The Anderson-Higgs Mechanism in Superconductors

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References


1 Introduction
   - Basic concepts
   - Cooper-pairing

2 Theoretical framework
   - Superfluids and superconductors
   - Order parameter of SFs and SCs

3 Fluctuation physics and the Higgs mechanism
   - Lattice gauge theory
   - Current loops
   - Vortex loops
Overview

- What is the Higgs mechanism?
- Definition of the superconducting state
- Theories of superconductors
- Higgs Mechanism I: Mean field theory physics
- Higgs Mechanism II: Fluctuation physics
What is the Higgs mechanism?

THE HIGGS BOSON WALKS INTO A CHURCH.
THE PRIEST SAYS WE DON'T ALLOW HIGGS BOSONS IN HERE.

THE HIGGS BOSON SAYS BUT WITHOUT ME HOW CAN YOU HAVE MASS?

- Brian Malow

@sciencecomedian

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The Anderson-Higgs Mechanism in Superconductors
What is the Higgs mechanism?

- The Higgs mechanism is the dynamical generation of a mass of a gauge-boson (mediating interactions) resulting from the coupling to some matter field.

- In superconductors, it is the generation of a mass of the photon.

- Superconductors have radically different electrodynamics compared to non-superconductors.

- The Higgs mass therefore distinguishes a metallic state from a superconducting state.
Defining properties of a superconductor

H. Kammerling-Onnes (1911). Nobel Prize 1913.

But: SC is much more than just perfect conductivity!
Defining properties of a superconductor

SC: Perfect diamagnet! Completely different from a perfect conductor!

W. Meissner, R. Oechsenfeld (1933)
All theories of superconductivity somehow involve the concept of Cooper-pairing: Two electrons experience an \textit{effective} attractive (!) interaction and form pairs.

NB!! No more than pairs! (L. Cooper (1956); J. Bardeen, L. Cooper, R. J. Schrieffer (1957)).
Cooper-pairing

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Introduction
Theoretical framework
Fluctuation physics and the Higgs mechanism

Generalized BCS theory

**Effective BCS Hamiltonian**

\[ H = \sum_{k, \sigma} \varepsilon_k C_{k, \sigma}^\dagger C_{k, \sigma} + \sum_{k, k', \sigma} V_{k, k'} \langle C_{-k', -\sigma} C_{k', \sigma} \rangle \]

- \( H \) is invariant if \( c_{k, \sigma} \rightarrow c_{k, \sigma} e^{i \theta} \). Theory is \( U(1) \)-symmetric. Note: Global \( U(1) \)-invariance!
- \( \Delta_k \neq 0 \rightarrow U(1) \)-symmetry is broken!
- If \( V_{k, k'} = V_{g_k} g_k' \), then \( \Delta_k = \Delta_0 (T) g_k \)
Normal metal in zero electric field

No net motion of electrons
Electrons (almost) do not see each other
Normal metal in electric field

Forced net motion of electrons
Electrons (almost) do not see each other
Superconductors in zero electric field

Unforced net motion of electrons
Electrons states are co-operatively highly organized
Outline

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2 Theoretical framework
   - Superfluids and superconductors
   - Order parameter of SFs and SCs

3 Fluctuation physics and the Higgs mechanism
   - Lattice gauge theory
   - Current loops
   - Vortex loops
Building a theory of Superconductivity

A theory of wave-function of Cooper-pairs

\[ \Psi(r) = |\Psi(r)| e^{i\theta(r)} \]

This theory needs to be able to somehow distinguish (by an order parameter OP) the high-temperature normal metallic state from the low-temperature superconducting state.

An OP is the expectation value of some operator that effectively distinguishes an ordered state from a disordered state, separated by a phase transition.

Two states separated by a phase transition cannot be analytically continued from one to the other.

What is the order parameter of a superconductor?
Conventional phase transitions are well described by the Landau-Ginzburg-Wilson (LGW) paradigm.

\[ F = \frac{1}{2} (\nabla \vec{M})^2 + \frac{r}{2} \vec{M}^2 + \frac{u}{8} (\vec{M}^2)^2 \]

\[ r \propto T - T_c \]

\[ r < 0 \implies \langle \vec{M} \rangle \neq 0 \]
Conventional phase transitions are well described by the Landau-Ginzburg-Wilson (LGW) paradigm.
Superfluids and superconductors

Superconductor

\[ \rho \]

[\[ T \]

Superfluid

\[ \mu \]

[\[ T \]

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Superfluids and superconductors

Superconductor

\[ \rho \]

Zero resistivity

\[ T_c \quad T \]

Superfluid

\[ \mu \]

Zero viscosity

\[ T_c \quad T \]

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Superfluids and superconductors

Phase transition

Superconductor

Superfluid

$\rho$

$\mu$

$T_c$

$T$

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Superfluids and superconductors

Order parameter of SFs and SCs
Superfluids and superconductors

Superconductor

Superfluid

Meissner effect
Superfluids and superconductors

Type I superconductor

- \( B \)
- \( T < T_c \)

- Meissner effect

Type II superconductor

- \( B \)
- \( T > T_c \)

- Mixed phase

- \( T < T_c \)
- \( T > T_c \)
Vortices suppress $\Psi$
Vortices suppress $\Psi$
Vortices suppress \( \psi \)
Effective models

- Construct effective models
- Complex matter field:
  \[ \psi(r) = |\psi(r)|e^{i\theta(r)} \]
- Neglect microscopic details
- Valid near the phase transition
Effective models

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Ginzburg-Landau model for superconductors

\[
H[\psi, A] = \int d^d r \left\{ \frac{g}{2} |[\nabla - ieA(r)]\psi(r)|^2 + \frac{\alpha t}{2} |\psi(r)|^2 + \frac{u}{4!} |\psi(r)|^4 ight. \\
+ \left. \frac{1}{2} [\nabla \times A(r)]^2 \right\}
\]
Effective models

- Construct effective models
- Complex matter field:
  \[ \psi(r) = |\psi(r)| e^{i\theta(r)} \]
- Neglect microscopic details
- Valid near the phase transition

Gross-Pitaevskii model for superfluids

\[
H[\psi] = \int d^d r \left[ \frac{1}{2m} |\nabla \psi(r)|^2 + V(r) |\psi(r)|^2 + \frac{U_0}{2} |\psi(r)|^4 \right]
\]
Gross-Pitaevskii model for superfluids

\[ H[\psi] = \int d^d r \left[ \frac{1}{2m} |\nabla \psi(r)|^2 + V(r) |\psi(r)|^2 + \frac{U_0}{2} |\psi(r)|^4 \right] \]

\[ Z = e^{-\beta F} = \int D\psi D\psi^* e^{-\beta H} \]

\[ \langle O \rangle = \frac{1}{Z} \int D\psi D\psi^* O(\psi, \psi^*) e^{-\beta H} \]

Candidate OPs

\[ |\langle \psi(r) \rangle|^2 = |\langle |\psi(r)| e^{i\theta(r)} \rangle|^2 \approx |\psi|^2 |\langle e^{i\theta(r)} \rangle|^2 \quad (Local) \]

\[ \gamma = \frac{\partial^2 F}{\partial \Delta \theta^2} \quad (Non-local) \]
Order parameter of a superfluid

\[ V(|\Psi|) = \alpha \left( \frac{T - T_c}{2T_c} \right) |\Psi|^2 + \frac{U}{4!} |\Psi|^4 \]

For the Higgs-mechanism to work at all, we need \( \langle |\Psi| \rangle \neq 0 \)!
Order parameter of a superfluid

\[
\langle |\psi'|^2 \rangle
\]

\[
\gamma_z^2
\]

\[
\langle |\psi|^2 \rangle
\]

Order parameter of a superconductor

Ginzburg-Landau model for superconductors

\[ H[\Psi, A] = \int d^d r \left\{ \frac{g}{2} \left| \nabla - ieA(r) \right| \Psi(r) \right|^2 + \frac{\alpha t}{2} |\Psi(r)|^2 + \frac{u}{4!} |\Psi(r)|^4 + \frac{1}{2} \left| \nabla \times A(r) \right|^2 \right\} \]

\[ \langle O \rangle = \frac{1}{Z} \int D\Psi D\Psi^* \ O(\Psi, \Psi^*) \ e^{-\beta H} \]

\[ |\langle \Psi(r) \rangle|^2 = |\langle |\Psi(r)| e^{i\theta(r)} \rangle|^2 \approx |\Psi|^2 |\langle e^{i\theta(r)} \rangle|^2 \quad (Local) \]

\[ \gamma = \frac{\partial^2 F}{\partial \Delta \theta^2} \quad (Non-local) \]
This will not work!
\[ \Psi(r) = |\Psi(r)| e^{i\theta(r)} \approx |\Psi_0| e^{i\theta(r)} \]

\[ H[\Psi, A] = \int d^d r \left\{ \frac{g |\Psi_0|^2}{2} (\nabla \theta - eA(r))^2 + \frac{1}{2} [\nabla \times A(r)]^2 \right\} \]

|\langle \Psi(r) \rangle|^2 = 0!
\[ \Upsilon = 0! \]

- Elitzur’s theorem: The expectation value of a gauge-invariant local operator can never acquire a non-zero expectation value in a gauge-theory.
- In a gauge-theory, the Goldstone modes are "eaten up" by the gauge-field \( \rightarrow \Upsilon = 0 \).
Order parameter of a superconductor

\[
H[\Psi, A] = \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} (\nabla \theta - eA(r))^2 + \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} \left( (\nabla \theta)^2 - 2e\nabla \theta \cdot A(r) + e^2 A^2 \right) \right. \\
+ \left. \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
= \int d^d r \left\{ \frac{g|\Psi_0|^2}{2} \left( (\nabla \theta)^2 - \frac{j \cdot A}{g|\Psi_0|^2} \right) \right. \\
+ \left. \frac{g|\Psi_0|^2 e^2}{2} A^2 + \frac{1}{2}[\nabla \times A(r)]^2 \right\} \\
\equiv m^2
\]
Consider in more detail the gauge-part of the theory (introducing Fourier-transformed fields)

\[ H_A = m^2 A_q A_{-q} + \frac{1}{2} q^2 A_q A_{-q} - \frac{1}{2} (S_q A_{-q} + S_{-q} A_q) \]

\[ = (m^2 + \frac{1}{2} q^2) A_q A_{-q} - \frac{1}{2} (S_q A_{-q} + j_{-q} A_q) \]

\[ \equiv D_0(q) \]

\[ = D_0(q) \left\{ \left( A_q - \frac{1}{2} D_0^{-1}(q) S_q \right) \cdot \left( A_{-q} - \frac{1}{2} D_0^{-1}(q) S_{-q} \right) \right\} \]

\[ - \frac{1}{4} D_0^{-1} S_q \cdot S_{-q} \]

\[ S_q \equiv g|\Psi_0|^2 e^2 \frac{\mathcal{F}(\nabla \theta)}{2} \]
Integrate out the shifted gauge fields $\tilde{A}_q = A_q - \frac{1}{2} D_0^{-1}(q) S_{-q}$:

$$H_{\text{eff}}(\Psi) = \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} S_q \cdot S_{-q}$$

$$m^2 \equiv \frac{g|\Psi_0|^2 e^2}{2}$$

Define a momentum-dependent phase-stiffness $\gamma_q$:

$$\gamma_q = \frac{d^2 H_{\text{eff}}}{dS_q dS_{-q}} = \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2}$$

Interested in long-distance physics $q \to 0$. 
Order parameter of a superconductor

\[
\lim_{q \to 0} \gamma_q = \lim_{q \to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} = \frac{g|\Psi_0|^2}{2}; \quad e = 0
\]

\[
\lim_{q \to 0} \gamma_q = \lim_{q \to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} = 0; \quad e \neq 0
\]
Order parameter of a superconductor

\[ \lim_{q \to 0} \gamma_q = \lim_{q \to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} = \frac{g|\Psi_0|^2}{2} ; e = 0 \]

\[ \lim_{q \to 0} \gamma_q = \lim_{q \to 0} \frac{m^2}{e^2} \frac{q^2}{2m^2 + q^2} = 0 ; e \neq 0 \]

The superfluid density is NOT an order parameter for a superconductor!

It IS an order parameter for a superfluid!
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Ginzburg-Landau lattice model

\[ Z = \int \mathcal{D}A \left( \prod_{\alpha} \int \mathcal{D}\psi_{\alpha} \right) e^{-S} \]

where the action is

\[ S = \beta \int d^3 r \left\{ \frac{1}{2} \sum_{\alpha} (|\nabla - ieA(r)|) \psi_{\alpha}(r) |^2 \right. \\
+ V (|\psi_{\alpha}(r)|) + \frac{1}{2} (\nabla \times A(r))^2 \left. \right\} \]
Ginzburg-Landau lattice model

London-approximation

$$\Psi(r) \approx |\Psi_0| e^{i\theta(r)}$$

$$S = \beta \sum_r \left\{ - \sum_{\mu,\alpha} \cos (\Delta_\mu \theta_{r,\alpha} - eA_{r,\mu}) + \frac{1}{2} (\Delta \times A_r)^2 \right\}$$
Current-loop formulation

\[ e^\beta \cos \gamma = \sum_{b=-\infty}^{\infty} l_b(\beta) e^{ib\gamma} \]

\[ Z = \int \mathcal{D}A \left( \prod_{\alpha} \int \mathcal{D}\theta_{\alpha} \right) \]
\[ \times \prod_{r,\mu,\alpha} \sum_{b_{r,\mu,\alpha}=-\infty}^{\infty} l_{b_{r,\mu,\alpha}}(\beta) e^{ib_{r,\mu,\alpha}(\Delta_\mu \theta_{r,\alpha} - eA_{r,\mu})} \]
\[ \times \prod_{r} e^{-\frac{\beta}{2}(\Delta \times A_{r})^2} \]

Current-loop formulation

\[
\mathcal{Z}_\theta = \prod_{r,\alpha} \int_0^{2\pi} d\theta_{r,\alpha} e^{-i\theta_{r,\alpha} \left( \sum_\mu \Delta_\mu b_{r,\mu,\alpha} \right)}
\]

\(\theta\)-integration gives:

\[
\Delta \cdot b_{r,\alpha} = 0 \ \forall \ r, \alpha
\]

The partition function is then given by

\[
\mathcal{Z}_J = \int \mathcal{D}(A) \sum_{\{b\}} \prod_{r,\alpha} \delta \Delta \cdot b_{r,\alpha,0} \prod_{r,\mu,\alpha} I_{b_{r,\mu,\alpha}}(\beta)
\]

\[\equiv S_J\]

\[
\prod_r e^{-\left[ ie \sum_\alpha b_{r,\alpha} \cdot A_r + J \cdot A + \frac{\beta}{2} (\Delta \times A_r)^2 \right]}
\]
Integrating out the $A$-field gives

$$Z = \sum \prod_{r,\alpha} \delta_{\Delta \cdot b_r,\alpha} \prod_{r,\mu,\alpha} I_{b_r,\alpha,\mu}(\beta) \sim e^{-(b_r,\alpha,\mu)^2/2\beta}$$

$$-\frac{e^2}{2\beta} \sum_{\alpha,\beta} b_{r,\alpha} \cdot b_{r',\beta} D(r - r') \prod_{r,r'} e^{1/|r-r'|}$$

This is a theory of supercurrents $b_{r,\alpha}$, forming closed loops, interacting via the potential $D(r - r')$ mediated by the gauge-field $A$, as well as contact interaction.

Gauge-field correlations and Higgs mass

\[ \langle A_{\mu} A_{\nu}^\parallel \rangle = \frac{1}{\mathcal{Z}_0} \frac{\delta^2 \mathcal{Z}_J}{\delta J_{-q,\mu} \delta J_{q,\nu}} \bigg|_{J=0} \]

\[ = \frac{1}{\mathcal{Z}_0} \sum_{\{b,m\}} \prod_{r,\alpha} \delta \Delta b_{r,\alpha,0} \prod_{r,\mu,\alpha} l_{br,\alpha,\mu}(\beta) \]

\[ \times \left( -\frac{\delta^2 S_J}{\delta J_{-q,\mu} \delta J_{q,\nu}} - \frac{\delta S_J}{\delta J_{-q,\mu}} \frac{\delta S_J}{\delta J_{q,\nu}} \right) e^{-S_J} \bigg|_{J=0} \]
Gauge-field correlations and Higgs mass

\[ \langle A_q \cdot A_{-q} \rangle \sim \frac{1}{q^2 + m_A^2} \]

Excitations \rightarrow Higgs mass

\[ \langle A_q \cdot A_{-q} \rangle = \frac{1}{\beta |q|^2} \left( 2 - \frac{e^2}{\beta |q|^2} \sum_{\alpha \beta} \langle b_{q,\alpha} \cdot b_{-q,\beta} \rangle \right) \]

\[ m_A^2 = \lim_{q \to 0} \frac{2}{\beta \langle A_q A_{-q} \rangle} \]
Gauge-field correlations and Higgs mass

\[
\lim_{q \to 0} \frac{e^2}{2\beta} \langle B_q \cdot B_{-q} \rangle \sim \begin{cases} 
(1 - C_2(T))q^2, & T > T_C. \\
q^2 - C_3(T)q^{2+n_A}, & T = T_C. \\
q^2 - C_4(T)q^4, & T < T_C.
\end{cases}
\]

\[
m_A^2 \sim \begin{cases} 
0, & T \geq T_C. \\
\frac{1}{C_4(T)}, & T < T_C.
\end{cases}
\]

Fluctuating supercurrents \( B_q \equiv \sum_\alpha b_{q,\alpha} \) create a Higgs mass!
Go back to vortices
Vortex-loop formulation

\[ Z = \int D(\mathbf{A}) \sum_{\{\mathbf{b}\}} \prod_{r,\alpha} \delta \Delta \cdot \mathbf{b}_{r,\alpha,0} \prod_{r,\mu,\alpha} l_{br,\mu,\alpha}(\beta) \prod_{r} e^{-i e \sum_{\alpha} \mathbf{b}_{r,\alpha} \cdot \mathbf{A}_{r} + \frac{\beta}{2} (\Delta \times \mathbf{A}_{r})^2} \]

Solve constraint

\[ \Delta \cdot \mathbf{b}_{r,\alpha} = 0 \ \forall \ r, \alpha \]

\[ \mathbf{b}_{r,\alpha} = \Delta \times \mathbf{K}_{r,\alpha} \]
Vortex-loop formulation

\[ Z = \int \mathcal{D}(A) \sum_{\{b\}} \prod_{r,\alpha} \delta_{\Delta \cdot b_r,\alpha,0} \prod_{r,\mu,\alpha} l_{br,\mu,\alpha}(\beta) \prod_{r} e^{-\left[i e \sum_{\alpha} b_{r,\alpha} \cdot A_r + \frac{\beta}{2} (\Delta \times A_r)^2\right]} \]

Use Poisson summation formula

\[ \sum_{n=-\infty}^{\infty} e^{i 2\pi n \cdot \Delta \times h} = \sum_{K=-\infty}^{\infty} \delta(\Delta \times h - \Delta \times K) \]

\[ \sum_{m=-\infty}^{\infty} e^{i 2\pi m \cdot h} = \sum_{K=-\infty}^{\infty} \delta(h - K) \]

\[ m = \Delta \times n \quad (\Delta \cdot m = 0!) \]
Vortex-loop formulation

\[ Z = \int \mathcal{D}(A) \sum_{\{b\}} \prod_{r,\alpha} \delta \Delta \cdot b_{r,\alpha,0} \prod_{r,\mu,\alpha} l_{b_{r,\mu,\alpha}}(\beta) \]
\[ \prod_{r} e^{-\left[ie \sum_{\alpha} b_{r,\alpha} \cdot A_{r} + \frac{\beta}{2} (\Delta \times A_{r})^2\right]} \]

\[ Z = \int \mathcal{D}(A) \int \mathcal{D}(h) \sum_{\{m\}} \prod_{r,\mu,\alpha} l_{\Delta \times h_{r,\mu,\alpha}}(\beta) \]
\[ \prod_{r} e^{-\left[i2\pi m \cdot h + ie \sum_{\alpha} \Delta \times h_{r,\alpha} \cdot A_{r} + \frac{\beta}{2} (\Delta \times A_{r})^2\right]} \]

Integrate out \( A \).
Vortex-loop formulation

$$Z = \int \mathcal{D}(A) \sum_{\{b\}} \prod_{r, \alpha} \delta(\Delta \cdot b_r, \alpha, 0) \prod_{r, \mu, \alpha} l_{br, \mu, \alpha}(\beta) \prod_{r} e^{-i\varepsilon \sum_{\alpha} b_r, \alpha \cdot A_r + \beta \frac{1}{2} (\Delta \times A_r)^2}$$

$$Z = \int \mathcal{D}(h) \sum_{\{m\}} \prod_{r, \alpha} l_{\Delta \times h_r, \alpha}(\beta) \prod_{r, \alpha} e^{i2\pi m_r, \alpha \cdot h_r, \alpha + \frac{e^2}{2} h_r^2, \alpha}$$

$h$ is a massive (dual) gauge-field mediating interactions between the $m$'s. Integrate out $h$. 

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The Anderson-Higgs Mechanism in Superconductors
Vortex-loop formulation

\[ Z = \int D(A) \sum_{\{b\}} \prod_{r,\alpha} \delta_{\Delta \cdot b_{r,\alpha},0} \prod_{r,\mu,\alpha} l_{br,\mu,\alpha}(\beta) \]

\[ \prod_r e^{-[ie \sum_\alpha b_{r,\alpha} \cdot A_r + \frac{\beta}{2}(\Delta \times A_r)^2]} \]

\[ Z = \sum_{\{m\}} \prod_{r,\alpha} \delta_{\Delta \cdot m_{r,\alpha},0} \theta^{4\pi^2\beta} \sum_{r,r'} m_{r,\alpha} \cdot m_{r',\alpha} D(r-r') \]

\[ D(q) = \frac{1}{q^2 + e^2\beta} \]
Gauge-field correlations and Higgs mass

\[ \langle A_q^\mu A_{-q}^\nu \rangle \sim \frac{1}{q^2 + m_A^2} \]

\[ \langle A_q \cdot A_{-q} \rangle = \frac{1}{\beta |q|^2 + e^2 \beta} \left( 1 + \frac{4\pi^2 e^2 \beta^2}{|q|^2 (|q|^2 + e^2 \beta)} \right) \sum_{\alpha\beta} \langle m_q,\alpha \cdot m_{-q},\beta \rangle \]

\[ m_A^2 = \lim_{q \to 0} \frac{2}{\beta \langle A_q A_{-q} \rangle} \]
Gauge-field correlations and Higgs mass

Vortex-loop blowout
Gauge-field correlations and Higgs mass

Vortex-loop blowout
Gauge-field correlations and Higgs mass

Vortex-loop blowout
Gauge-field correlations and Higgs mass

Vortex correlator $G(q) = \langle m_q \cdot m_{-q} \rangle$

Gauge-field correlator

\[ \langle A_q^\mu A_{-q}^\nu \rangle \sim \frac{1}{q^2 + m_A^2} \]

\[ m_A \sim \frac{1}{\lambda} \]

- \( \lambda \): London penetration length.
- Anderson-Higgs mechanism yields the Meissner-effect, the defining property of a superconductor. Manifestation of a broken local (gauge) \( U(1) \)-invariance.
3D superconductors and superfluids are dual

- Vortices in a three-dimensional superconductor behave like supercurrents in a superfluid, and vice versa, with inverted temperature axes.

- 3D superfluids and superconductors are dual to each other. The field theory of one is the field theory of the topological defects (vortices) of the matter-field of the other.

- Order parameter for a superfluid is the superfluid density (non-local) or the condensate fraction (local). OP for a superconductor is the Higgs mass (non-local).

- Higgs mass is created by supercurrent-loop proliferation, and is destroyed by vortex-loop proliferation.