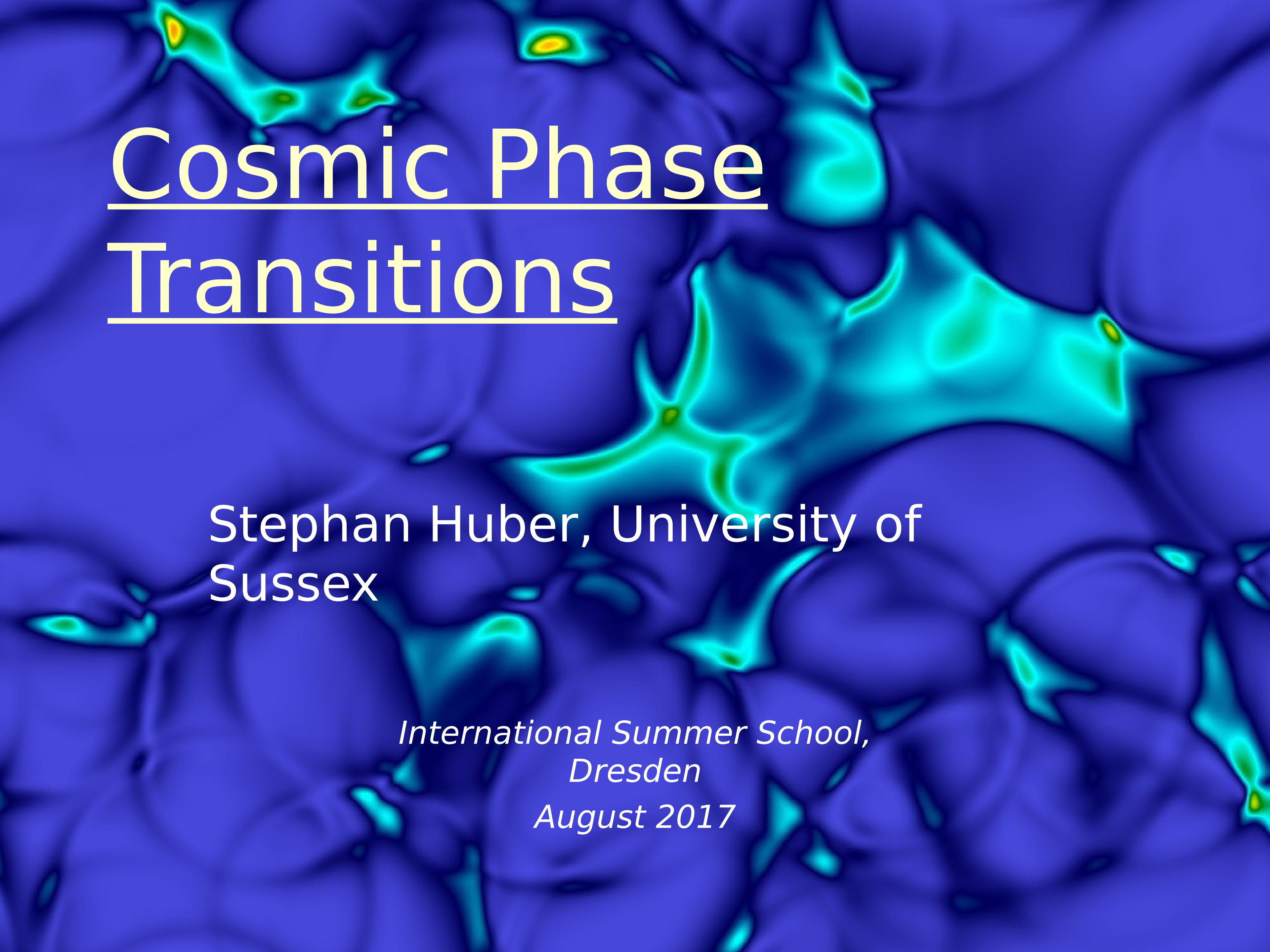

Cosmic Phase Transitions

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International Summer School, Dresden

August 2017

Overview:

Brief history of the universe and the history of that

Some theory basics for phase transitions in particle physics

Electroweak baryogenesis

Example: Two Higgs doublet model

Dynamics of a phase transition

Gravitational waves from phase transitions

Does the universe evolve?

From Balkonsternwarte.at

A few milestones:

1915: General Relativity (GR), theory for a dynamic spacetime (A. Einstein)

1922: GR solutions for an expanding universe (A. Friedmann)

1927 (Lamaitre), 1929 (Hubble): distance-velocity relation

"Hubble law"

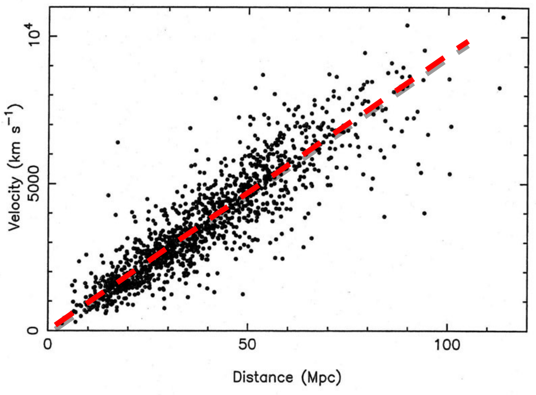
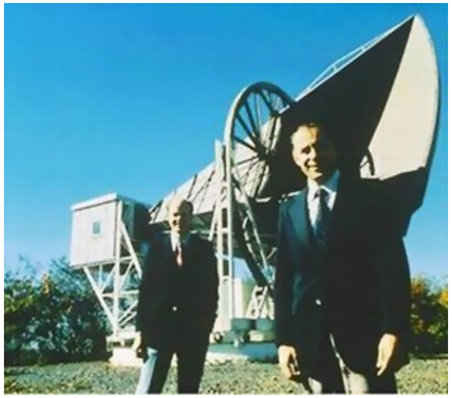


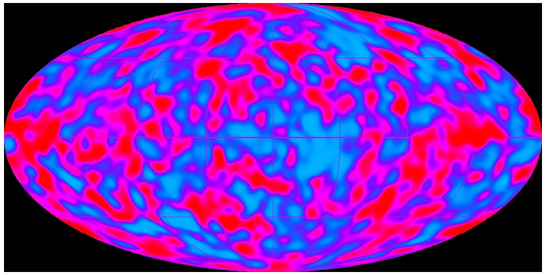
Figure 2.5 A plot of velocity versus estimated distance for a set of 1355 galaxies. A straight-line relation implies Hubble's law. The considerable scatter is due to observational uncertainties and random galaxy motions, but the best-fit line accurately gives Hubble's law. [The x-axis scale assumes a particular value of H_0.]

1964: Penzias and Wilson discover the cosmic microwave background radiation (CMB), corresponding to a black body radiation of temperature 2.7 K



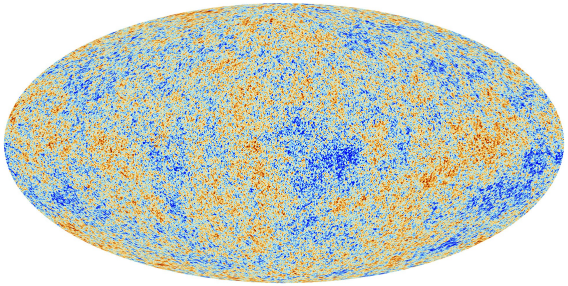
Satellite missions to study the CMB:

1989-1993: COBE discovers anisotropies



2001-2010: WMAP

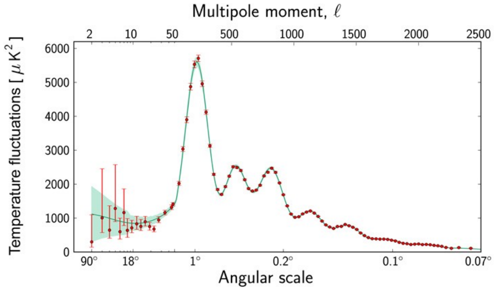
2009-2013: Planck



These maps encode valuable information on the early universe, eg. the

Baryon to photon ratio:

$$\eta_B = \frac{n_B}{n_\gamma} = (6.047 \pm 0.074) \times 10^{-10}$$

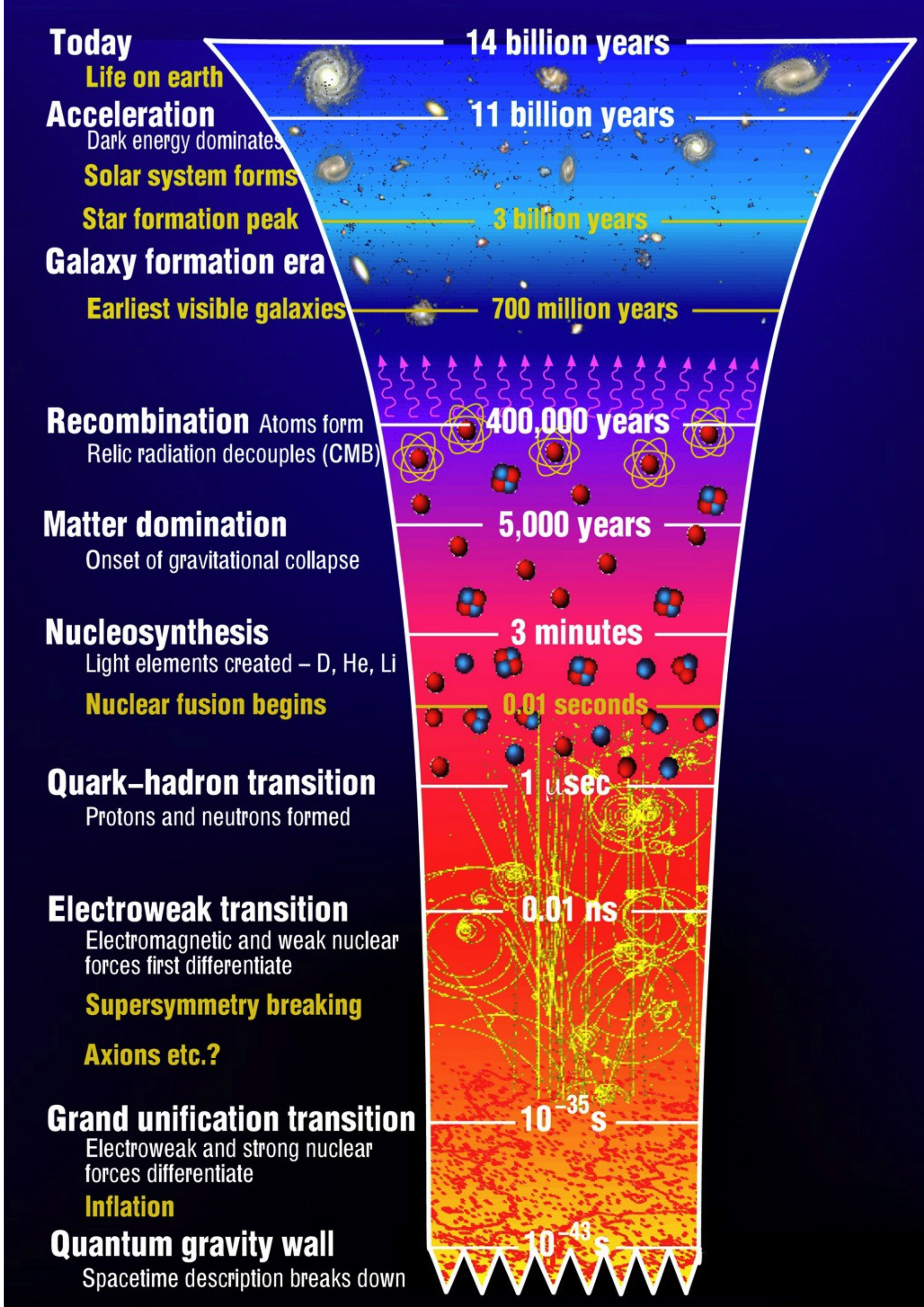


Summary: the universe is expanding and had a hot and dense beginning: “Big Bang”

Thermal history of The universe:

(No direct observational evidence Before Nucleosynthesis)

Source: CTC Cambridge



Mathematical foundations

Particle physics:

Theoretical framework: quantum field theory

In particular nonabelian gauge theories based on a

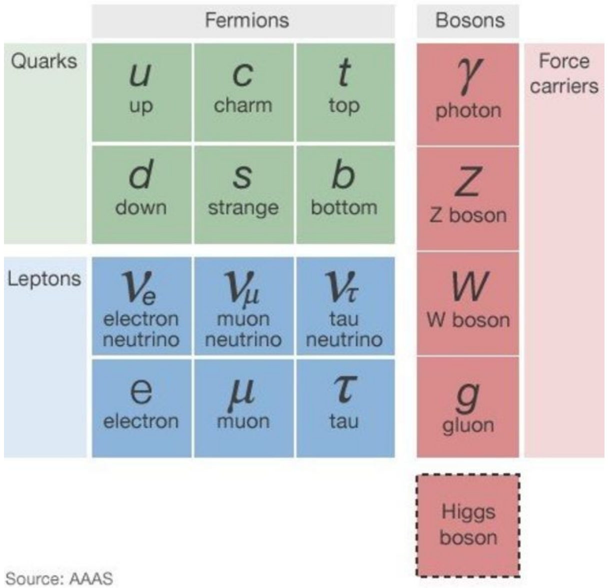
gauge group: in the standard model (SM) SU(3) x SU(2) x U(1)

describing

Gauge bosons: in the SM the gluons, W's, Z and the photon

Fermions: in the SM the quarks and leptons

Scalar fields: in the SM the Higgs



General form of the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{\mu\nu\,(a)}$$
Gauge field kinetic term

$$+ \bar{\psi}_I (i\gamma^\mu D_\mu - m)\psi_I$$
Dirac Lagrangian for fermions

$$+ (D_\mu \phi_A)^+ (D^\mu \phi_A)$$
Kinetic terms for scalars

$$- V(\phi_A)$$
Scalar potential

$$- y_{ABC} \bar{\psi}_A \psi_B \phi_C$$
Yukawa interactions

$$D_\mu = \partial_\mu - ig T^a A^a_\mu$$
Covariant derivative

$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu \ldots$$
Field strength tensor

Symmetries

Such a system typically has various symmetries: (in addition to the Lorentz symmetry)

Gauge symmetry:

$$\phi^\alpha \to U^{\alpha\beta} \phi^\beta$$

$U \in G$ (gauge group)

$\alpha = 1, \ldots, \dim(\text{repr.})$

Global symmetries, eg.

$$\phi_I \to V_{IJ} \phi_J$$

$V \in H$

I: flavor index

Discrete symmetries, eg.

$$\phi \to -\phi \quad (\mathbb{Z}_2)$$

Spontaneous symmetry breaking:

Crucial: the scalar potential
Its generic form for one complex field is

$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$$

The lowest energy configuration of the system is a constant scalar field (all other fields zero)

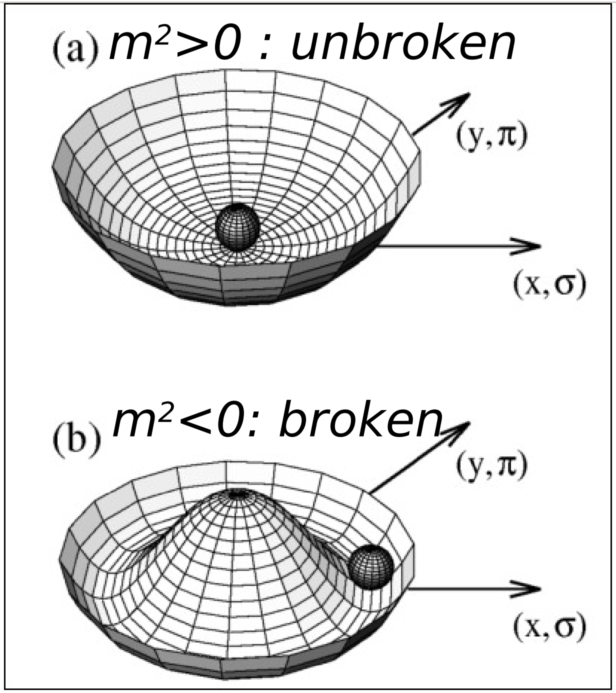
Φ=v

$$|v|^2 = -\frac{m^2}{2\lambda}$$

where *V(v)* is the minimum of the potential

for *v* different from zero, the minimum is not invariant under gauge transformations:

$$v \longrightarrow U v \neq v$$



V. Koch, *Introduction to Chiral Symmetry* ,
ort 38000, 1995

This process also generates masses for gauge bosons and fermions

$$y \bar{\psi} \psi \langle\phi\rangle \rightarrow m_\psi = y \langle\phi\rangle$$

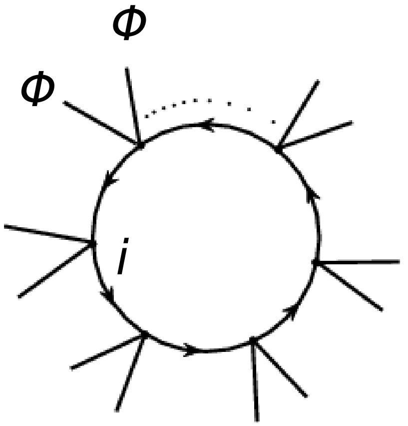
$$g^2 A_\mu A^\mu |\langle\phi\rangle|^2 \rightarrow m^2_A = g^2 |\langle\phi\rangle|^2$$

Quantum corrections to the scalar potential:

Interactions with other particles modify the scalar potential

Often this can be computed in perturbation theory, organized
In a loop expansion.

$$V^{(1)}(\phi) = \pm \sum_i \frac{k_i}{64\pi^2} m_i^4(\phi) \left[\ln\left(\frac{m_i^2(\phi)}{Q^2} \right) - (const) \right]$$



+(-) is bosons (fermions)

k_i: number of degrees of freedom eg k_t=3x2x2=12

$m_i(\Phi)$ are the scalar field dependent masses

Eg. $m_t = y\Phi$, $m_W = (1/2)g\Phi$

Q: renormalization scale *(Q ~ v)*

Total potential:

$$V_{tot}(\phi) = V_{tree}(\phi) + V^{(1)}(\phi) + \ldots$$

Effective Higgs potential of the Standard model:

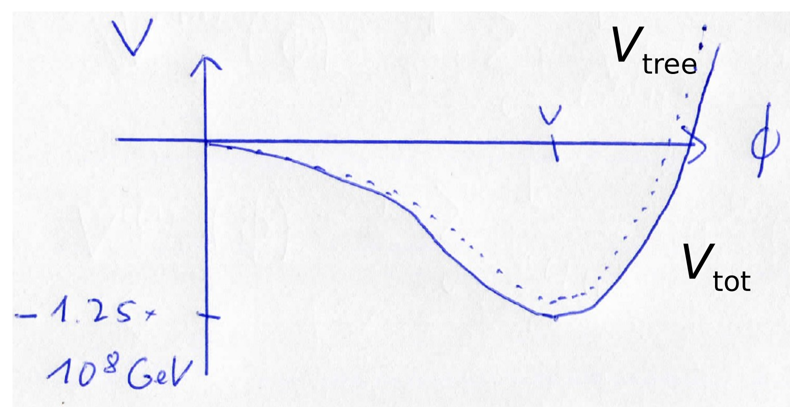
Relevant are the particles which couple strongly to the Higgs, ie. the heavy ones:

top, W-bosons, Z-boson

$$V_{SM}(\phi) = m^2|\phi|^2 + \lambda|\phi|^4 + \frac{2\cdot 3}{64\pi^2} m_W^4(\phi)\left[\ln\frac{m_W^2(\phi)}{Q^2} - \mathrm{const}\right] + \frac{1\cdot 3}{64\pi^2} m_Z^4(\phi)\left[\ln\frac{m_Z^2(\phi)}{Q^2} - \mathrm{const}\right] - \frac{4\cdot 2\cdot 2}{64\pi^2} m_t^4(\phi)\left[\ln m_t^2/Q^2 - \mathrm{const}\right] + \ldots$$

Measured: Higgs vev v=246 GeV

Higgs mass m_h=125 GeV



Depth of the Higgs potential is unknown,
But predicted by the SM
Unknown further contributions:

$$V_{tree}^{(SM)} = m^2|\phi|^2 + \lambda|\phi|^4 + \frac{1}{M^2}|\phi|^6 + $$

Metastability of the SM Higgs potential:

The top quark log will destabilize the previous potential at a Higgs field value of a couple of TeV

This is just a computational artefact; the large logs mess up perturbation theory

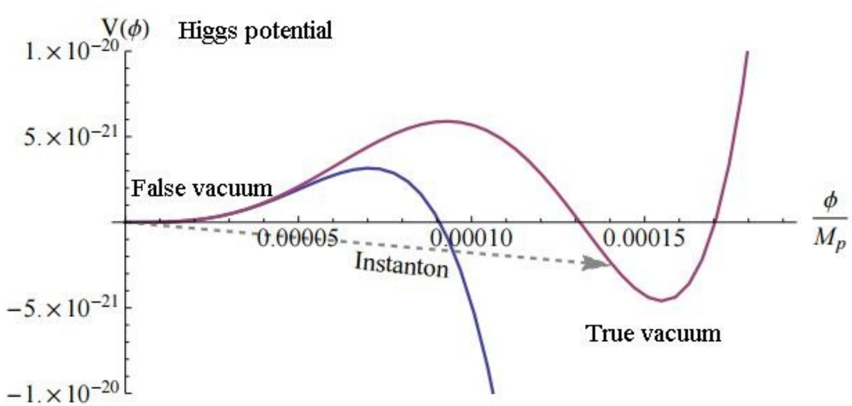
ie. these need to be resummed, equivalently “define” the theory at a much larger scale:

Scale dependence of couplings in a quantum field theory are governed by

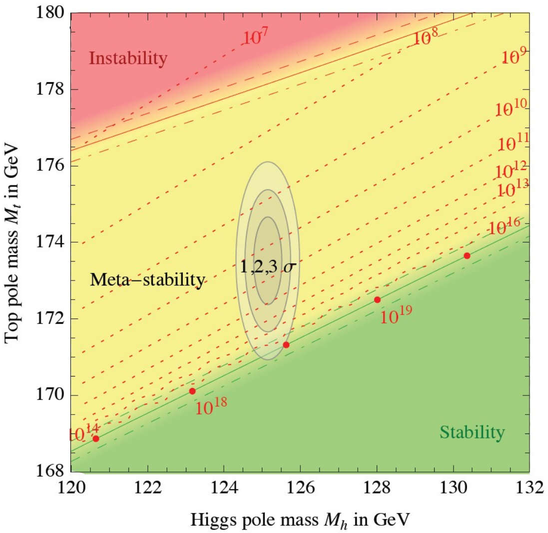
Renormalization group equations:

RGE for the Higgs quartic coupling:

$$\frac{d\lambda}{d\ln\bar{\mu}^2} = \frac{1}{(4\pi)^2}\left[\lambda\left(12\lambda + 6y_t^2 + 6y_b^2 + 2y_\tau^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 - 3y_b^4 - y_\tau^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2g_1^2}{40}\right]$$



Source: https://mappingignorance.org

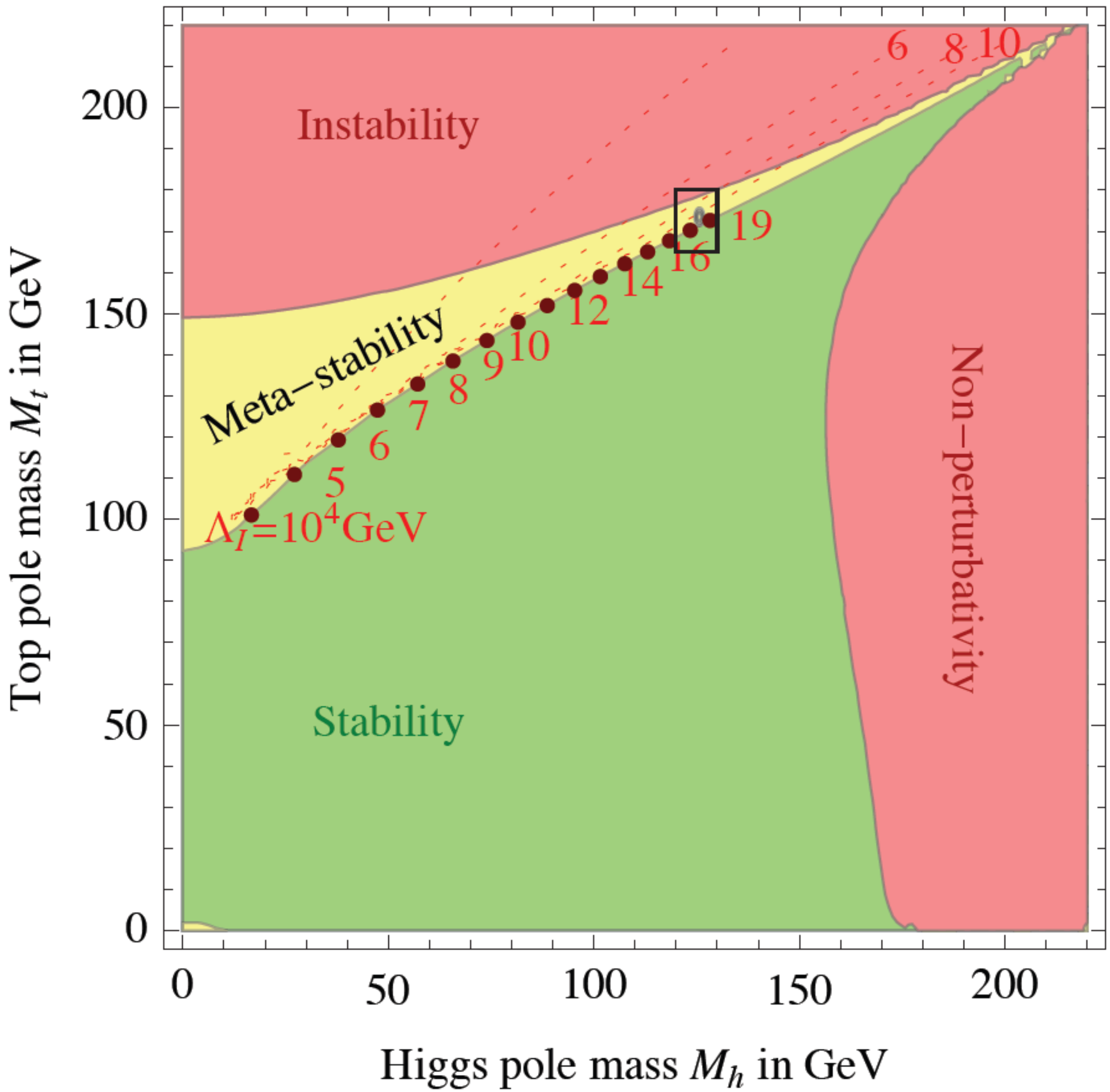


arXiv: 1307.3536 (Buttazzo et al.)

3-loop RGE equations, see arXiv: 1307.3536 (Buttazzo et al.)

$$
\begin{aligned}
\frac{d\lambda}{d\ln\bar{\mu}^2} = {} & \frac{1}{(4\pi)^2}\left[\lambda\left(12\lambda+6y_t^2+6y_b^2+2y_\tau^2-\frac{9g_2^2}{2}-\frac{9g_1^2}{10}\right)-3y_t^4-3y_b^4-y_\tau^4+\frac{9g_2^4}{16}+\frac{27g_1^4}{400}+\frac{9g_2^2g_1^2}{40}\right]+ \\
& +\frac{1}{(4\pi)^4}\left[\lambda^2\left(-156\lambda-72y_t^2-72y_b^2-24y_\tau^2+54g_2^2+\frac{54g_1^2}{5}\right)+\lambda y_t^2\left(-\frac{3y_t^2}{2}-21y_b^2+40g_3^2+\right.\right. \\
& \left.+\frac{45g_2^2}{4}+\frac{17g_1^2}{4}\right)+\lambda y_b^2\left(-\frac{3y_b^2}{2}+40g_3^2+\frac{45g_2^2}{4}+\frac{5g_1^2}{4}\right)+\lambda y_\tau^2\left(-\frac{y_\tau^2}{2}+\frac{15g_2^2}{4}+\frac{15g_1^2}{4}\right)+ \\
& \lambda\left(-\frac{73g_2^4}{16}+\frac{1887g_1^4}{400}+\frac{117g_2^2g_1^2}{40}\right)+y_t^4\left(15y_t^2-3y_b^2-16g_3^2-\frac{4g_1^2}{5}\right)+ \\
& +y_t^2\left(-\frac{9g_2^4}{8}-\frac{171g_1^4}{200}+\frac{63g_2^2g_1^2}{20}\right)+y_b^4\left(-3y_t^2+15y_b^2-16g_3^2+\frac{2g_1^2}{5}\right)+ \\
& +y_b^2\left(-\frac{9g_2^4}{8}+\frac{9g_1^4}{40}+\frac{27g_2^2g_1^2}{20}\right)+y_\tau^4\left(5y_\tau^2-\frac{6g_1^2}{5}\right)+y_\tau^2\left(-\frac{3g_2^4}{8}-\frac{9g_1^4}{8}+\frac{33g_2^2g_1^2}{20}\right)+ \\
& \left.+\frac{305g_2^6}{32}-\frac{3411g_1^6}{4000}-\frac{289g_2^4g_1^2}{160}-\frac{1677g_2^2g_1^4}{800}\right]+ \\
& +\frac{1}{(4\pi)^6}\left[\lambda^3\left(6011.35\lambda+873y_t^2-387.452g_2^2-77.490g_1^2\right)+\lambda^2y_t^2\left(1768.26y_t^2+160.77g_3^2+\right.\right. \\
& \left.-359.539g_2^2-63.869g_1^2\right)+\lambda^2\left(-790.28g_2^4-185.532g_1^4-316.64g_2^2g_1^2\right)+\lambda y_t^4\left(-223.382y_t^2+\right. \\
& \left.-662.866g_3^2-5.470g_2^2-21.015g_1^2\right)+\lambda y_t^2\left(356.968g_3^4-319.664g_2^4-74.8599g_1^4+15.1443g_3^2g_2^2+\right. \\
& \left.+17.454g_3^2g_1^2+5.615g_2^2g_1^2\right)+\lambda g_2^4\left(-57.144g_3^2+865.483g_2^2+79.638g_1^2\right)+\lambda g_1^4\left(-8.381g_3^2+\right. \\
& \left.+61.753g_2^2+28.168g_1^2\right)+y_t^6\left(-243.149y_t^2+250.494g_3^2+74.138g_2^2+33.930g_1^2\right)+ \\
& +y_t^4\left(-50.201g_3^4+15.884g_2^4+15.948g_1^4+13.349g_3^2g_2^2+17.570g_3^2g_1^2-70.356g_2^2g_1^2\right)+ \\
& +y_t^2g_3^2\left(16.464g_2^4+1.016g_1^4+11.386g_2^2g_1^2\right)+y_t^2g_2^4\left(62.500g_2^2+13.041g_1^2\right)+ \\
& +y_t^2g_1^4\left(10.627g_2^2+11.117g_1^2\right)+g_3^2\left(7.536g_2^6+0.663g_1^6+1.507g_2^4g_1^2+1.105g_2^2g_1^4\right)+ \\
& \left.-114.091g_2^8-1.508g_1^8-37.889g_2^6g_1^2+6.500g_2^4g_1^4-1.543g_2^2g_1^6\right]. \qquad (99)
\end{aligned}
$$

Full parameter space:



Finite temperature:

So far we were looking at a system in vacuum

But the early universe is hot, ie. is filled with a plasma at finite temperature

There is a full treatment in quantum field theory of systems in or out of equilibrium:
Close-time-path or Schwinger-Keldysh formalism

Often it is sufficient to consider systems in a semi-classical approximation, where we can encode the information on the system in terms of

scalar fields Φ_i
and particle distribution functions $f_i(p,x)$ (encodes the plasma or fluid)

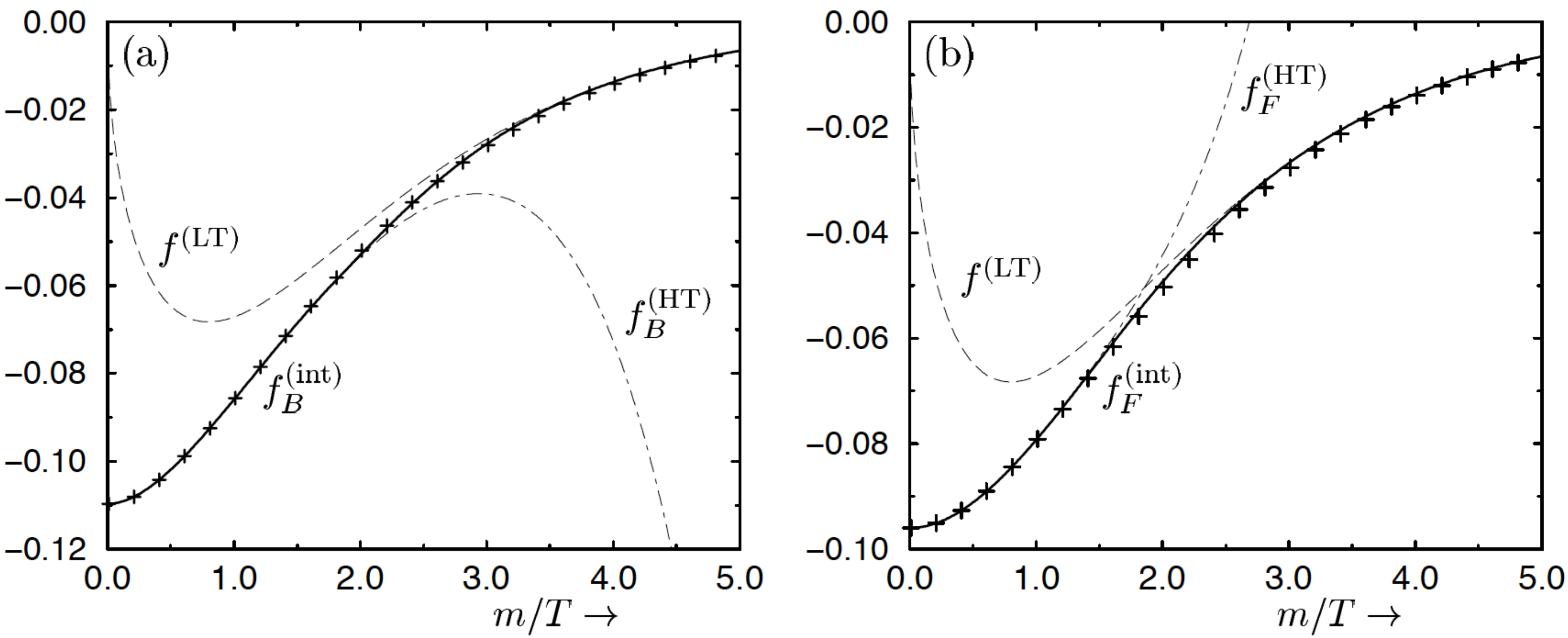
The resulting scalar potential at temperature T is

$$V_{tot}(\phi, T) = V_{Tree}(\phi) + V^{(1)}(\phi) + \Delta V_T(\phi, T)$$

with

$$\Delta V_T = \sum_{\text{bosons}} \frac{k_i T^4}{2\pi^2} \int_0^\infty dx\, x^2 \ln\left[1 - \exp\left(-\sqrt{x^2 + \frac{m_i^2(\phi)}{T^2}}\right)\right]$$

$$- \sum_{\text{fermions}} \frac{k_i T^4}{2\pi^2} \int_0^\infty dx\, x^2 \ln\left[1 + \exp\left(- x^2 + \frac{m_i^2(\phi)}{T^2}\right)\right]$$



This makes sense:

$$\frac{d}{d\phi} \Delta V_T = \frac{T}{2\pi^2} \frac{d}{d\phi} \int_0^\infty dp\, p^2 \ln\left[1 - e^{-E/T}\right]$$

$$\text{with} \quad p = T \cdot x, \quad E = \sqrt{p^2 + m^2(\phi)}$$

$$= \int_0^\infty \frac{dp}{2\pi^2}\, p^2 \, \frac{dE/d\phi}{1 - e^{-E/T}}$$

$$= \int \frac{d^3p}{(2\pi)^3}\, f_{bose} \frac{dE}{d\phi}$$

$$\text{with} \quad f_{bose} = \frac{1}{1 - e^{E/T}}$$

So the change in the thermal contribution is given by a sum over the change in the Individual particle energies in the plasma

The total potential is the free energy per unit volume, ie. the pressure of the system

at high temperature (small mass) we can expand these functions as

$$\Delta V_T \simeq \sum_{\text{bosons}} k_i \left(-\frac{\pi^2}{90} T^4 + \frac{1}{24} m_i^2(\phi) T^2 - \frac{1}{12\pi} T \left(m_i^2(\phi)\right)^{3/2} + \ldots \right)$$

$$+ \sum_{\text{fermions}} k_i \left(-\frac{7\pi^2}{720} T^4 + \frac{1}{48} m_i^2(\phi) + \ldots \right)$$

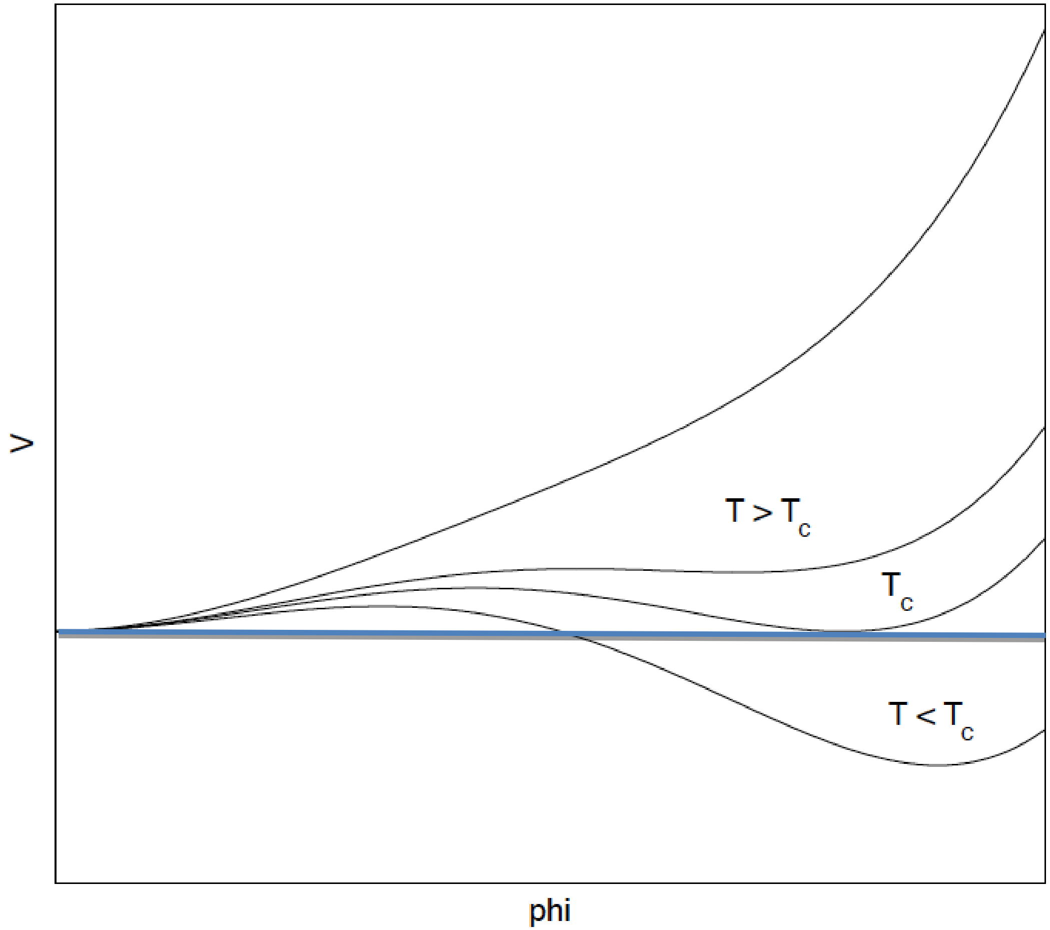
T^4 part is simply minus the pressure of a gas of massless bosons/fermions

T^2 part contributes to the thermal mass of the scalar and leads

to restoration of broken symmetries at high temperature

First order phase transitions:

The T part ($-m^3$ part, "cubic term", bosons only!) induces a bump in the scalar potential, which allows two minima ("phases") to be degenerate at a certain critical temperature T_c



The universe started with zero Higgs field!

A first order phase transition proceeds by nucleation and grows of bubbles of the new phase

Electroweak baryogenesis

Original motivation: baryon asymmetry

$$\eta_B = \frac{n_B}{n_\gamma} = (6.047 \pm 0.074) \times 10^{-10}$$

[Planck 2013]

Good agreement between CMB and primordial nucleosynthesis

$\rightarrow$ we understand the universe up to Temp~MeV

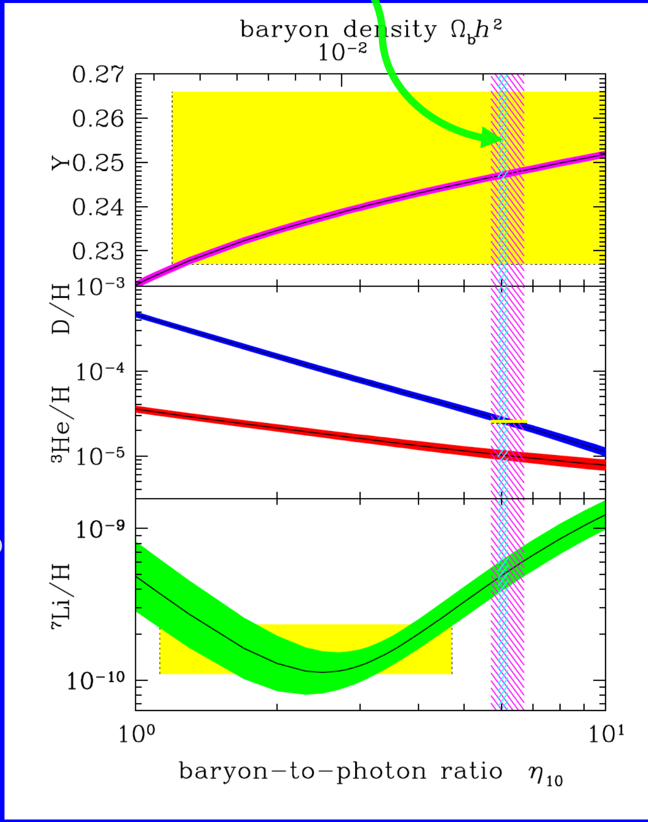
Can we repeat this success for the baryon asymmmetry?

Problem: only 1 observable

$\rightarrow$ How to be convinced by a specific mechanism?

Theory?, Experiment? (inspiration??) …

Temp < TeV scale? $\rightarrow$ EWBG



[Particle Data Group]

Sakharov criteria

$$\eta_B = \frac{n_B}{n_\gamma} = (6.047 \pm 0.074) \times 10^{-10}$$

Baryon number

C

CP

Equilibrium

Sakharov ‘67

SM

Sphalerons +

Gauge interactions +

Yukawa interactions ?

Electroweak phase transition ?

Kuzmin, Rubakov, Shaposhnikov ‘85

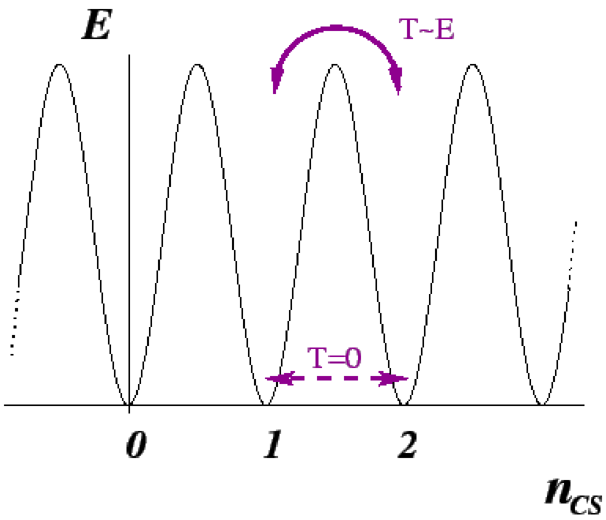
Baryon number violation in the Standard model:

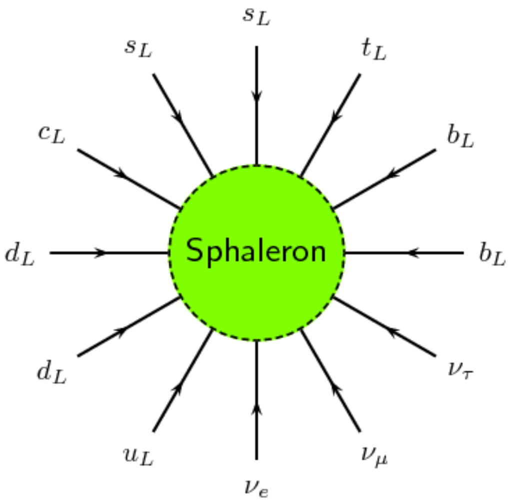
Has never been observed (accidental anomalous global symmetry of the SM)

Cannot happen in perturbation theory

But the electroweak theory has a non-trivial vacuum structure

Higgs-gauge field configurations exist with different winding number

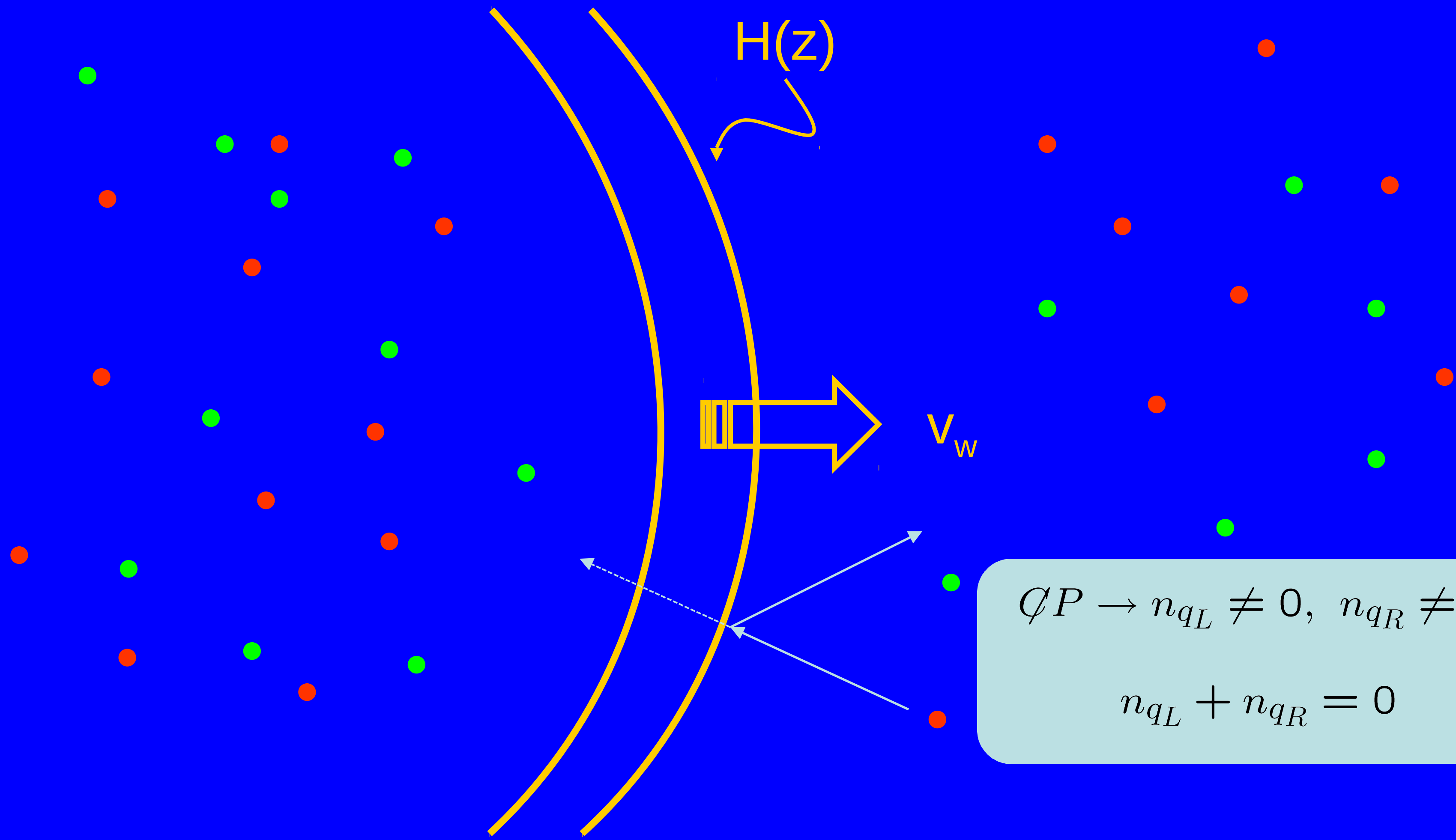




Going from one to another of these vacua changes baryon and lepton number (conserves B-L)

Highly suppressed at T=0, but rapid at T~100 GeV

The mechanism



$$\not{CP} \rightarrow n_{q_L} \neq 0, \; n_{q_R} \neq 0$$

$$n_{q_L} + n_{q_R} = 0$$

The mechanism

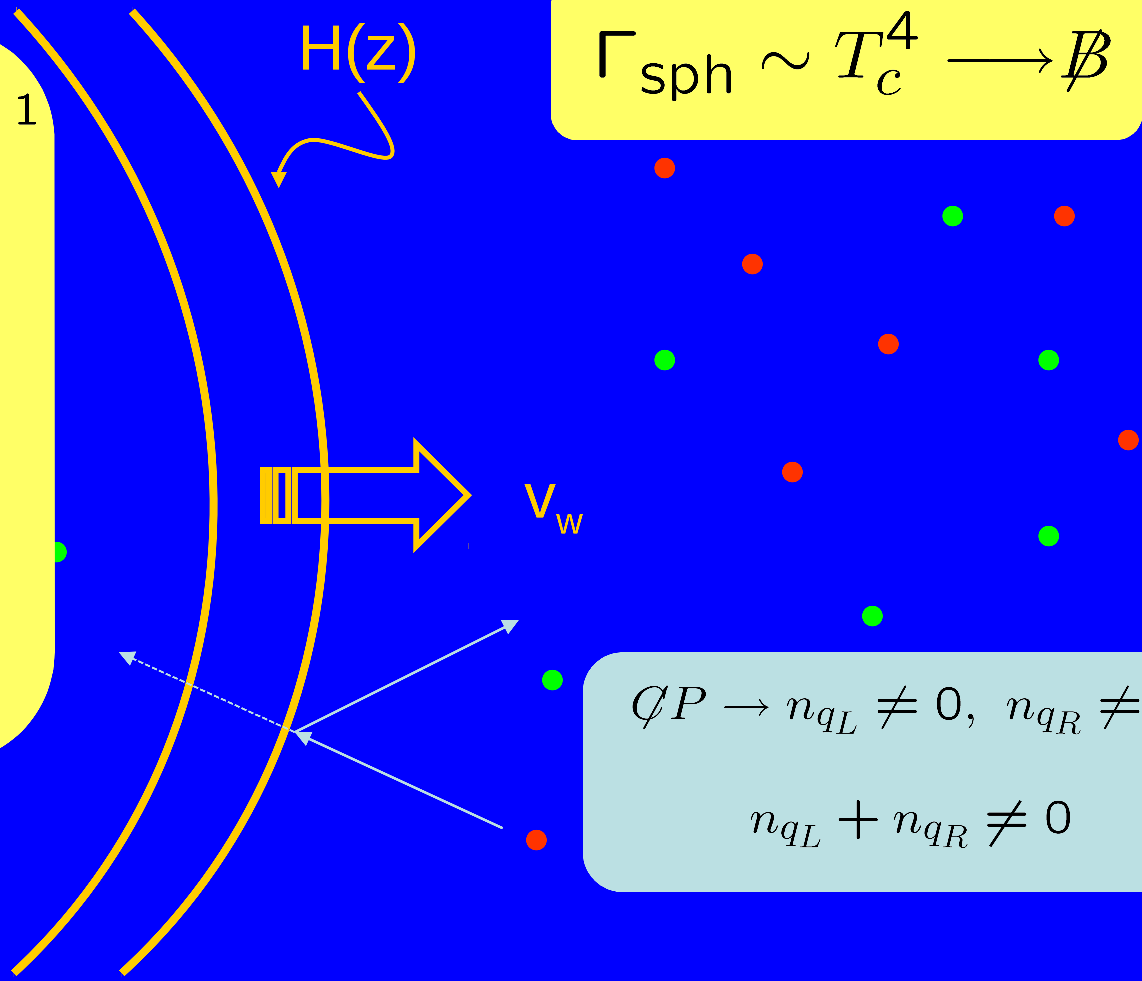
$\Gamma_{\text{sph}} \sim T_c^4 \exp(-\kappa v_c/T_c) \ll 1$

$\Downarrow$

$$\frac{v_c}{T_c} > 1.0$$

$\Updownarrow$

“strong PT”



$\Gamma_{\text{sph}} \sim T_c^4 \longrightarrow \not{B}$

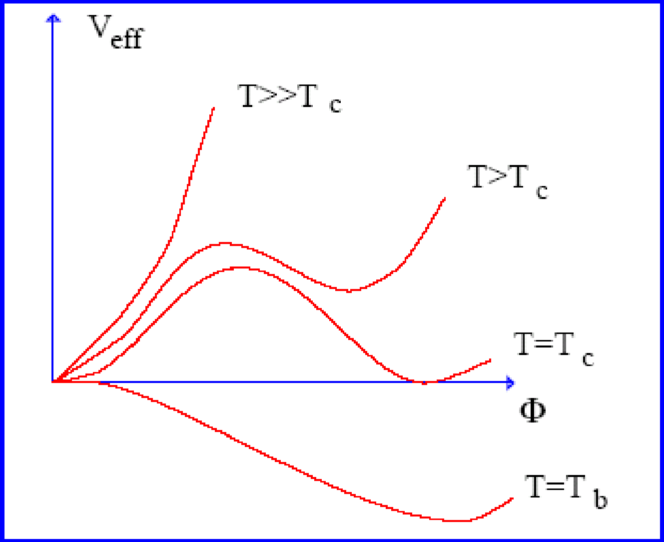
$\not{C}P \rightarrow n_{q_L} \neq 0, \; n_{q_R} \neq 0$

$n_{q_L} + n_{q_R} \neq 0$

The strength of the PT

Thermal potential:

$$V(H,T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4$$



- Bosons in the plasma:

SM: gauge bosons

strong PT: m_h<40 GeV (no top)

never (with realistic top mass)

Lattice: crossover for m_h>80 GeV → the SM fails: NEW PHYSICS!

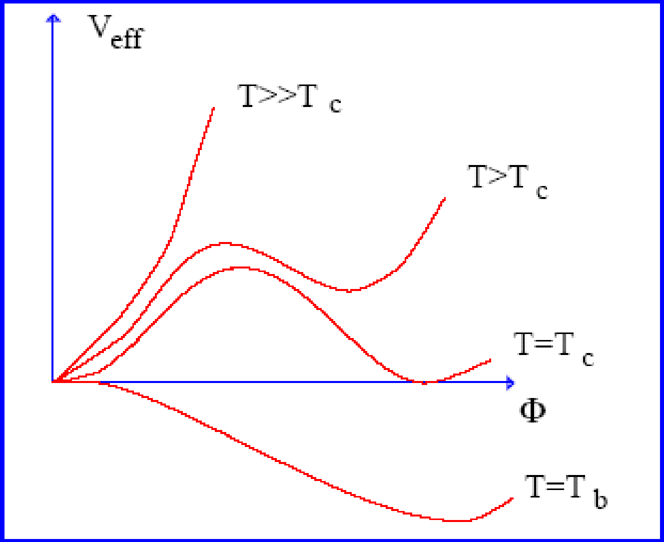
Kajantie, Laine, Rummukainen, Shaposhnikov 1996

Csikor, Fodor, Heitger 1998

The strength of the PT

Thermal potential:

$$V(H,T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4$$



- Bosons in the plasma:

  SM: gauge bosons

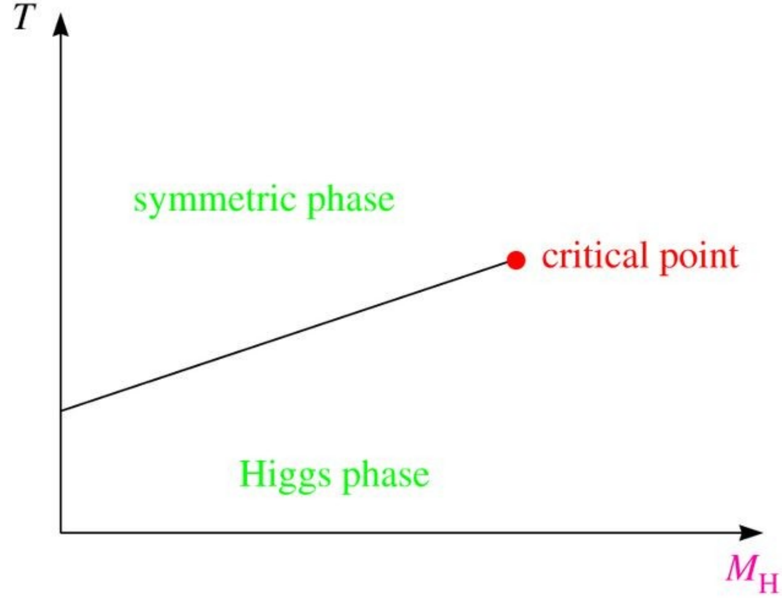
  SUSY: light stops [Laine, Nardini, Rummumainen '12]

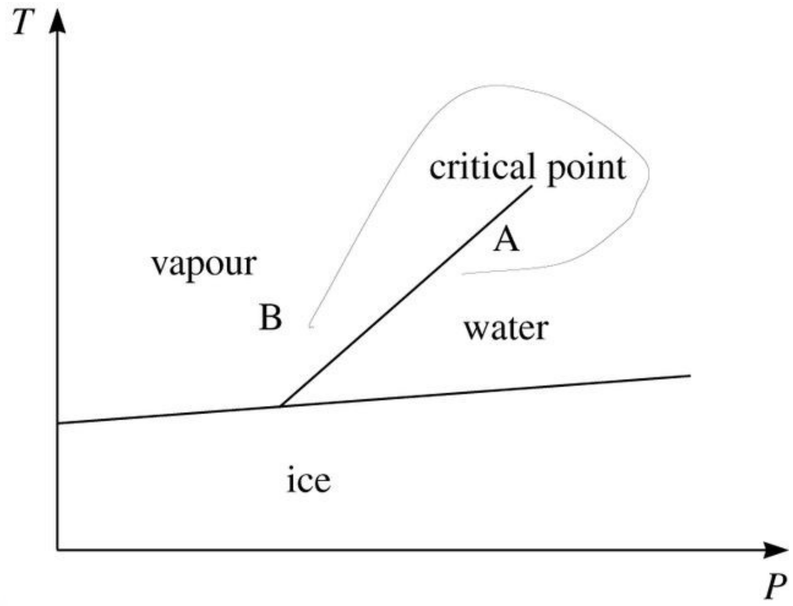
  2HDM: heavy Higgses [Dorsch, SJH, No '13]

- tree-level: extra singlets: $\lambda S H^2$, NMSSM, etc. [Kozaczuk et al.'14]

- replace H^4 by H^6 or introduce $H^2 \log(H^2)$, etc. [Dorsch, SJH, No '14]

Phase diagram of the electroweak theory





Source: Shaposhnikov 2014

For the observed Higgs mass of 125 GeV, the electroweak transition is

a smooth crossover (needs lattice methods to study)

How to compute the baryon asymmetry?

Transport equations

We want to write down a set of Boltzmann equations

The interaction with the bubble wall induces a force on the particles, which is different for particles and antiparticles if CP is broken

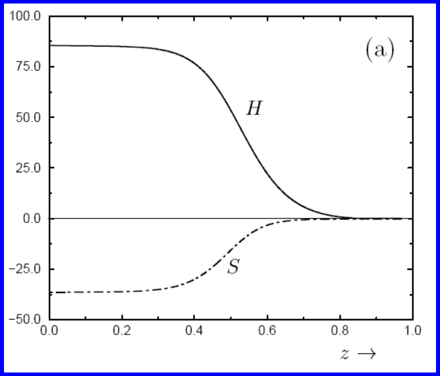
$$(\partial_t + \dot{z}\partial_z + \dot{p}_z \partial_{p_z}) f = \mathcal{C}[f]$$

collision terms

z is the coordinate along the wall profile
H(z)~tanh(z/L_w) with wall width L_w

Compute the force term from dispersion relations

$$\dot{p}_z = -\partial_z E(z, p_z)$$



WKB approximation

Elektroweak bubbles have typically thick walls, i.e. $L_w T_c >> 1$

$(L_w)^{-1} << p$ for a typical particle in the plasma

Compute the dispersion relation via an expansion in $1/(L_w T_c)$

Consider a free fermion with a complex mass

$$M(z) = m(z) e^{i\theta(z)}$$

$$(i\partial - P_L M(z) - P_R M^*(z))\psi = 0$$

$$\psi \sim \exp(-iEt - i\int^z p_z(z')dz')$$

$$\begin{aligned} E_\pm &= E_0 \pm \Delta E_0 \\ &= \sqrt{p^2+m^2} \pm \theta' \frac{m^2}{2(p^2+m^2)} \end{aligned}$$

Joyce, Prokopec, Turok '95
Cline, Joyce, Kainulainen '00

more rigorous, using the Schwinger-Keldysh formalism:
Kainulainen, Prokopec, Schmidt, Weinstock '01-'04
Konstandin, Prokopec, Schmidt, Seco '05

alternative:
Carena, Moreno, Quiros, Seco, Wagner '00

only a varying θ contributes!

no effect for scalars in LO!

Diffusion equations

Fluid ansatz for the phase space densities: $f_i = \frac{1}{e^{(E_i - v_i p_z - \mu_i)/T} \pm 1}$

to arrive at diffusion equations for the μ's

$$-(D_i \mu_i'' + v_w \mu_i') + \Gamma_{ij} \mu_j = S_i$$

diffusion constant — wall velocity — interaction rates — CP violating source terms

relevant particles: top, Higgs, super partners,…

interactions: top Yukawa interaction
strong sphalerons
top helicity flips (broken phase)
super gauge interactions (equ.)

Step 1: compute $n_{B_L} \; (= -n_{B_R})$

Step 2: switch on the weak sphalerons

$$\eta_B \sim \Gamma_{\text{ws}} \int^{\infty} dz \; n_{B_L}(z)$$

Example: Two Higgs doublet model

(with Dorsch, Mimasu, No)

The 2HDM

$$V(H_1, H_2) = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \mu_3^2 e^{i\phi} H_1^\dagger H_2 + \lambda_1 |H_1|^4 + \ldots$$

→ 4 extra physical Higgs degrees of freedom: 2 neutral, 2 charged

→ CP violation, phase Φ (μ_3 breaks Z_2 symmetry softly)

→ there is a phase induced between the 2 Higgs vevs

$$v_1 = \langle H_1 \rangle, \qquad v_2 e^{i\theta} = \langle H_2 \rangle$$

early work:

Turok, Zadrozny ’91

Davies, Froggatt, Jenkins, Moorhouse ’94

Cline, Kainulainen, Vischer ’95

Cline, Lemieux ‘96

simplified parameter choice:

1 light Higgs m_h → SM-like

3 degenerate heavy Higgses m_H → keeps EW corrections small

The phase transition

Evaluate 1-loop thermal potential:

loops of heavy Higgses generate a cubic term

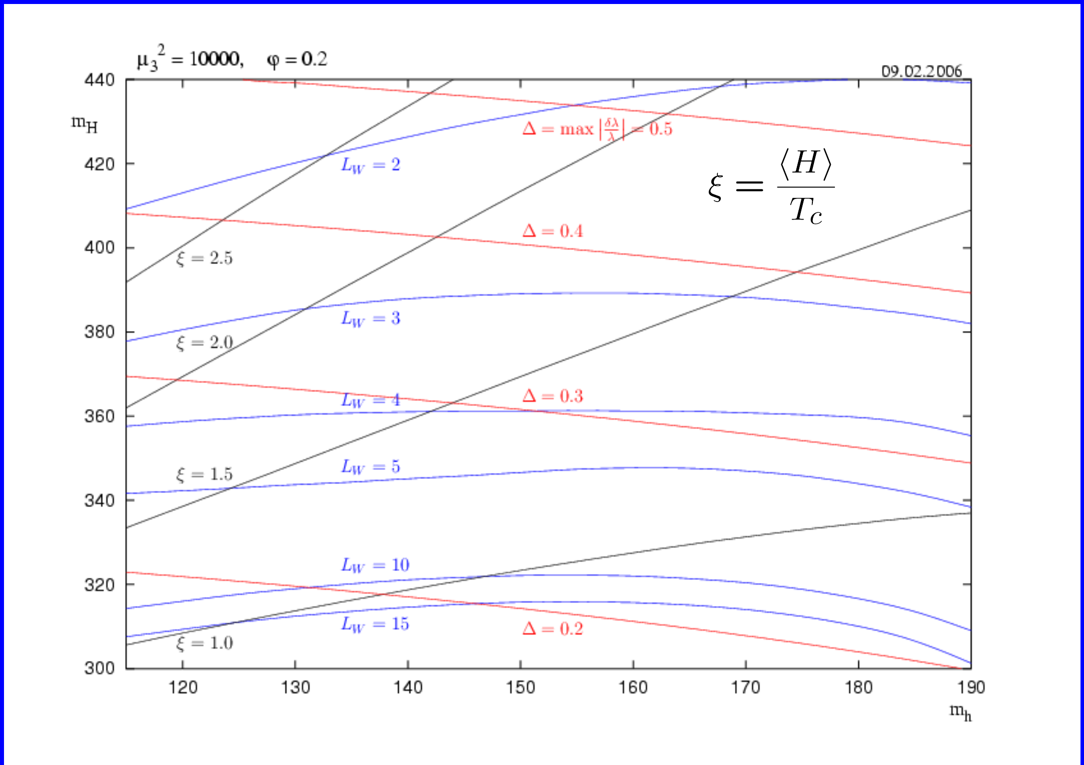
→ strong PT for

m_H>300 GeV

m_h up to 200 GeV

→ PT ~ independent of Φ

→ thin walls only for very strong PT (agrees with Cline, Lemieux ’96)



[Fromme, S.H., Senuich ’06]

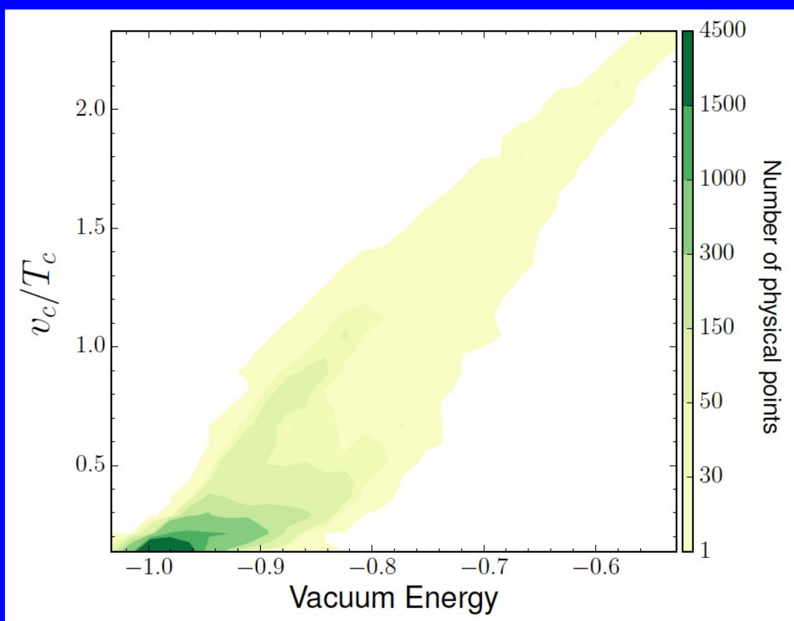
missing: 2-loop analysis of the thermal potential; lattice; wall velocity

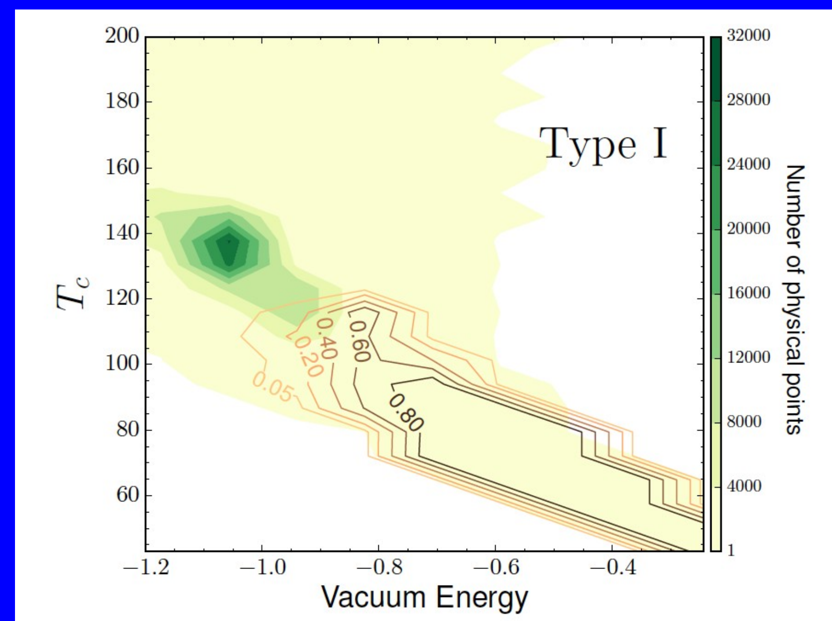
Impact of the vacuum energy:

[Dorsch, S.H., Mimasu, No 2017]

One loop zero temperature corrections to the Higgs potential

$$V_1 = \sum_\alpha n_\alpha \frac{m_\alpha^4(h_1,h_2)}{64\pi^2}\left(\log\frac{|m_\alpha^2(h_1,h_2)|}{Q^2} - C_\alpha\right)$$

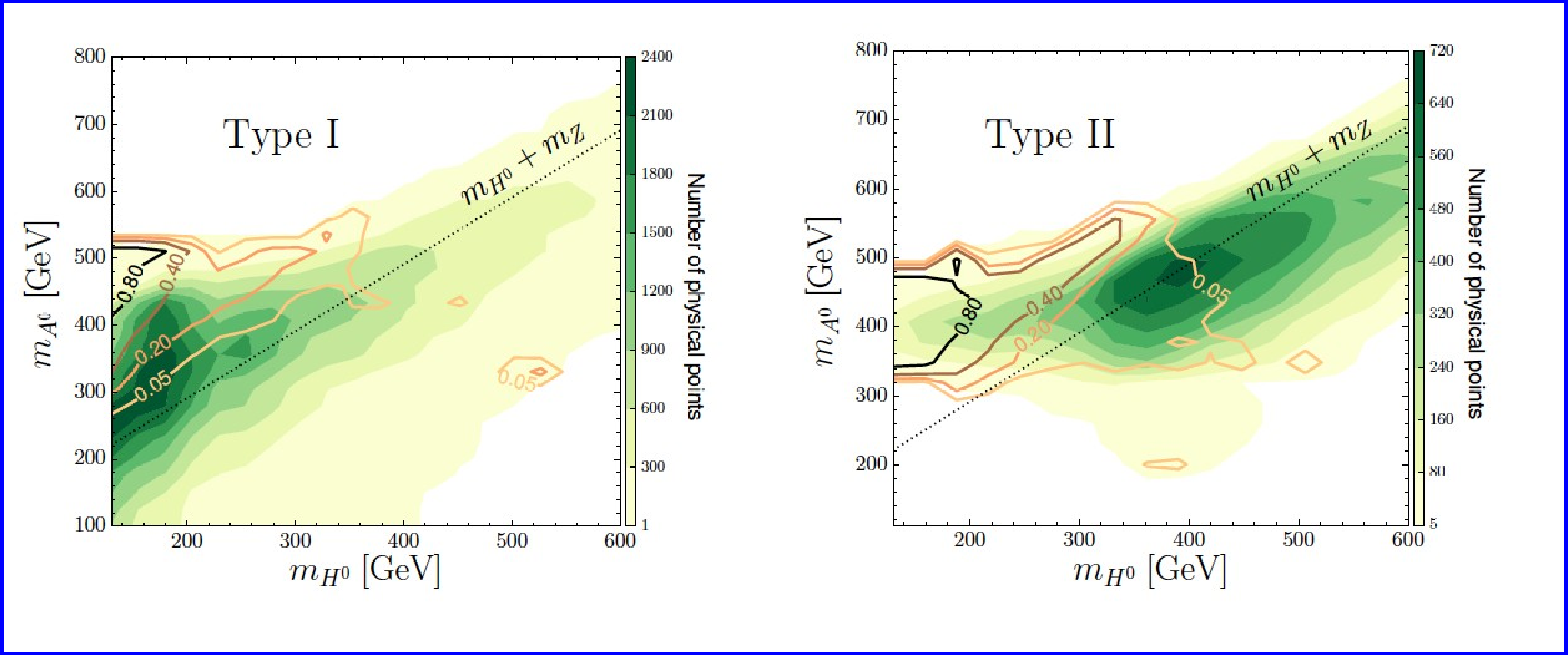




Big change in Higgs masses between symmetric and broken phase:

EW minimium is uplifted, PT at lower T, Higgs field moves less, stronger PT

Summary: strong PT prefers a hierarchical Higgs spectrum



<u>Search for $A_0 \to H_0 Z \to ll\ bb$</u>

at the LHC

The dynamics of the phase transition

Dynamics of the transition

At the critical temperature T_c the two minima are degenerate

Bubble nucleation starts at $T < T_c$ with a rate

$$\Gamma = A\, T^4\, e^{-S_3/T},$$

Where the bubble energy is

$$S_3 = 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{d\varphi}{dr}\right) + V(\varphi, T)\right]$$

The bubble configuration follows from

(static, radially symmetric solution)

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V}{\partial \phi}$$

This bounce solution is a saddle point, not a minimum
$\rightarrow$ difficult to compute for multi field models (one field: shooting)

Compute the bubble configuration as function of T

$\rightarrow S_3(T)$

Define nucleation temperature T_n when the probability for nucleating one bubble per horizon volume becomes 1

This happens for $S_3(T)/T \sim 130\text{-}150$ (close to T_c the bubbles would have to be very large with huge energy)

The bubbles expand and fill space to a fraction 1-exp(-f):

$$f(T_x) \simeq \frac{4\pi H^3}{3} \int_{T_x}^{\infty} R^3(T_x, T) dP.$$

$$dP = A \frac{T^4}{H^4} e^{-S_3/T} \frac{dT}{T}.$$

$$R(T_x, T) = v_b \frac{T_x}{H(T_x)} \left(\frac{1}{T_x} - \frac{1}{T} \right)$$

Define the end of the phase transition T_f (i.e. when the bubbles bubbles collide) to occur when f=1

Bubble radius:

Define $\langle R \rangle$ from the bubble volume distribution at T_f (Dolgov et al. '02)

alternative:

$$\frac{\beta}{H_*} = -T_* \frac{d}{dT}\left(\frac{S_3}{T}\right)\Bigg|_{T_*}$$

$$\langle R \rangle \approx 3\frac{v_b}{\beta}.$$

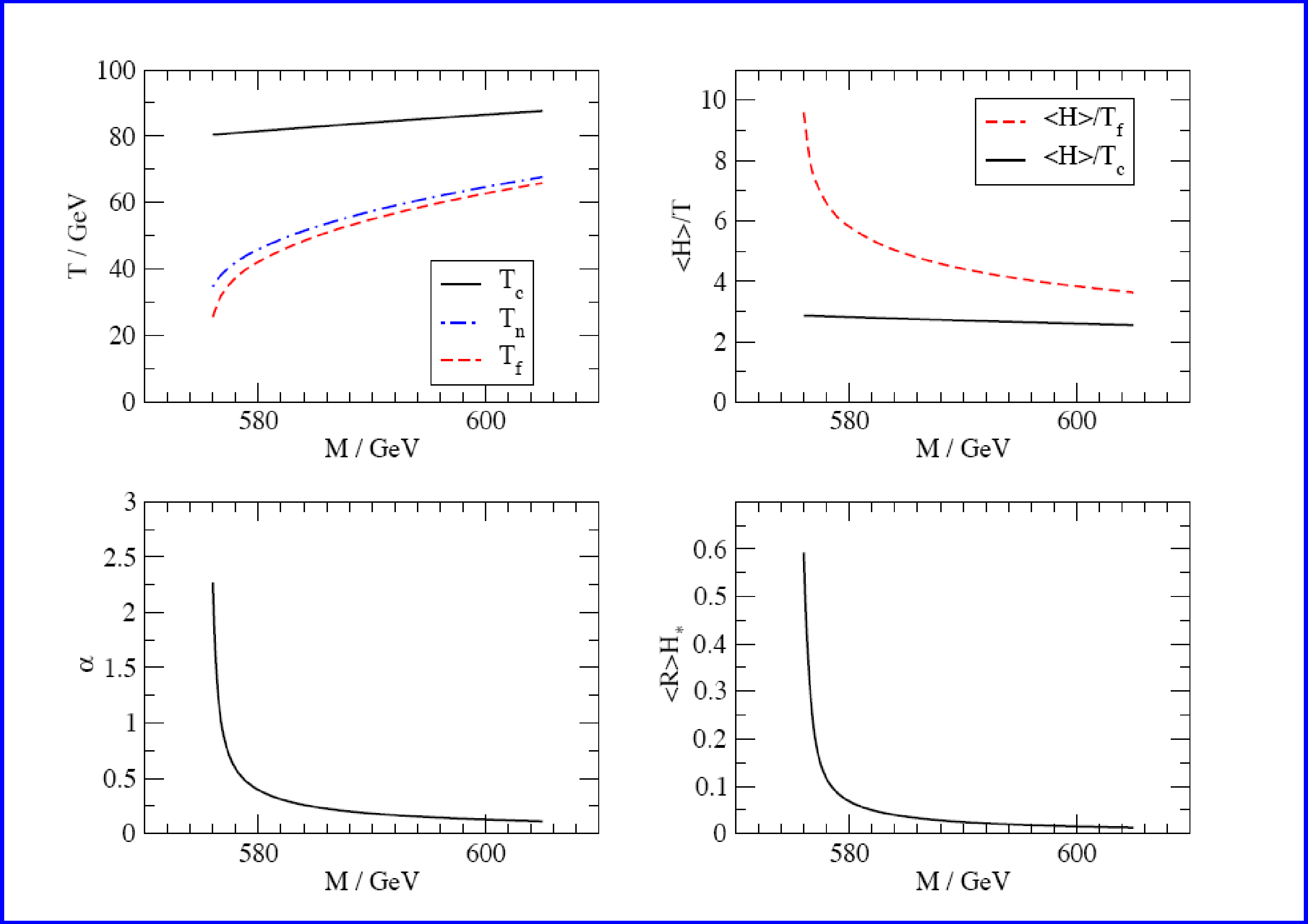
Electroweak bubbles grow a lot and become of "macroscopic" size when they collide (horizon size at 100 GeV is about 10 cm)

Latent heat:

$$\epsilon_* = -\Delta V + T_* \frac{\partial V}{\partial T}\Bigg|_{T_*}$$

$$\alpha = \frac{30\epsilon_*}{\pi^2 g_* T_*^4}.$$

Key parameteres of the phase transition: Φ^6 model, m_h=120 GeV



S. H. & Konstandin '07

Compute as function of temperature: bubble configurations $\rightarrow$ E

nucleation rate Γ~exp(-E)

The wall velocity:

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic (v_s^2=1/3)

Standard model: v_w~ 0.35 - 0.45 for low Higgs masses [Moore, Prokopec '95]

MSSM: v_w~0.05 [John, Schmidt '00]

All other models: no detailed computations

**Recently: walls can run away, i.e. approach v_w=1 [Bodeker, Moore '09]

How to compute the wall velocity?

Main ingredients: pressure difference vs. plasma friction

Also important: reheating due to release of latent heat

Microscopic description: Moore, Prokopec '95

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3\, 2E}\delta f(p,x) = 0$$

$$\frac{df}{dt} = \partial_t f + \dot{\vec{x}}\cdot\partial_{\vec{x}} f + \dot{\vec{p}}\cdot\partial_{\vec{p}} f = -C[f]$$

$$\begin{aligned}
a v_w \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3}v' + F_1 &= -\Gamma_{\mu 1}\frac{\mu}{T} - \Gamma_{T1}\frac{\delta T}{T} \\
b v_w \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3}v' + F_2 &= -\Gamma_{\mu 2}\frac{\mu}{T} - \Gamma_{T2}\frac{\delta T}{T} \\
b\frac{\mu'}{T} + \frac{\delta T'}{T} + v_w v' + 0 &= -\Gamma_v v
\end{aligned}$$

$$\dot{\vec{p}_z} = -\frac{\partial E}{\partial z}\vec{u_z} = -\frac{1}{2E}\frac{d(m^2)}{dz}\vec{u_z}$$

$$f = \frac{1}{1 + \exp\frac{E - E\delta T/T - p_z v - \mu}{T}}$$

(fluid ansatz)

→ Complicated set of coupled field equations

and Boltzmann equations

need many scattering rates

$$F_1 = -\frac{v_w \ln 2}{9\zeta_3}\frac{(m^2)'}{T^2}, \qquad F_2 = -\frac{v_w \zeta_2}{42\zeta_4}\frac{(m^2)'}{T^2}$$

(force terms)

SM: v ~ 0.35 - 0.45

Simplified approach: (Ignatius, Kajantie, Kurki-Suonio, Laine '94)

1) describe friction by a friction coefficient η

2) model the fluid by a fluid velocity and temperature

$$\frac{d^2\phi(x)}{dx^2} = \frac{\partial V(\phi,T)}{\partial \phi} + \eta \frac{\phi^2}{T_{s1}} v\gamma \frac{d\phi(x)}{dx}$$

$$(4aT^4 - T\frac{\partial V(\phi,T)}{\partial T})\gamma^2 v = C_1$$

$$(4aT^4 - T\frac{\partial V(\phi,T)}{\partial T})\gamma^2 v^2 + P_r - V(\phi,T) + \frac{1}{2}(\frac{d\phi}{dx})^2 = C_2$$

← 1 + 1 dimensions

3) Determine η from fitting the to the full result by Moore and Prokopec

[with Miguel Sopena, see also Megevand, Sanchez '09]

→ the formalism should describe situations with SM friction well

→ study models with SM friction, but different potential, e.g. phi^6 model

→ same for the MSSM

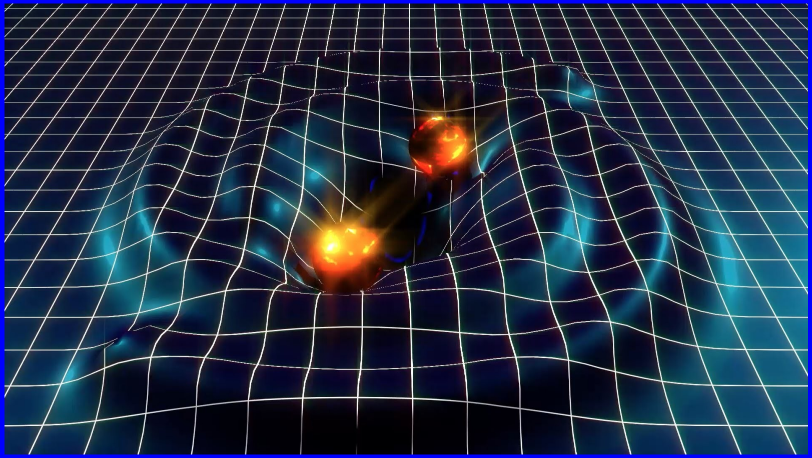
→ such a model can be used for numerical simulations of the phase transition

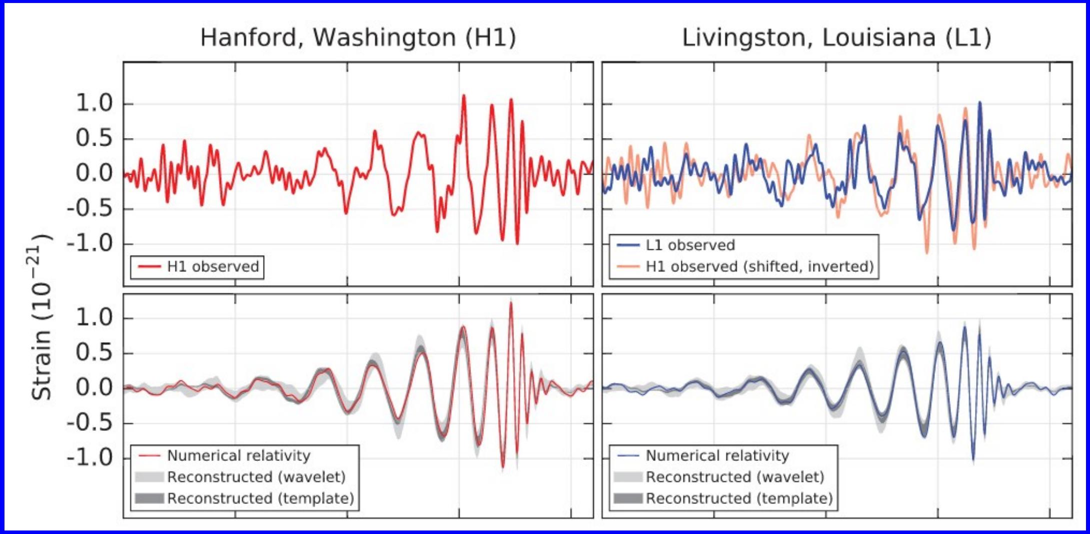
[work in progress with Hindmarsh, Rummukainen, Weir]

Numerical Simulations

of a first-order phase transition
and gravitational waves
(with Hindmarsh, Rummukainen, Weir)

Gravitational wave discovery at LIGO





Merger of two two black holes, having about 30 solar masses

Frequency is in the kHz range

Gravitational waves from phase transitions

[see eLISA Cosmo working group report '15]

sources of GW's: direct bubble collisions

turbulence

(magnetic fields)

sound waves

key parameters: available energy

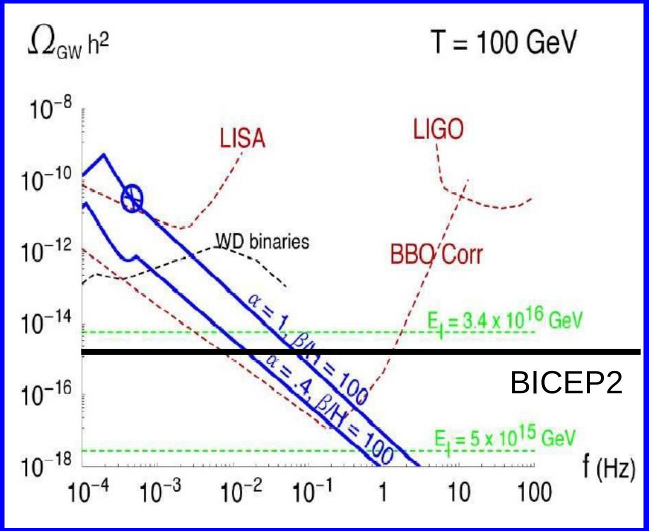
$$\alpha = \frac{\text{latent heat}}{\text{radiation energy}}$$

typical bubble radius

$$\langle R \rangle \propto v_b \, \tau \approx \frac{v_b}{\beta}.$$

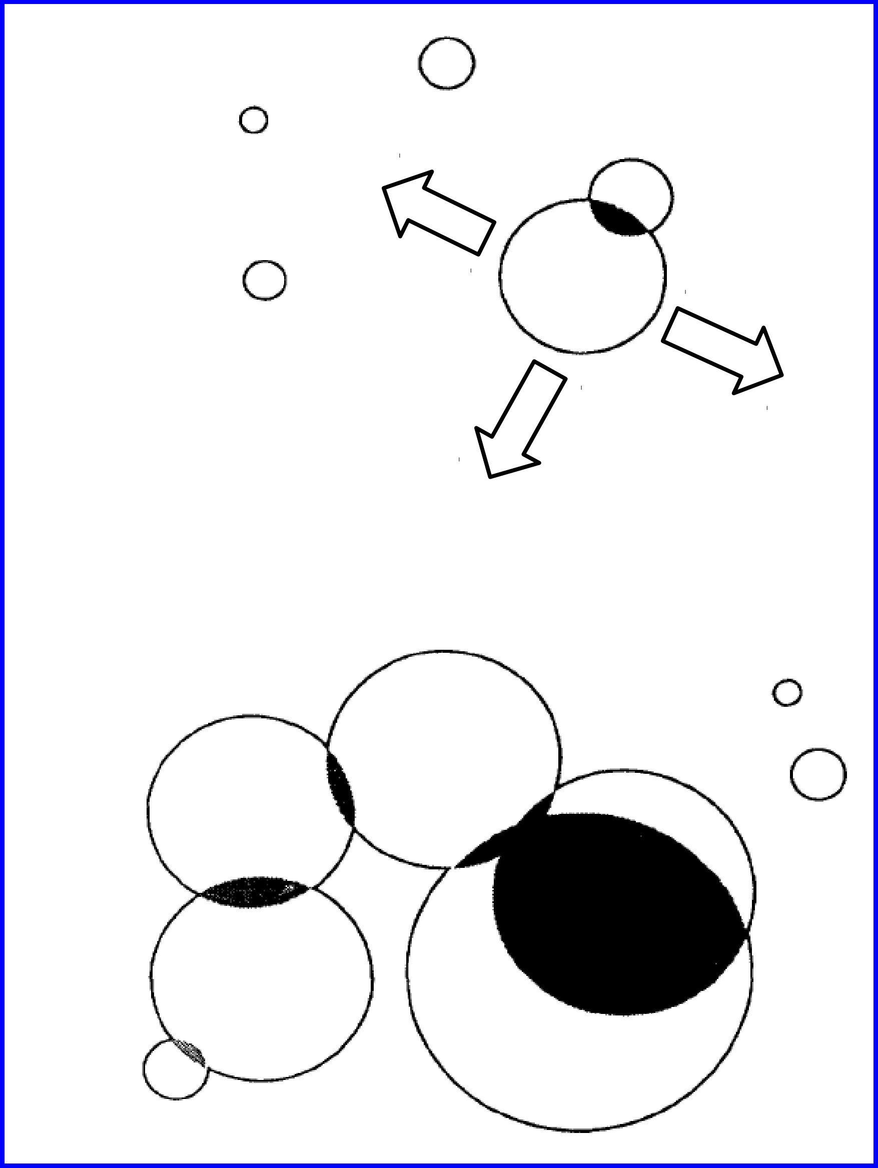
v_b wall velocity

eLISA: 2034?



[Grojean, Servant '06]

The envelope approximation: Kosowsky, Turner 1993



Energy momentum tensor of expanding bubbles modelled by expanding infinitely thin shells,

cutting out the overlap

 very non-linear!

Tested by colliding two pure scalar bubbles

Recent scalar field theory simulation: Child, Giblin 2012

What happens if the fluid is relevant?

Turbulence??

We performed the first 3d simulation of a scalar + relativistic fluid system:

$$V(\phi,T)=\frac{1}{2}\gamma(T^2-T_0^2)\phi^2-\frac{1}{3}\alpha T\phi^3+\frac{1}{4}\lambda\phi^4.$$

(Thermal scalar potential)

phenom. friction parameter

$$-\ddot{\phi}+\nabla^2\phi-\frac{\partial V}{\partial\phi}=\eta W(\dot{\phi}+V^i\partial_i\phi)$$

(Scalar eqn. of motion)

$$\dot{E}+\partial_i(EV^i)+P[\dot{W}+\partial_i(WV^i)]-\frac{\partial V}{\partial\phi}W(\dot{\phi}+V^i\partial_i\phi) = \eta W^2(\dot{\phi}+V^i\partial_i\phi)^2. \quad (7)$$

(eqn. for the energy density)

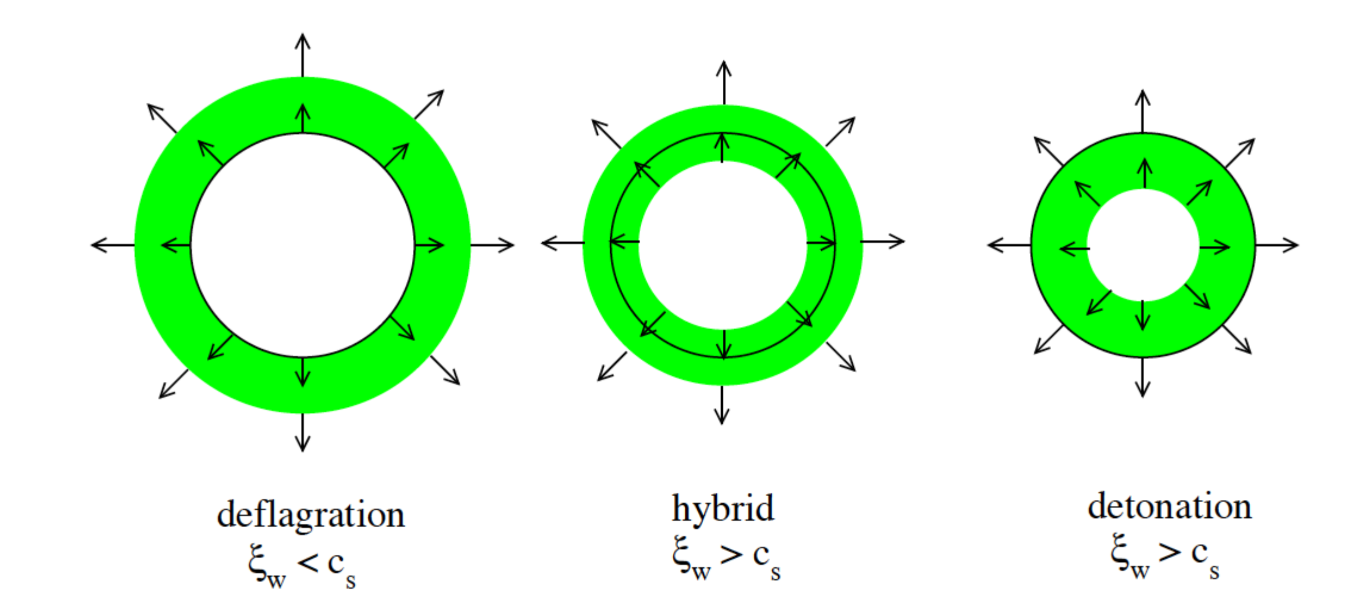
$$\dot{Z}_i+\partial_j(Z_iV^j)+\partial_iP+\frac{\partial V}{\partial\phi}\partial_i\phi=-\eta W(\dot{\phi}+V^j\partial_j\phi)\partial_i\phi.$$

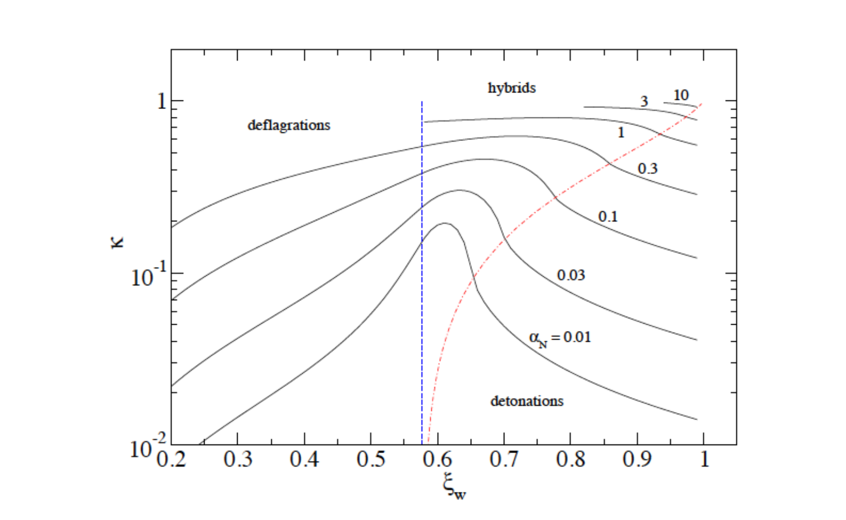
(eqn. for the momentum density)

$$\ddot{u}_{ij}-\nabla^2u_{ij}=16\pi G(\tau^{\phi}_{ij}+\tau^{\mathrm{f}}_{ij}),$$

(eqn. for the metric perturbations)

Types of single bubble solutions:





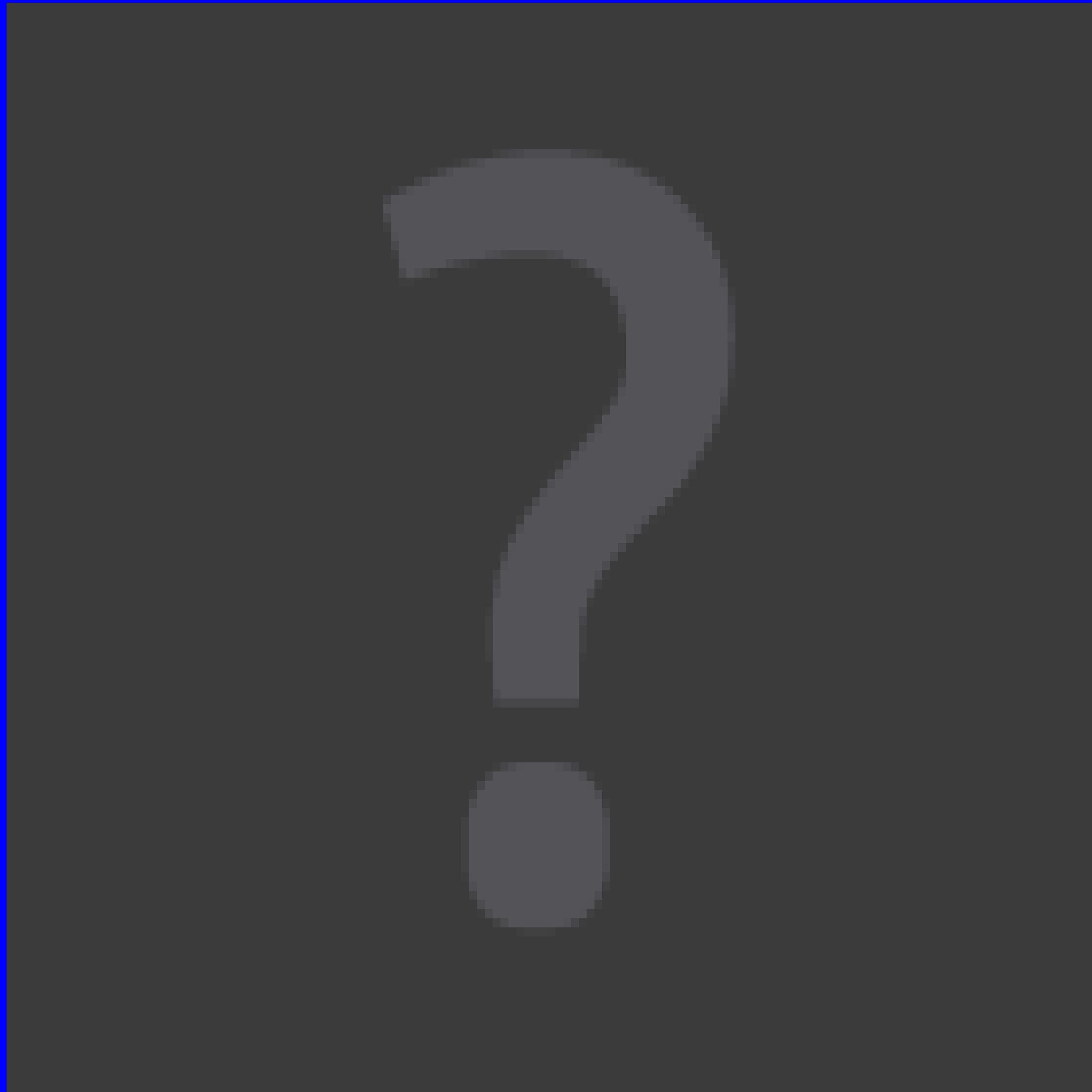
Espinosa, Konstandin, No, Servant'10

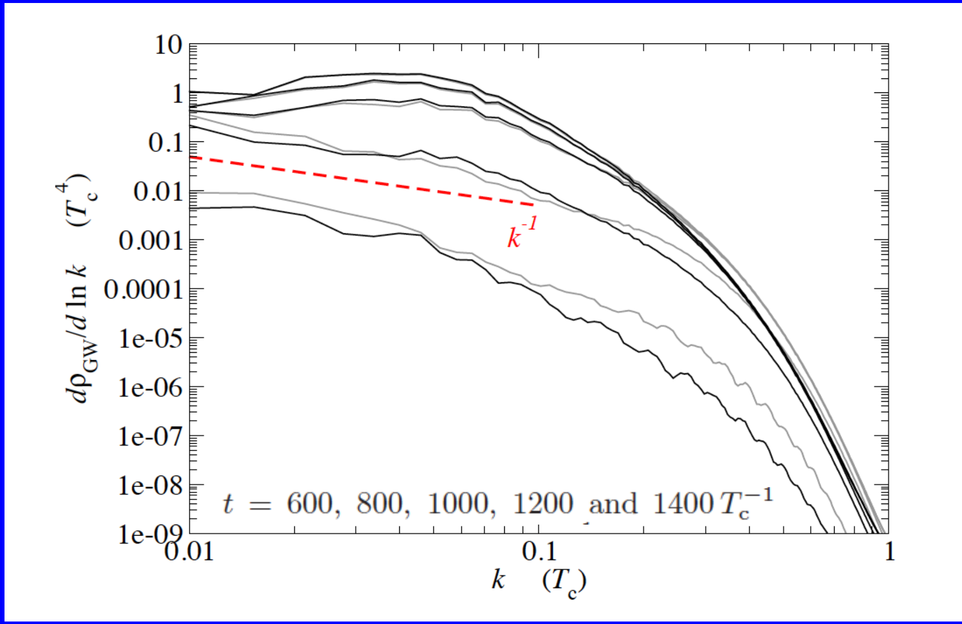
Efficiency κ for turning latent heat into fluid motion

1024^3

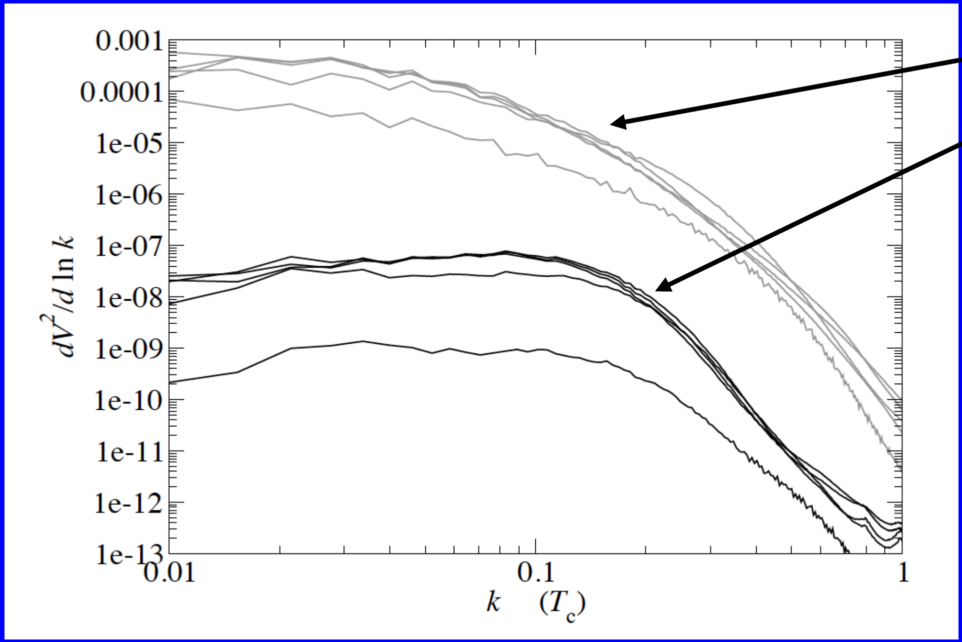
Lattice

Fluid energy





GW Spectrum



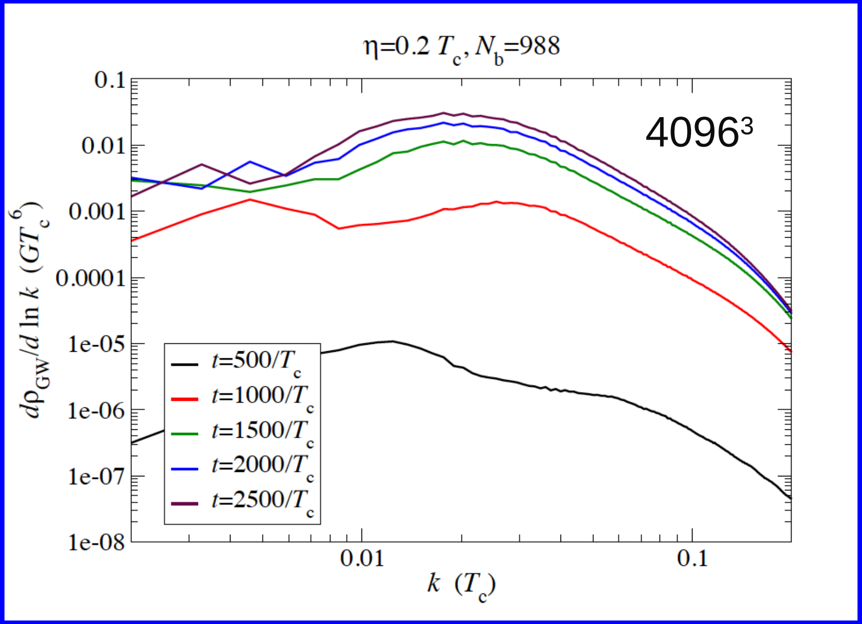
- longitudinal and
- transverse part of the fluid stress

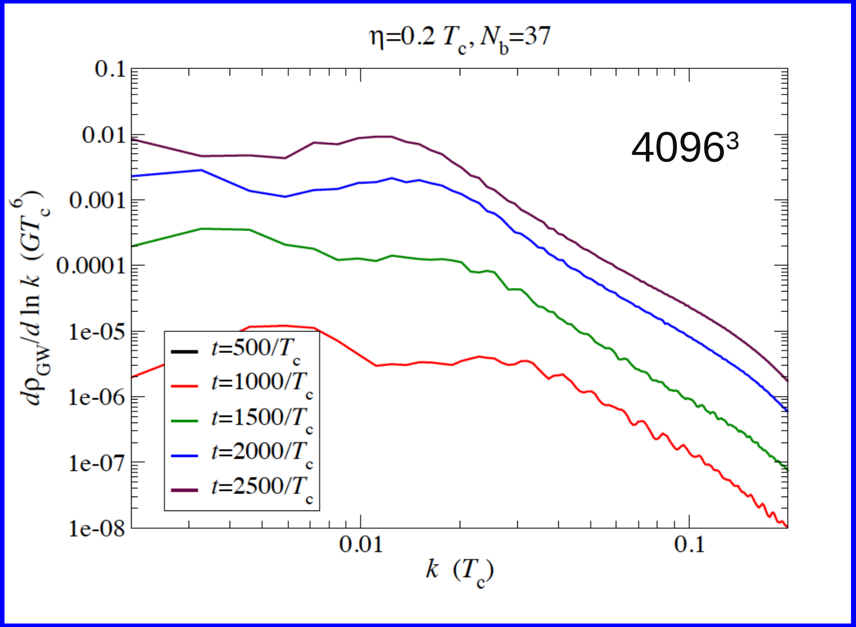
Logitudinal part dominates
Basically sound waves

For very strong transitions turbulence will develop ??

Power laws:

[Hindmarsh, SH, Rummukainen, Weir ’15]



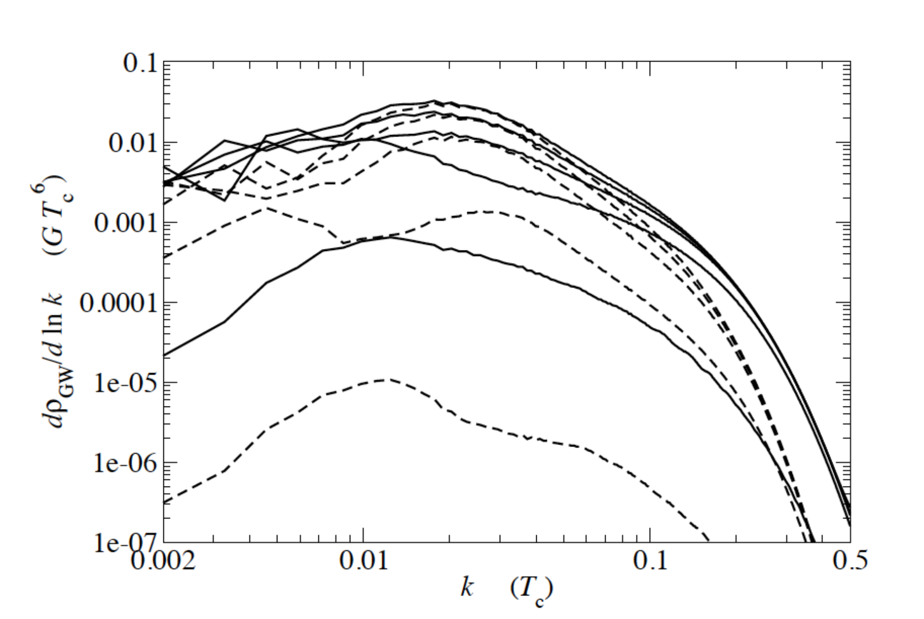


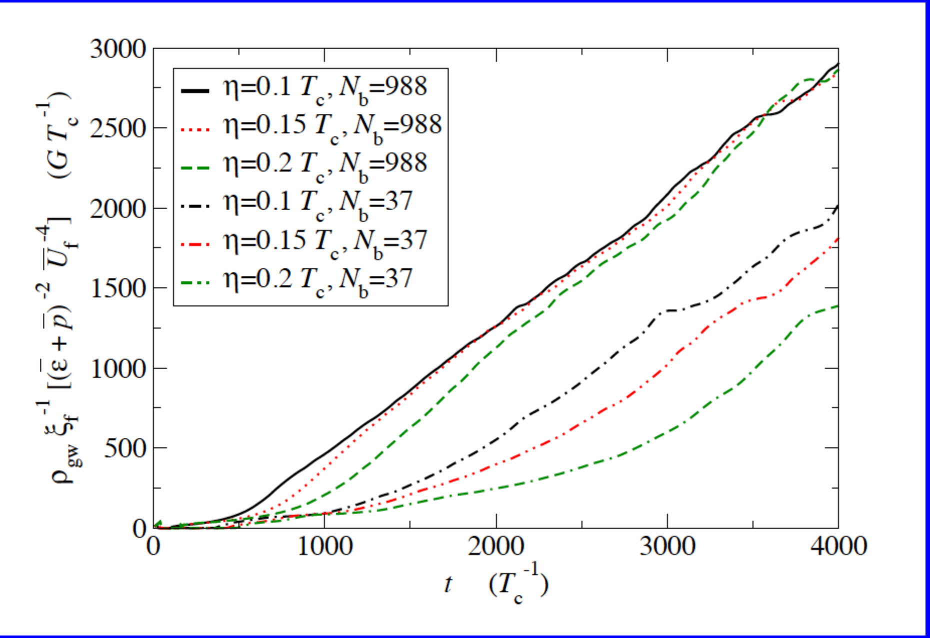
Clear k^{-3} power law fall off in the UV (different from pure scalar!)

Observations will be able to distinguish between a thermal and a scalar-only transition

Maybe also other information hidden in the spectrum?

Time evolution:





Strength of the GW signal:

$$\Omega_{\mathrm{GW}} \simeq \frac{3\bar{\Pi}^2}{4\pi^2}(H_*\tau_{\mathrm{s}})(H_*R_*)(1+w)^2\overline{U}_{\mathrm{f}}^4,$$

Simulation (sound)

$$\Omega_{\mathrm{GW}} \simeq \frac{0.11 v_{\mathrm{w}}^3}{0.42+v_{\mathrm{w}}^2}\left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2\alpha_T^2}{(\alpha_T+1)^2}$$

env. appr. (scalar)

Enhancement by $\tau_{\mathrm{s}}/R_* v_{\mathrm{w}}$

What sets τ_s? Hubble time?

GW's in the SUSY with singlets

General Next-to-MSSM: no discrete symmetries

 no domain wall problem, rich Higgs phenomenology

$$W = L_1 \hat{S} + \mu \hat{H}_u \hat{H}_d + \frac{1}{2} M_S \hat{S}^2 + \lambda \hat{H}_u \hat{H}_d \hat{S} + \frac{1}{3} \kappa \hat{S}^3$$

[SH, Konstandin, Nardini, Rues '15]

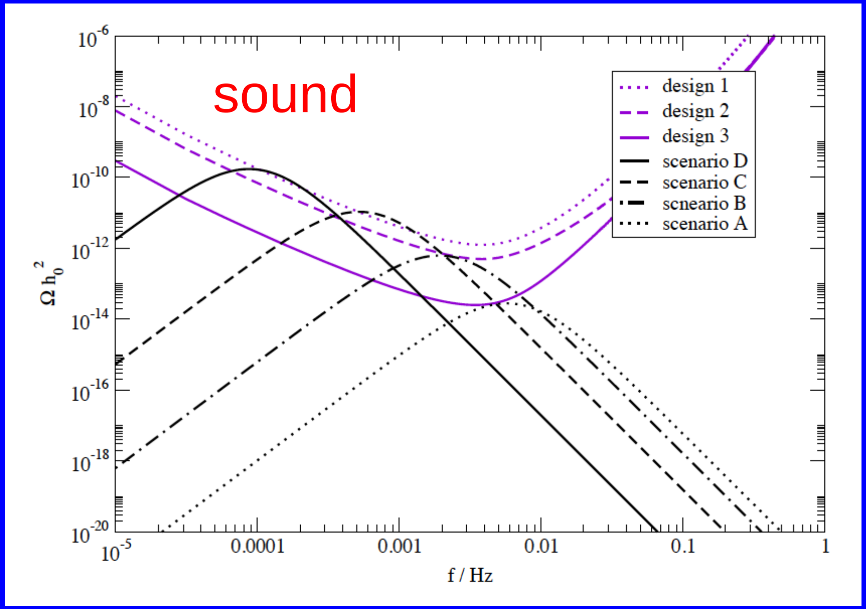
Look for parameter points with a <u>very strong phase transition</u> (substantially lifted electroweak vacuum): 4 benchmarks A-D

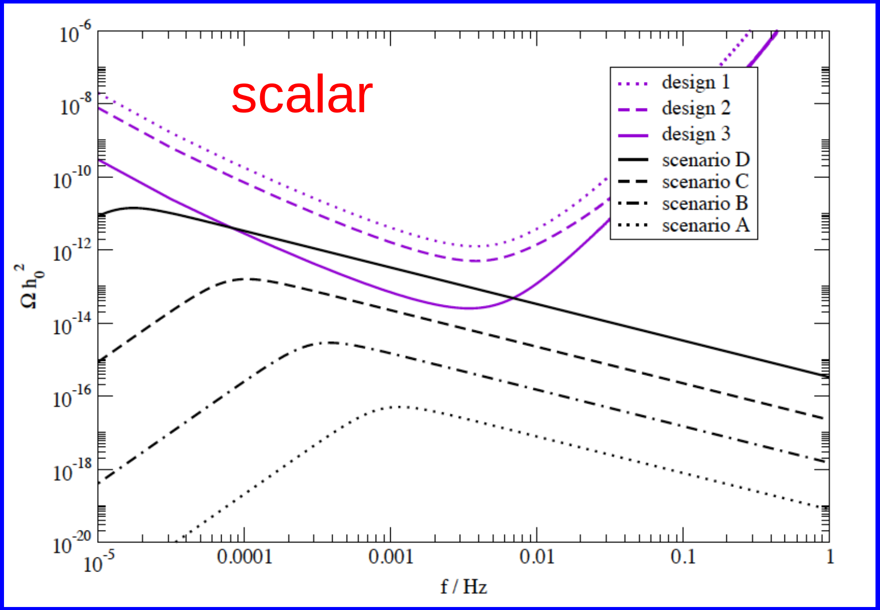
	A - D
$\tan\beta$	5
λ	0.7
κ	0.015
L_1	0
B_S [GeV^2]	-250^2
μ [GeV]	300

	A	B	C	D
T_n [GeV]	112.3	94.7	82.5	76.4
α	0.037	0.066	0.105	0.143
β/H	277	105.9	33.2	6.0
$v_h(T_n)/T_n$	1.89	2.40	2.83	3.12

1-loop	A - D
m_{h_1}	91
m_{h_2}	125.6
$\sin^2\gamma$	10^{-3}

Gravitational wave signal:





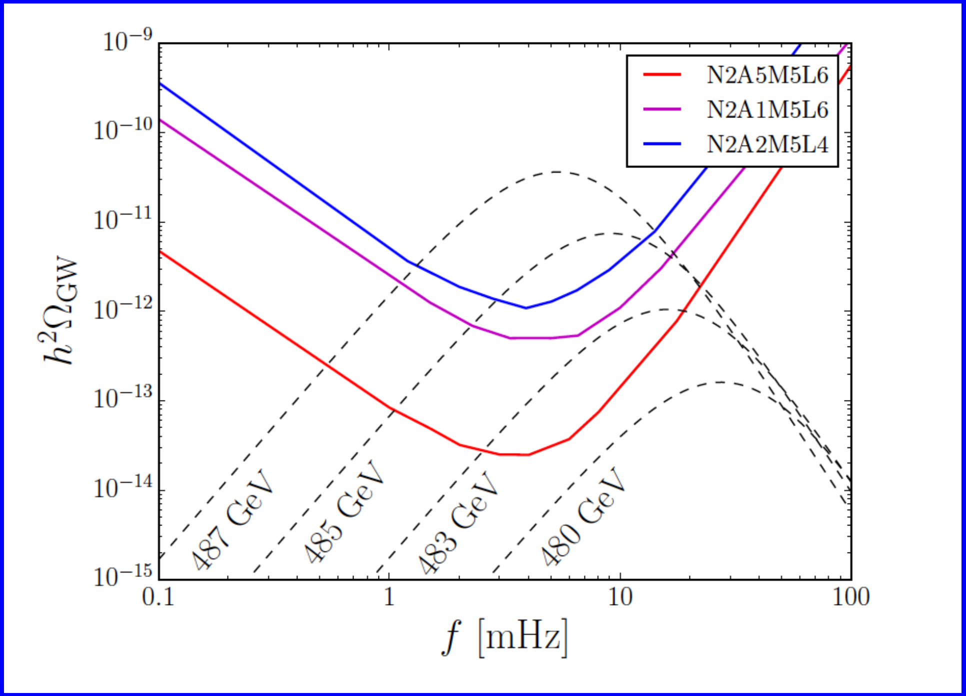
Very strong transitions in the GNMSSM lead to an observable GW signal in eLISA

The spectrum from sound (fluid) clearly different from that of scalar only

GWs in the 2HDM

Consider the 2HDM from the first part: [Dorsch, SH, Konstandin, No ’16]

One can at the same time have successful baryogenesis and observational GWs:



m_{A^0} [GeV]	T_n	v_n/T_n	$L_w T_n$	$\Delta\Theta_t$	α_n	β/H_*	v_w
450	83.665	2.408	3.169	0.0126	0.024	3273.41	0.15
460	76.510	2.770	2.632	0.0083	0.035	2282.42	0.20
480	57.756	3.983	1.714	0.0037	0.104	755.62	0.30
483	53.549	4.349	1.556	0.0031	0.140	557.77	0.35
485	50.297	4.668	1.441	—	0.179	434.80	0.45
487	46.270	5.120	1.309	—	0.250	306.31	$\approx c_s$

Summary

Cosmic phase transitions provide various connections

Between particle physics and cosmology:

Baryogenesis

Gravitational waves

Magnetic fields

Topological defects (monopoles, strings, domain walls??)

Some of the models can also be studied with collider experiments, such as the LHC