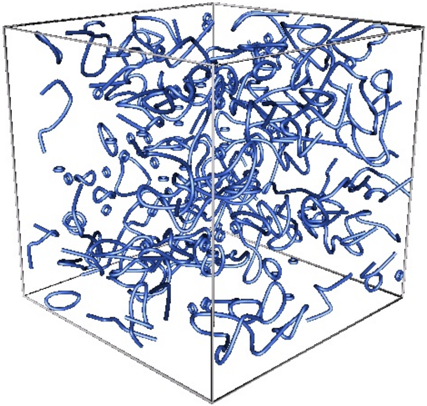
Vortices suppress Ψ



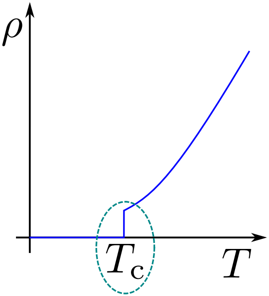
Vortices suppress Ψ



Vortices suppress Ψ



Effective models

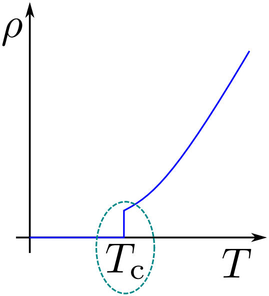


- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)|e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Effective models

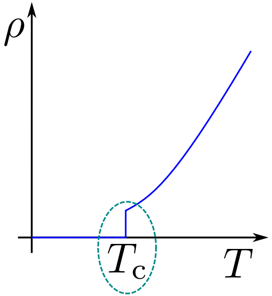


- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)|e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Effective models

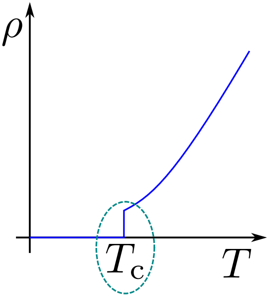


- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)| e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Effective models

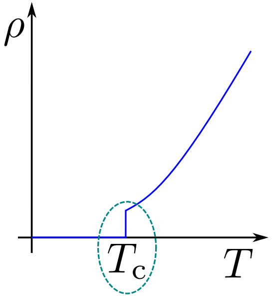


- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)| e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Effective models

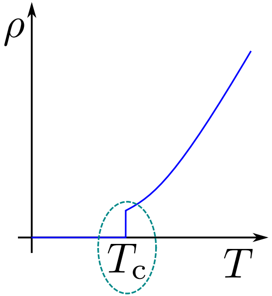


- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)|e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Effective models



- Construct effective models
- Complex matter field:

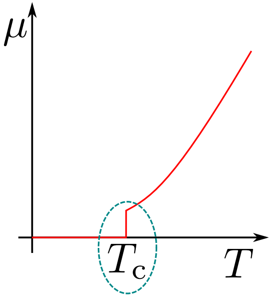
$$\psi(r) = |\psi(r)|e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Ginzburg-Landau model for superconductors

$$H[\psi, A] = \int d^d r \left\{ \frac{g}{2} |[\nabla - ieA(r)]\psi(r)|^2 + \frac{\alpha t}{2} |\psi(r)|^2 + \frac{u}{4!} |\psi(r)|^4 + \frac{1}{2} [\nabla \times A(r)]^2 \right\}$$

Effective models



- Construct effective models
- Complex matter field:

$$\psi(r) = |\psi(r)| e^{i\theta(r)}$$

- Neglect microscopic details
- Valid near the phase transition

Gross-Pitaevskii model for superfluids

$$H[\psi] = \int d^d r \left[\frac{1}{2m} |\nabla \psi(r)|^2 + V(r) \, |\psi(r)|^2 + \frac{U_0}{2} |\psi(r)|^4 \right]$$

Order parameter of a superfluid

Gross-Pitaevskii model for superfluids

$$H[\psi] = \int d^d r \left[\frac{1}{2m} |\nabla \psi(r)|^2 + V(r) |\psi(r)|^2 + \frac{U_0}{2} |\psi(r)|^4 \right]$$

$$Z = e^{-\beta F} = \int D\Psi D\Psi^* \; e^{-\beta H}$$

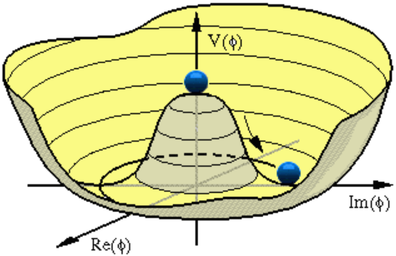
$$\langle O \rangle = \frac{1}{Z} \int D\Psi D\Psi^* \; O(\Psi, \Psi^*) \; e^{-\beta H}$$

Candidate OPs

$$|\langle \Psi(r) \rangle|^2 = |\langle |\Psi(r)| \; e^{i\theta(r)} \rangle|^2 \approx |\Psi|^2 |\langle e^{i\theta(r)} \rangle|^2 \quad (Local)$$

$$\Upsilon = \frac{\partial^2 F}{\partial \Delta\theta^2} \quad (Non-local)$$

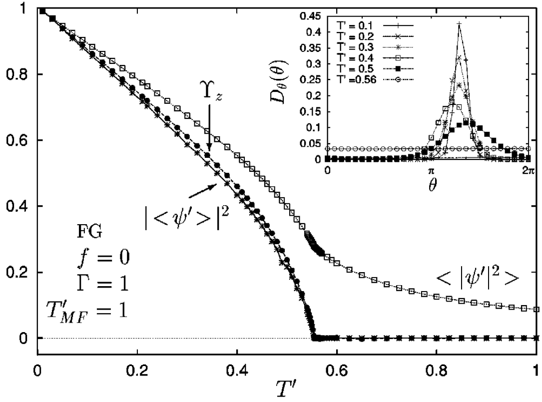
Order parameter of a superfluid



$$V(|\Psi|) = \alpha \left(\frac{T - T_c}{2T_c} \right) |\Psi|^2 + \frac{u}{4!} |\Psi|^4$$

For the Higgs-mechanism to work at all, we need $\langle |\Psi| \rangle \neq 0$!

Order parameter of a superfluid



A. K. Nguyen and A. Sudbø, Phys. Rev **B 60**, 15307 (1999).

Order parameter of a superconductor

Ginzburg-Landau model for superconductors

$$
\begin{aligned}
H[\Psi, A] &= \int d^d r \left\{ \frac{g}{2} \left|[\nabla - ieA(r)]\Psi(r)\right|^2 + \frac{\alpha t}{2} |\Psi(r)|^2 + \frac{u}{4!} |\Psi(r)|^4 \right. \\
&+ \left. \frac{1}{2} [\nabla \times A(r)]^2 \right\} \\
\langle O \rangle &= \frac{1}{Z} \int D\Psi D\Psi^* \; O(\Psi, \Psi^*) \; e^{-\beta H}
\end{aligned}
$$

$$
\begin{aligned}
|\langle \Psi(r) \rangle|^2 &= |\langle |\Psi(r)| \; e^{i\theta(r)} \rangle|^2 \approx |\Psi|^2 |\langle e^{i\theta(r)} \rangle|^2 \quad (Local) \\
\Upsilon &= \frac{\partial^2 F}{\partial \Delta\theta^2} \quad (Non - local)
\end{aligned}
$$

Order parameter of a superconductor

This will not work!

$$\Psi(r) = |\Psi(r)|e^{i\theta(r)} \approx |\Psi_0|e^{i\theta(r)}$$

$$\begin{aligned}
H[\Psi, A] &= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} (\nabla\theta - eA(r))^2 + \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
|\langle\Psi(r)\rangle|^2 &= 0! \\
\Upsilon &= 0!
\end{aligned}$$

- Elitzur's theorem: The expectation value of a gauge-invariant local operator can never acquire a non-zero expectation value in a gauge-theory.
- In a gauge-theory, the Goldstone modes are "eaten up" by the gauge-field $\to \Upsilon = 0$.

Order parameter of a superconductor

$$
\begin{aligned}
H[\Psi, A] &= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} (\nabla\theta - eA(r))^2 + \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
&= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} \Big((\nabla\theta)^2 - 2e\nabla\theta \cdot A(r) + e^2 A^2 \Big) \right. \\
&+ \left. \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
&= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} (\nabla\theta)^2 - \overbrace{g|\Psi_0|^2 \nabla\theta \cdot A(r)}^{\boldsymbol{j}\cdot\boldsymbol{A}} \right. \\
&+ \left. \underbrace{\frac{g|\Psi_0|^2 e^2}{2}}_{\equiv {\color{red} m^2}} A^2 + \frac{1}{2}[\nabla \times A(r)]^2 \right\}
\end{aligned}
$$

Order parameter of a superconductor

Consider in more detail the gauge-part of the theory (introducing Fourier-transformed fields)

$$
\begin{aligned}
H_A &= m^2 A_q A_{-q} + \frac{1}{2} q^2 A_q A_{-q} - \frac{1}{2}\left(S_q A_{-q} + S_{-q} A_q\right) \\
&= \underbrace{\left(m^2 + \frac{1}{2} q^2\right)}_{\equiv D_0(q)} A_q A_{-q} - \frac{1}{2}\left(S_q A_{-q} + j_{-q} A_q\right) \\
&= D_0(q) \left\{ \left(A_q - \frac{1}{2} D_0^{-1}(q) S_q \right) \cdot \left(A_{-q} - \frac{1}{2} D_0^{-1}(q) S_{-q} \right) \right. \\
&- \frac{1}{4} D_0^{-1} S_q \cdot S_{-q} \\
S_q &\equiv \frac{g |\Psi_0|^2 e^2}{2} \mathcal{F}(\nabla \theta)
\end{aligned}
$$

Order parameter of a superconductor

Integrate out the shifted gauge fields $\tilde{A}_q = A_q - \frac{1}{2} D_0^{-1}(q) S_{-q}$:

$$H_{eff}(\Psi) = \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} S_q \cdot S_{-q}$$

$$m^2 \equiv \frac{g|\Psi_0|^2 e^2}{2}$$

Define a momentum-dependent phase-stiffness Υ_q:

$$\Upsilon_q = \frac{d^2 H_{eff}}{dS_q dS_{-q}} = \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2}$$

Interested in long-distance physics $q \to 0$.

Order parameter of a superconductor

$$\lim_{q\to 0} \Upsilon_q = \lim_{q\to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2+q^2} = \frac{g|\Psi_0|^2}{2}; e = 0$$

$$\lim_{q\to 0} \Upsilon_q = \lim_{q\to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2+q^2} = 0; e \neq 0$$

Order parameter of a superconductor

$$\lim_{q\to 0} \Upsilon_q = \lim_{q\to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2+q^2} = \frac{g|\Psi_0|^2}{2}; e = 0$$

$$\lim_{q\to 0} \Upsilon_q = \lim_{q\to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2+q^2} = 0; e \neq 0$$

The superfluid density is NOT an order parameter for a superconductor!

It IS an order parameter for a superfluid!

Outline

Ginzburg-Landau lattice model

$$\mathcal{Z} = \int \mathcal{D}\boldsymbol{A} \left(\prod_{\alpha} \int \mathcal{D}\psi_{\alpha} \right) e^{-S}$$

where the action is

$$S = \beta \int d^3 r \left\{ \frac{1}{2} \sum_{\alpha} \left(|\nabla - ie\boldsymbol{A}(\boldsymbol{r}) \right) \psi_{\alpha}(\boldsymbol{r})|^2 + V\left(\{|\psi_{\alpha}(\boldsymbol{r})|\}\right) + \frac{1}{2} \left(\nabla \times \boldsymbol{A}(\boldsymbol{r})\right)^2 \right\}$$

Ginzburg-Landau lattice model

London-approximation

$$\Psi(r) \approx |\Psi_0|\, e^{i\theta(r)}$$

$$S = \beta \sum_{\mathbf{r}} \left\{ - \sum_{\mu,\alpha} \cos\left(\Delta_\mu \theta_{\mathbf{r},\alpha} - eA_{\mathbf{r},\mu}\right) + \frac{1}{2}(\Delta \times \mathbf{A}_{\mathbf{r}})^2 \right\}$$

Current-loop formulation

$$e^{\beta \cos \gamma} = \sum_{b=-\infty}^{\infty} I_b(\beta) e^{ib\gamma}$$

$$\mathcal{Z} = \int \mathcal{D}\boldsymbol{A} \left(\prod_{\alpha} \int \mathcal{D}\theta_\alpha \right) \times \prod_{\boldsymbol{r},\mu,\alpha} \sum_{b_{\boldsymbol{r},\mu,\alpha}=-\infty}^{\infty} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) e^{ib_{\boldsymbol{r},\mu,\alpha}(\Delta_\mu \theta_{\boldsymbol{r},\alpha} - eA_{\boldsymbol{r},\mu})} \times \prod_{\boldsymbol{r}} e^{-\frac{\beta}{2}(\Delta \times \boldsymbol{A}_{\boldsymbol{r}})^2}$$

P. N. Galteland and A. S., Phys. Rev **B94**, 054518 (2016).

Current-loop formulation

$$\mathcal{Z}_\theta = \prod_{\boldsymbol{r},\alpha} \int_0^{2\pi} d\theta_{\boldsymbol{r},\alpha} e^{-i\theta_{\boldsymbol{r},\alpha}\left(\sum_\mu \Delta_\mu b_{\boldsymbol{r},\mu,\alpha}\right)}$$

θ-integration gives:

$$\boldsymbol{\Delta} \cdot \boldsymbol{b}_{\boldsymbol{r},\alpha} = 0 \;\forall\; \boldsymbol{r}, \alpha$$

The partition function is then given by

$$\mathcal{Z}_J = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta)$$

$$\prod_{\boldsymbol{r}} e^{-\overbrace{\left[ie\sum_\alpha \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \boldsymbol{J}\cdot\boldsymbol{A} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}^{\equiv S_J}}$$

Current-loop formulation

Integrating out the A-field gives

$$\mathcal{Z} = \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} \underbrace{I_{b_{\boldsymbol{r},\alpha,\mu}}(\beta)}_{\sim e^{-(b_{\boldsymbol{r},\alpha,\mu})^2/2\beta}} \prod_{\boldsymbol{r},\boldsymbol{r}'} e^{-\frac{e^2}{2\beta}\sum_{\alpha,\beta} \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{b}_{\boldsymbol{r}',\beta} \underbrace{D(\boldsymbol{r}-\boldsymbol{r}')}_{1/|\boldsymbol{r}-\boldsymbol{r}'|}}$$

This is a theory of supercurrents $\boldsymbol{b}_{\boldsymbol{r},\alpha}$, forming closed loops, interacting via the potential $D(\boldsymbol{r}-\boldsymbol{r}')$ mediated by the gauge-field A, as well as contact interaction.

P. N. Galteland and A. S., Phys. Rev **B94**, 054518 (2016).

Gauge-field correlations and Higgs mass

$$\langle A^{\mu}_{\boldsymbol{q}} A^{\nu}_{-\boldsymbol{q}} \rangle = \frac{1}{\mathcal{Z}_0} \frac{\delta^2 \mathcal{Z}_J}{\delta J_{-\boldsymbol{q},\mu} \delta J_{\boldsymbol{q},\nu}} \bigg|_{\boldsymbol{J}=0}$$

$$= \frac{1}{\mathcal{Z}_0} \sum_{\{\boldsymbol{b},m\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\alpha,\mu}}(\beta)$$

$$\times \left(-\frac{\delta^2 S_J}{\delta \boldsymbol{J}^{\mu}_{-\boldsymbol{q}} \delta \boldsymbol{J}^{\nu}_{\boldsymbol{q}}} - \frac{\delta S_J}{\delta \boldsymbol{J}^{\mu}_{-\boldsymbol{q}}} \frac{\delta S_J}{\delta \boldsymbol{J}^{\nu}_{\boldsymbol{q}}} \right) e^{-S_J} \bigg|_{\boldsymbol{J}=0}$$

Gauge-field correlations and Higgs mass

$$\langle \boldsymbol{A}_{\boldsymbol{q}} \cdot \boldsymbol{A}_{-\boldsymbol{q}} \rangle \sim \frac{1}{q^2 + m_{\boldsymbol{A}}^2}$$

Excitations $\rightarrow$ Higgs mass

$$\langle \boldsymbol{A}_{\boldsymbol{q}} \cdot \boldsymbol{A}_{-\boldsymbol{q}} \rangle = \frac{1}{\beta \, |\boldsymbol{q}|^2} \left(2 - \frac{e^2}{\beta \, |\boldsymbol{q}|^2} \sum_{\alpha\beta} \langle \boldsymbol{b}_{\boldsymbol{q},\alpha} \cdot \boldsymbol{b}_{-\boldsymbol{q},\beta} \rangle \right)$$

$$m_{\boldsymbol{A}}^2 = \lim_{\boldsymbol{q} \to 0} \frac{2}{\beta \langle \boldsymbol{A}_{\boldsymbol{q}} \boldsymbol{A}_{-\boldsymbol{q}} \rangle}$$

Gauge-field correlations and Higgs mass

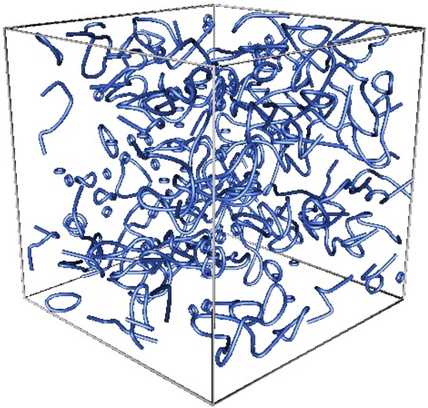
$$\lim_{\boldsymbol{q}\to 0} \frac{e^2}{2\beta}\langle \boldsymbol{B_q}\cdot\boldsymbol{B_{-q}}\rangle \sim \begin{cases} (1-C_2(T))q^2, & T > T_C. \\ q^2 - C_3(T)q^{2+\eta_{\boldsymbol{A}}}, & T = T_C. \\ q^2 - C_4(T)q^4, & T < T_C. \end{cases}$$

$$m_{\boldsymbol{A}}^2 \sim \begin{cases} 0, & T \geq T_C. \\ \frac{1}{C_4(T)}, & T < T_C. \end{cases}$$

Fluctuating supercurrents $\boldsymbol{B_q} \equiv \sum_\alpha \boldsymbol{b}_{\boldsymbol{q},\alpha}$ create a Higgs mass!

Vortex-loop formulation

Go back to vortices



Vortex-loop formulation

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[ie\sum_{\alpha} \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

Solve constraint

$$\begin{aligned} \boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha} &= 0 \;\forall\; \boldsymbol{r},\alpha \\ \boldsymbol{b}_{\boldsymbol{r},\alpha} &= \Delta\times\boldsymbol{K}_{\boldsymbol{r},\alpha} \end{aligned}$$

Vortex-loop formulation

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[ie\sum_\alpha \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

Use Poisson summation formula

$$\sum_{\boldsymbol{n}=-\infty}^{\infty} e^{i2\pi\boldsymbol{n}\cdot\Delta\times\boldsymbol{h}} = \sum_{\boldsymbol{K}=-\infty}^{\infty} \delta(\Delta\times\boldsymbol{h} - \Delta\times\boldsymbol{K})$$

$$\sum_{\boldsymbol{m}=-\infty}^{\infty} e^{i2\pi\boldsymbol{m}\cdot\boldsymbol{h}} = \sum_{\boldsymbol{K}=-\infty}^{\infty} \delta(\boldsymbol{h} - \boldsymbol{K})$$

$$\boldsymbol{m} = \Delta\times\boldsymbol{n} \quad (\Delta\cdot\boldsymbol{m} = 0!)$$

Vortex-loop formulation

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[ie\sum_\alpha \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \int \mathcal{D}(\boldsymbol{h}) \sum_{\{\boldsymbol{m}\}} \prod_{\boldsymbol{r},\mu,\alpha} I_{\Delta\times h_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[i2\pi\boldsymbol{m}\cdot\boldsymbol{h} + ie\sum_\alpha \Delta\times\boldsymbol{h}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

Integrate out $\boldsymbol{A}$.

Vortex-loop formulation

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[ie\sum_\alpha \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{h}) \sum_{\{\boldsymbol{m}\}} \prod_{\boldsymbol{r},\alpha} I_{\Delta\times h_{\boldsymbol{r},\alpha}}(\beta) \prod_{\boldsymbol{r},\alpha} e^{\left[i2\pi \boldsymbol{m}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{h}_{\boldsymbol{r},\alpha} + \frac{e^2}{2}\boldsymbol{h}^2_{\boldsymbol{r},\alpha}\right]}$$

h is a massive (dual) gauge-field mediating interactions between the ***m***'s. Integrate out ***h***.

Vortex-loop formulation

$$\mathcal{Z} = \int \mathcal{D}(\boldsymbol{A}) \sum_{\{\boldsymbol{b}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{b}_{\boldsymbol{r},\alpha},0} \prod_{\boldsymbol{r},\mu,\alpha} I_{b_{\boldsymbol{r},\mu,\alpha}}(\beta) \prod_{\boldsymbol{r}} e^{-\left[ie\sum_\alpha \boldsymbol{b}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{A}_{\boldsymbol{r}} + \frac{\beta}{2}(\boldsymbol{\Delta}\times\boldsymbol{A}_{\boldsymbol{r}})^2\right]}$$

$$\mathcal{Z} = \sum_{\{\boldsymbol{m}\}} \prod_{\boldsymbol{r},\alpha} \delta_{\boldsymbol{\Delta}\cdot\boldsymbol{m}_{\boldsymbol{r},\alpha},0} \quad e^{4\pi^2\beta\sum_{\boldsymbol{r},\boldsymbol{r}'} \boldsymbol{m}_{\boldsymbol{r},\alpha}\cdot\boldsymbol{m}_{\boldsymbol{r}',\alpha} D(\boldsymbol{r}-\boldsymbol{r}')}$$

$$D(q) = \frac{1}{q^2 + e^2\beta}$$

Gauge-field correlations and Higgs mass

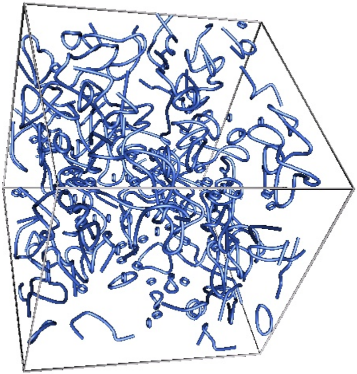
$$\langle A^{\mu}_{\boldsymbol{q}} A^{\nu}_{-\boldsymbol{q}} \rangle \sim \frac{1}{q^2 + m_{\boldsymbol{A}}^2}$$

$$\langle \boldsymbol{A}_{\boldsymbol{q}} \cdot \boldsymbol{A}_{-\boldsymbol{q}} \rangle = \frac{1}{\beta\,|\boldsymbol{q}|^2 + e^2\beta} \left(1 + \frac{4\pi^2 e^{2\beta^2}}{|\boldsymbol{q}|^2\,(|\boldsymbol{q}|^2 + e^2\beta)} \underbrace{\sum_{\alpha\beta} \langle \boldsymbol{m}_{\boldsymbol{q},\alpha} \cdot \boldsymbol{m}_{-\boldsymbol{q},\beta} \rangle}_{\text{Destroys Higgs mass}} \right)$$

$$m_{\boldsymbol{A}}^2 = \lim_{\boldsymbol{q} \to 0} \frac{2}{\beta \langle \boldsymbol{A}_{\boldsymbol{q}} \boldsymbol{A}_{-\boldsymbol{q}} \rangle}$$

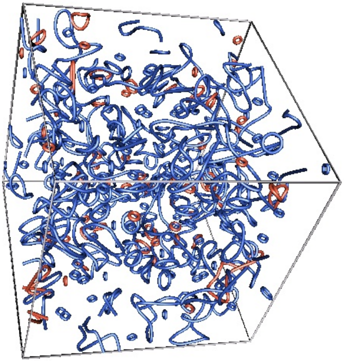
Gauge-field correlations and Higgs mass

Vortex-loop blowout



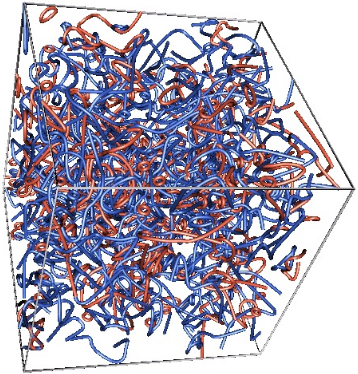
Gauge-field correlations and Higgs mass

Vortex-loop blowout



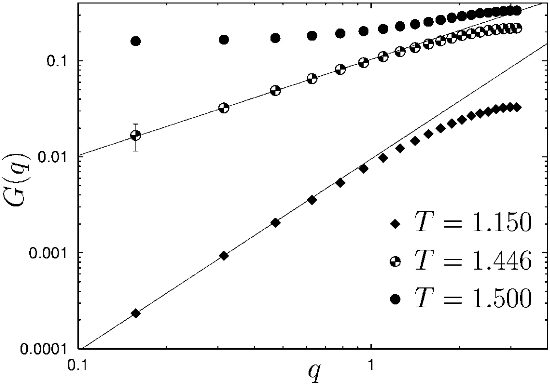
Gauge-field correlations and Higgs mass

Vortex-loop blowout



Gauge-field correlations and Higgs mass

Vortex correlator $G(q) = \langle \boldsymbol{m}_q \cdot \boldsymbol{m}_{-q} \rangle$



J. Hove and A. Sudbø, Phys. Rev. Lett. **84**, 3426 (2000).

Physical interpretation of the Higgs mass

Gauge-field correlator

$$\langle A^{\mu}_{\boldsymbol{q}} A^{\nu}_{-\boldsymbol{q}} \rangle \sim \frac{1}{q^2 + m_{\boldsymbol{A}}^2}$$

$$m_{\boldsymbol{A}} \sim \frac{1}{\lambda}$$

- λ: London penetration length.
- Anderson-Higgs mechanism yields the Meissner-effect, the defining property of a superconductor. Manifestation of a broken *local* (gauge) $U(1)$-invariance.

$3D$ superconductors and superfluids are dual

- Vortices in a three-dimensional superconductor behave like supercurrents in a superfluid, and vice versa, with inverted temperature axes.
- 3D superfluids and superconductors are dual to each other. The field theory of one is the field theory of the topoligical defects (vortices) of the matter-field of the other.
- Order parameter for a superfluid is the superfluid density (non-local) or the condensate fraction (local). OP for a superconductor is the Higgs mass (non-local).
- Higgs mass is created by supercurrent-loop proliferation, and is destroyed by vortex-loop proliferation.