Majorana Fermions (Neutrinos) in Particle Physics

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Plan of the Lectures

1. Preliminary Remarks.

2. Massive Neutrinos, Neutrino Mixing and Oscillations: Brief Overview.

3. The Three Neutrino Mixing: what we have learned.

4. Open Questions in the Physics of Massive Neutrinos and Future Progress.

5. The Nature of Massive Neutrinos I:
Massive Majorana versus Massive Dirac Neutrinos.

6. The Nature of Massive Neutrinos II:
Origins of Dirac and Majorana Massive Neutrinos (Deneral Discussion).

7. Determining the Nature of Massive Neutrinos:
Neutrinoless Double Beta Decay.

8. The Nature of Massive Neutrinos III:
The Seesaw Mechanisms of Neutrino Mass Generation.

9. Leptogenesis Scenario of Generation of the Baryon Asymmetry of the Universe.

10. Future LBL Neutrino Oscillation Experiments on $\mathrm{sgn}(\Delta m^2_{31})$ and CP Violation (brief overview).

11. Conclusions.

3 Families of Fundamental Particles: Quarks+Leptons

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- $|p> = |uud>, \quad |n> = |ddu>, ...$
- Electromagnetic interaction: the photon γ (q's,e,μ,τ)
- Strong interaction: 8 Gluons G (only q's)
- Weak interaction: $W^{\pm}$, Z^0 (all: q's, $l^{\pm}$, ν_l)
- **Gravitational interaction: the graviton g (all)**

EM, Strong and Gravitational Interactions have symmetries: *particle $\leftrightarrow$ antiparticle (C), Left–Right or Mirror (P), and Combined CP* (Strong CP Problem?) not respected by Weak Interactions.

All Fundamental Interactions obey *CPT-symmetry: particle $\leftrightarrow$ antiparticle, Left–Right, $t \to -t$*.

Universe: there are no antiparticles.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ & \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ & \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ & \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active $\nu' s$ and $\tilde{\nu}' s$
- The notion of "type" ("flavour") - dynamical;

ν_e: $\nu_e + n \rightarrow e^- + p$; ν_μ: $\pi^+ \rightarrow \mu^+ + \nu_\mu$; etc.

- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}$, $\tilde{\nu}_l \neq \tilde{\nu}_{l'}$, $l \neq l' = e, \mu, \tau$; $\nu_l \neq \tilde{\nu}_{l'}$, $l, l' = e, \mu, \tau$.

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu'_l | \nu_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}'_l | \tilde{\nu}_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}'_l | \nu_l \rangle = 0$.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}_{\text{lep}}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x)\gamma_{\alpha}\,(1-\gamma_5)\,\nu_{l\text{L}}(x)\,W^{\alpha}(x) + \text{h.c.}\,,$$

$W^{\pm}$: $\quad M_{\text{W}} = 80$ GeV, $g(\cong 0.6)$ – $SU(2)_L$.

The Neutral Current Weak Interaction Lagrangian:

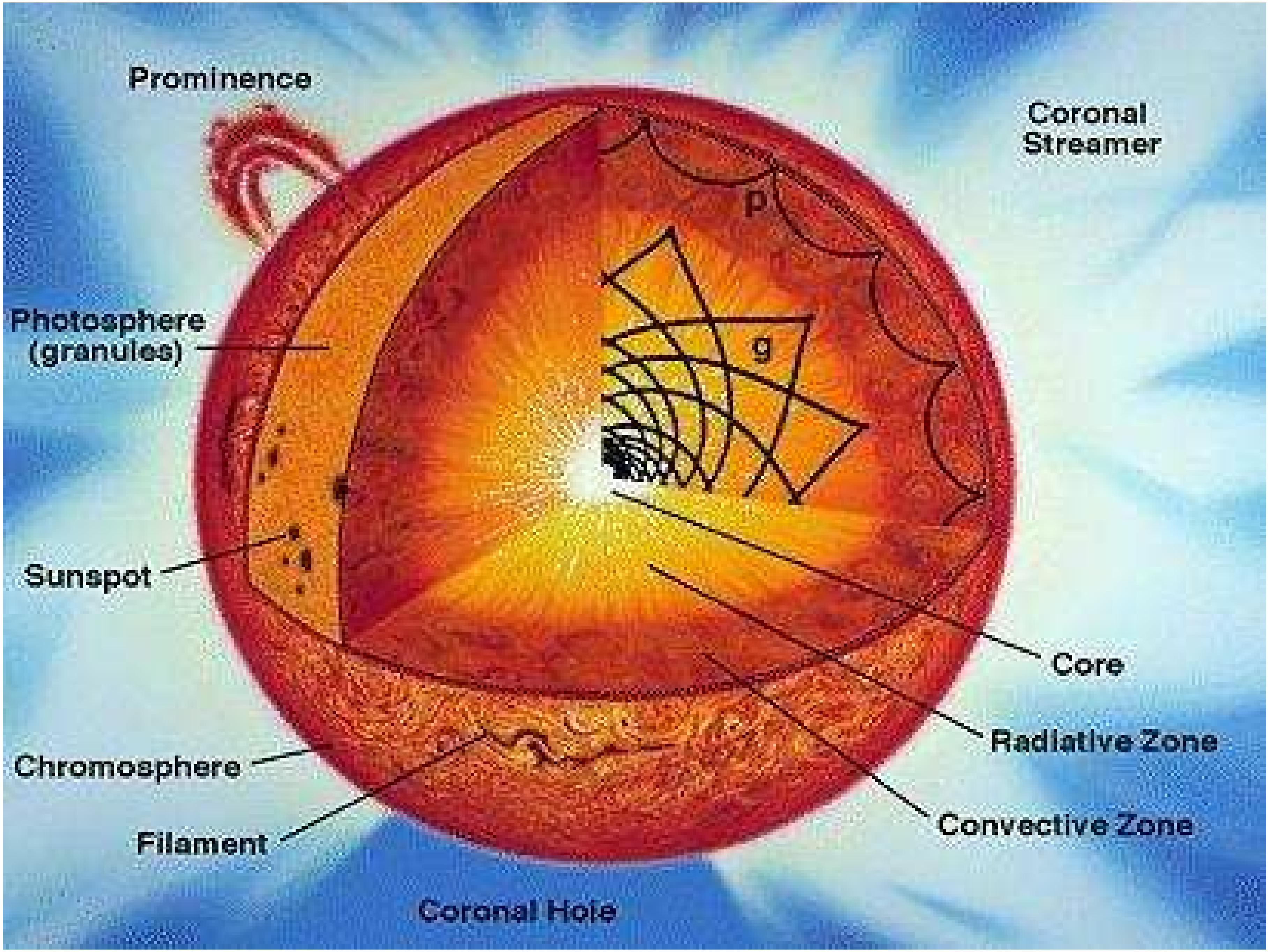
$$\mathcal{L}^{NC}_{\nu}(x) = -\frac{\sqrt{g^2+(g')^2}}{2} \sum_{l=e,\mu,\tau} \bar{\nu}_{l\text{L}}(x)\gamma_{\alpha}\,\nu_{l\text{L}}(x)\,Z^{\alpha}(x)\,,$$

Z^0–boson: $\quad M_{\text{Z}} = 92$ GeV, $g'(\cong 0.4)$ – $U(1)_{Y_W}$

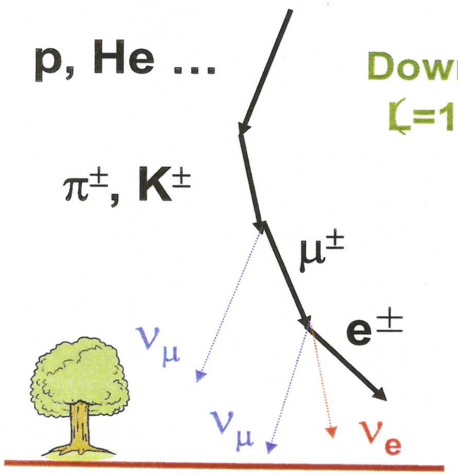
For example, $\mu^- - decay$, $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$:

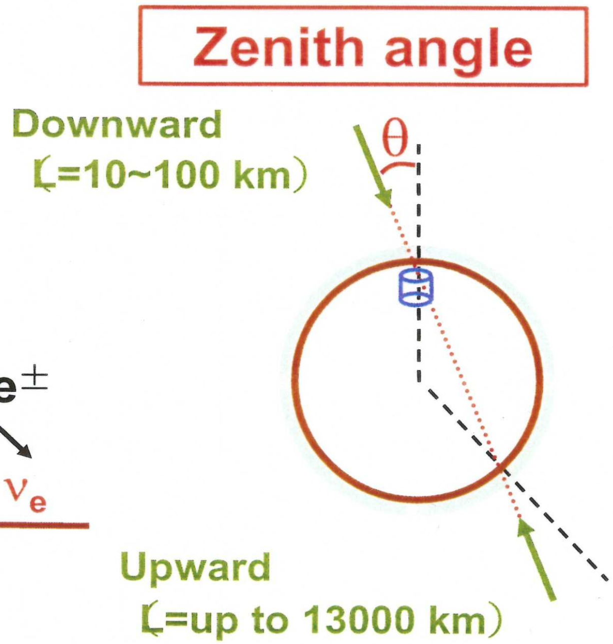
$\mu^- \rightarrow \nu_\mu +$ virtual $W^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$

We live in a sea of neutrinos.

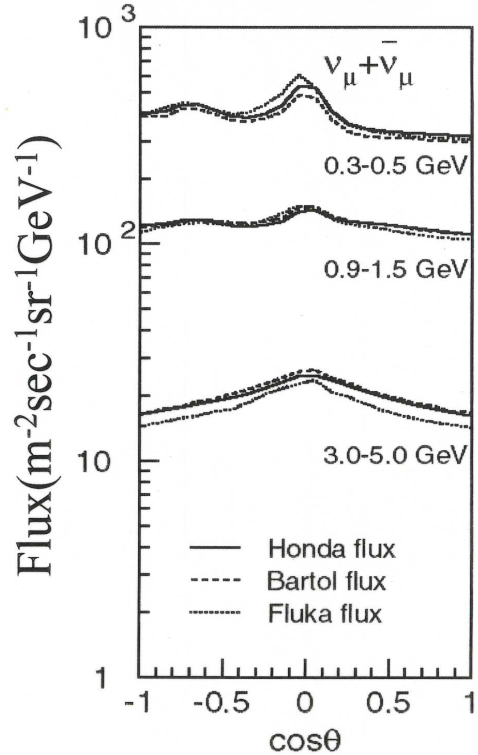

Prominence
Coronal Streamer
p
g
Photosphere (granules)
Sunspot
Core
Radiative Zone
Chromosphere
Convective Zone
Filament
Coronal Hole

Atmospheric neutrinos



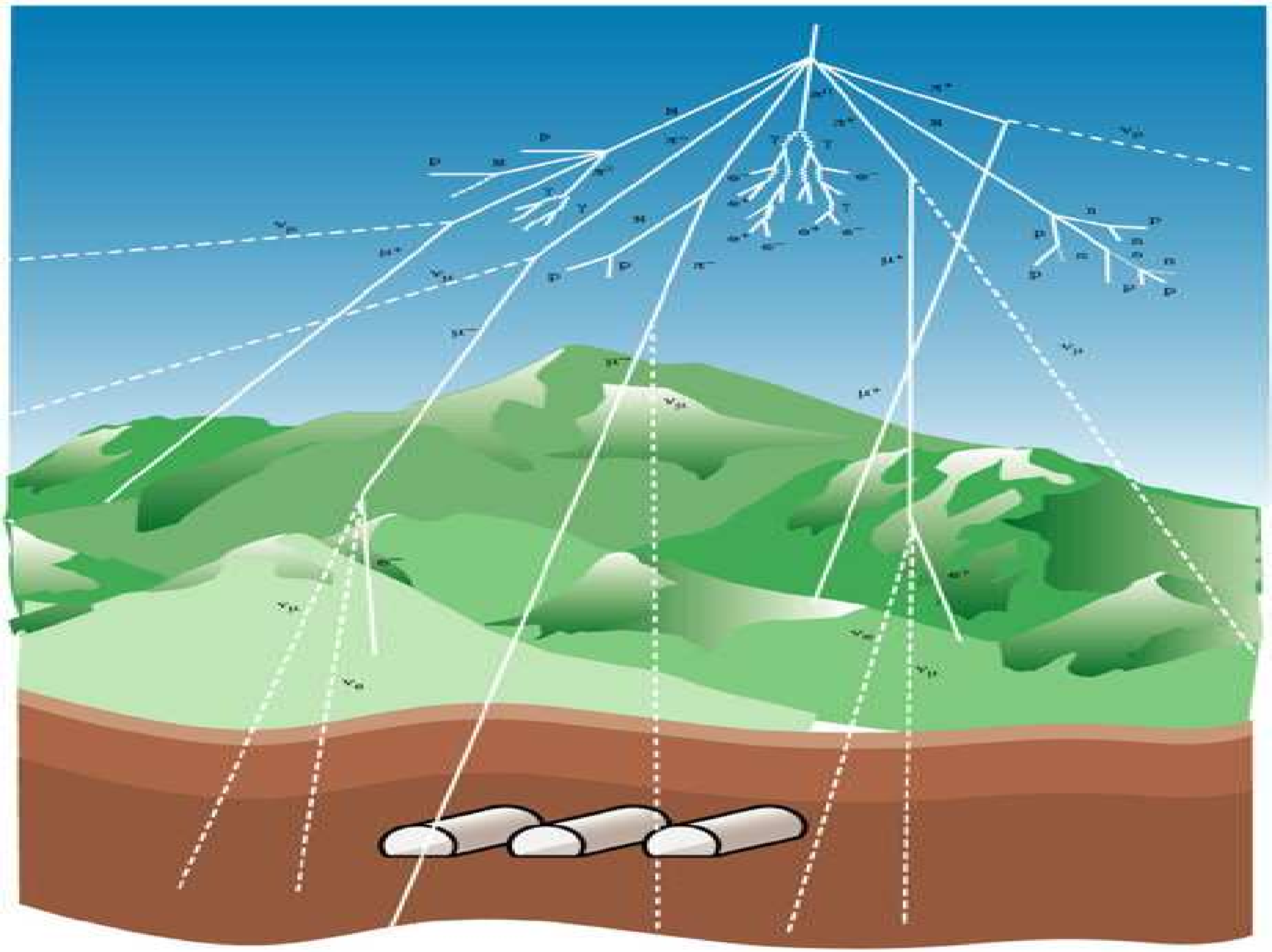


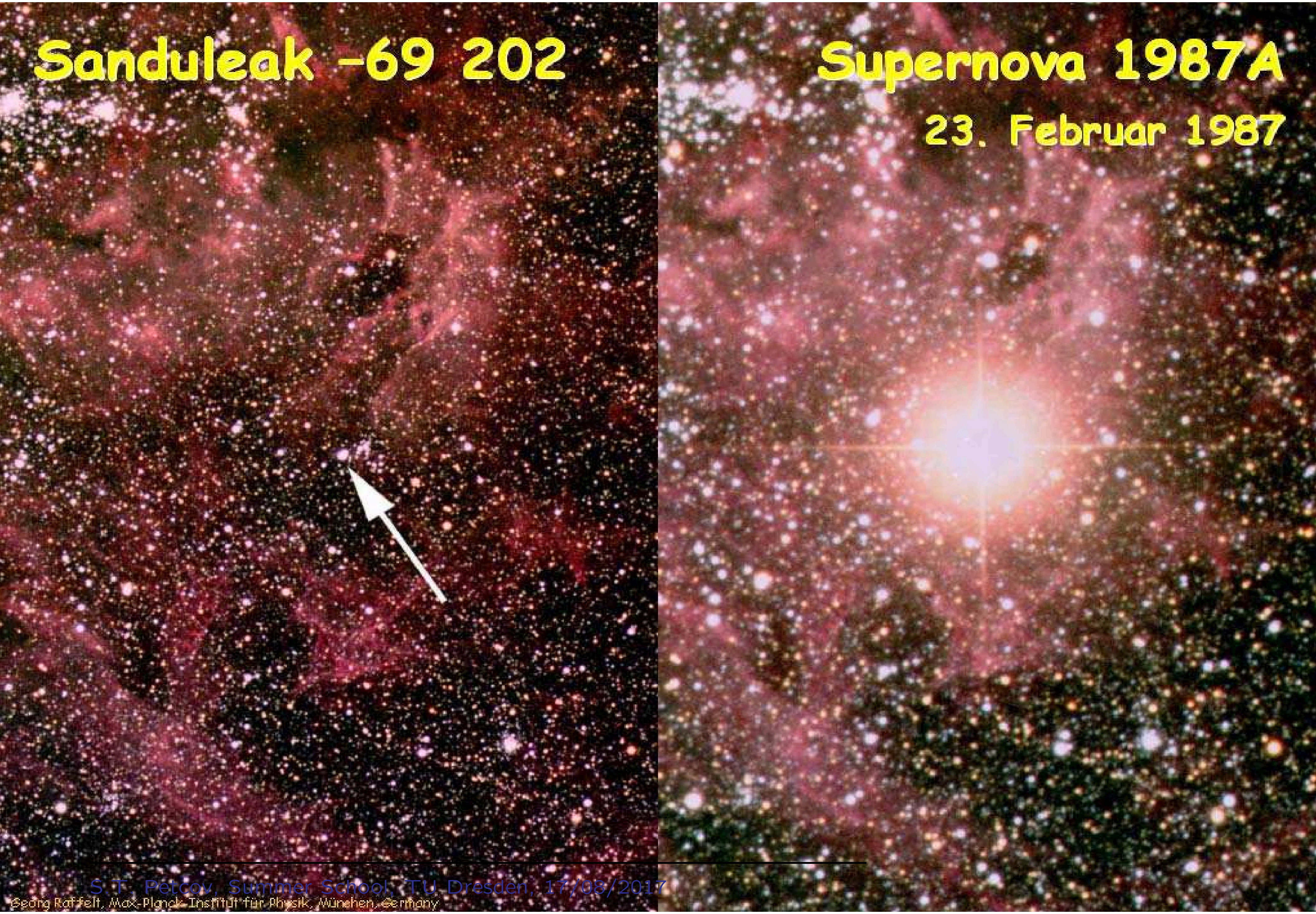
Zenith angle dist. of Atmospheric ν flux

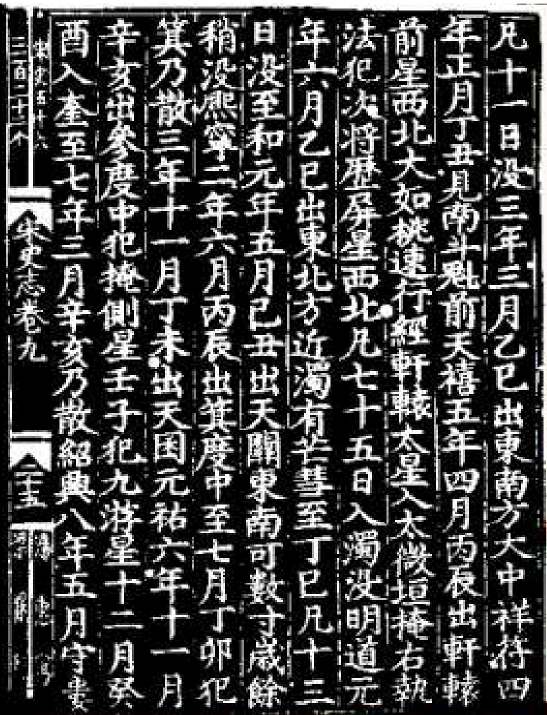


Eν > a few GeV

Up/Down Symmetry

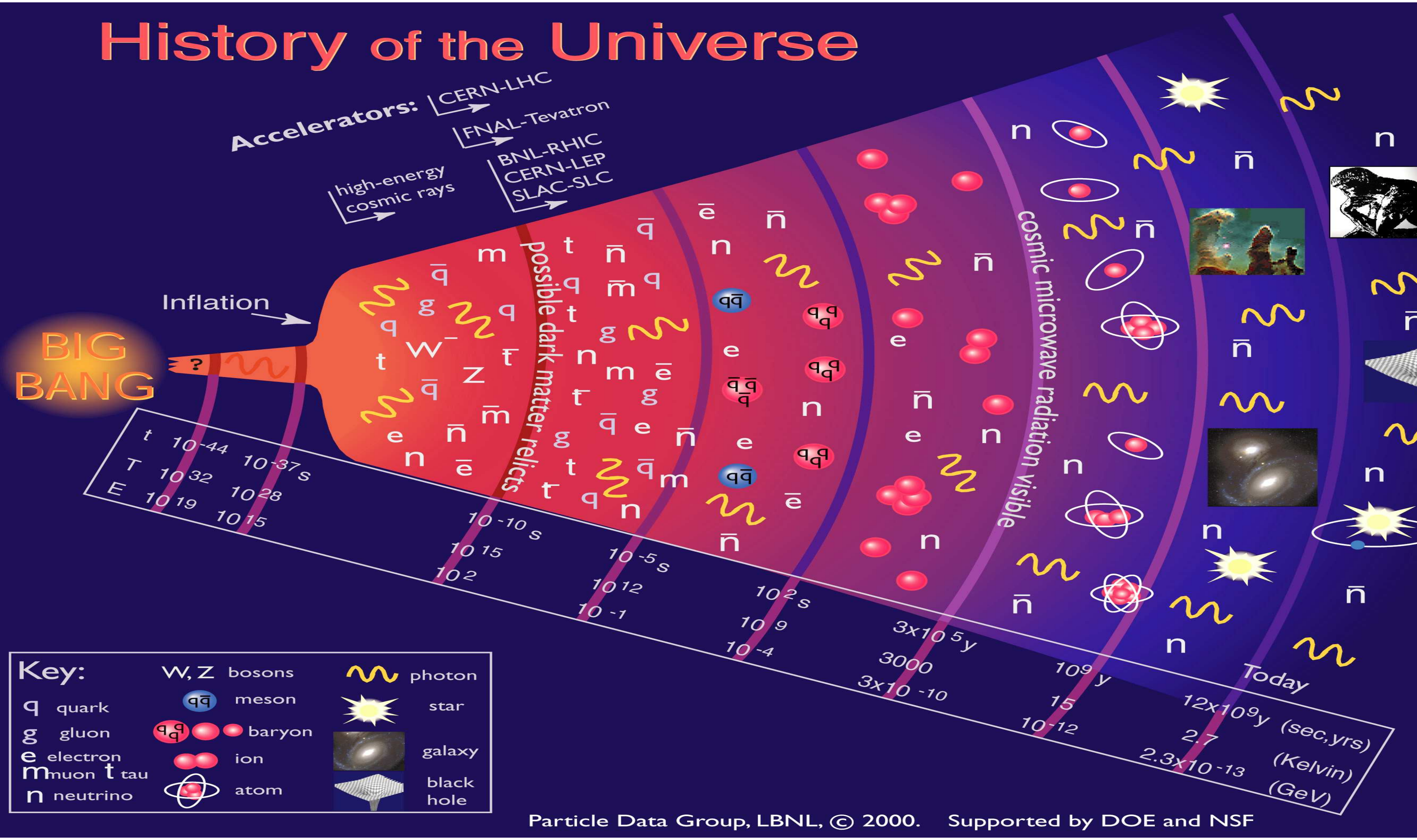



Sanduleak -69 202
Supernova 1987A
23. Februar 1987
Georg Raffelt, Max-Planck-Institut für Physik, München, Germany



凡十一日没三年三月乙巳出東南方大中祥符四
年正月丁丑見南斗魁前天禧五年四月丙辰出軒轅
前星西北大如桃速行經軒轅太星入太微垣掩右執
法犯次將歷屏星西北凡七十五日入濁没明道元
年六月乙巳出東北方近濁有芒彗至丁巳凡十三
日没至和元年五月己丑出天關東南可數寸歲餘
稍没熙寧二年六月丙辰出箕度中至七月丁卯犯
箕乃散三年十一月丁未出天囷元祐六年十一月
辛亥出參度中犯掩側星壬子犯九游星十二月癸
酉入奎至七年三月辛亥乃散紹興八年五月守婁

宋史志卷九



History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Solar Neutrinos: $\Phi_{\rm Earth} \cong 6.5 \times 10^{10} \frac{\nu_\odot}{\rm cm^2\,s}$, $\bar{E} \sim 1$ **MeV.**

SN Neutrinos: $\bar{E} \sim 10$ **MeV**

$\Phi_{\rm SN}$ **carries 99% of the released E** ($\sim 10^{53}$ **ergs).**

Relic Neutrinos (Cosmic Background ν**s):**

$t_{\rm U} \sim 1$ **sec,** $E \sim 10^{-4}$ **eV,** $n_{CB} \sim 330 \frac{\nu}{\rm cm^2}$.

Atmospheric Neutrinos, ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$, $E \sim 0.2 - 10$ **GeV.**

Reactor $\bar{\nu}_e$, $E \sim 3$ **MeV:**

NPS 1 GWT$_{\rm thermal}$: $\sim 10^{20}\ \frac{\bar{\nu}_e}{sec}$.

Accelerator ν_μ, $\bar{\nu}_\mu$, $E \sim 1 - 100$ **GeV.**

• Data (relativistic ν's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y_W}}$ Theory: ν_l, $\tilde{\nu}_l$ - $\nu_{lL}(x)$;

$\nu_{lL}(x)$ form $SU(2)_{\mathrm{L}}$ doublets with $l_L(x)$, $l = e, \mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} ; \quad l_R(x) - SU(2)_{\mathrm{L}} \text{ singlets}, \quad l = e, \mu, \tau \, .$$

• No (compelling) evidence for existence of (relativistic) ν's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)
If ν_R, $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l, $\tilde{\nu}_l$: ν_R, $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν's can play crucial role
i) in the generation of $m(\nu) \neq 0$,
ii) in understanding why $m(\nu) \ll m_l, m_q$,
iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = const.$, $l = e, \mu, \tau$;
$L \equiv L_e + L_\mu + L_\tau = const.$

There have been remarkable discoveries in neutrino physics in the last ~ 19 years.

Compellings Evidence for $\nu-$Oscillations

$-\nu_{\rm atm}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of $\mu-$like events**

Dominant $\nu_\mu \rightarrow \nu_\tau$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS, NOνA (accelerators ν_μ, $\bar{\nu}_\mu$): $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U_{lj}^{*} \, |\nu_j> \,, \quad \nu_j : \; m_j \neq 0 \,; \quad l = e, \mu, \tau \,; \quad n \geq 3 \,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj} \, \nu_{j\mathrm{L}}(x) \,, \quad \nu_{j\mathrm{L}}(x) : \; m_j \neq 0 \,; \quad l = e, \mu, \tau \,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

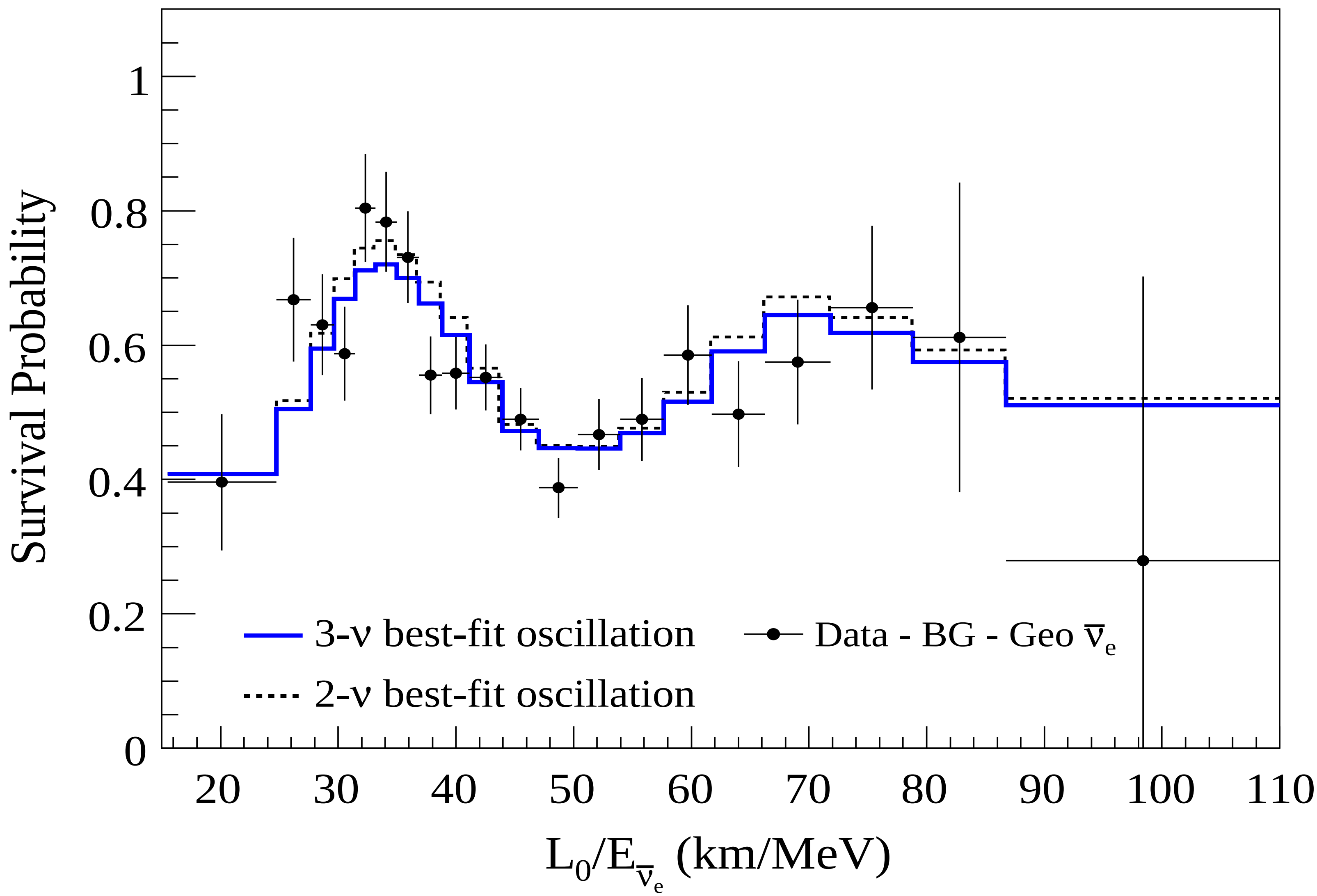
ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

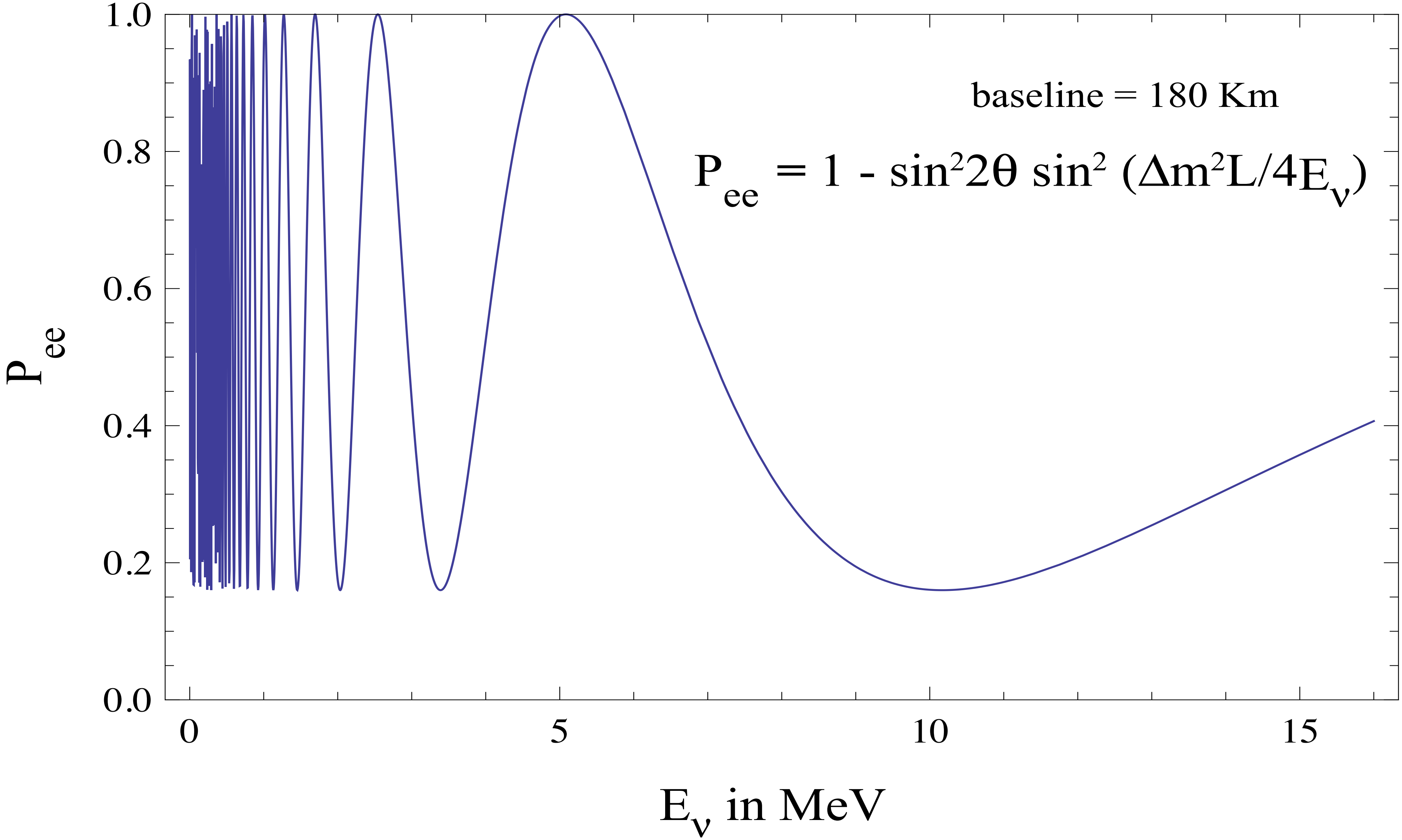
$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x)\gamma_\alpha\,(1-\gamma_5)\,\nu_{l\mathrm{L}}(x)\,W^{\alpha}(x) + \mathrm{h.c.}\,,$$

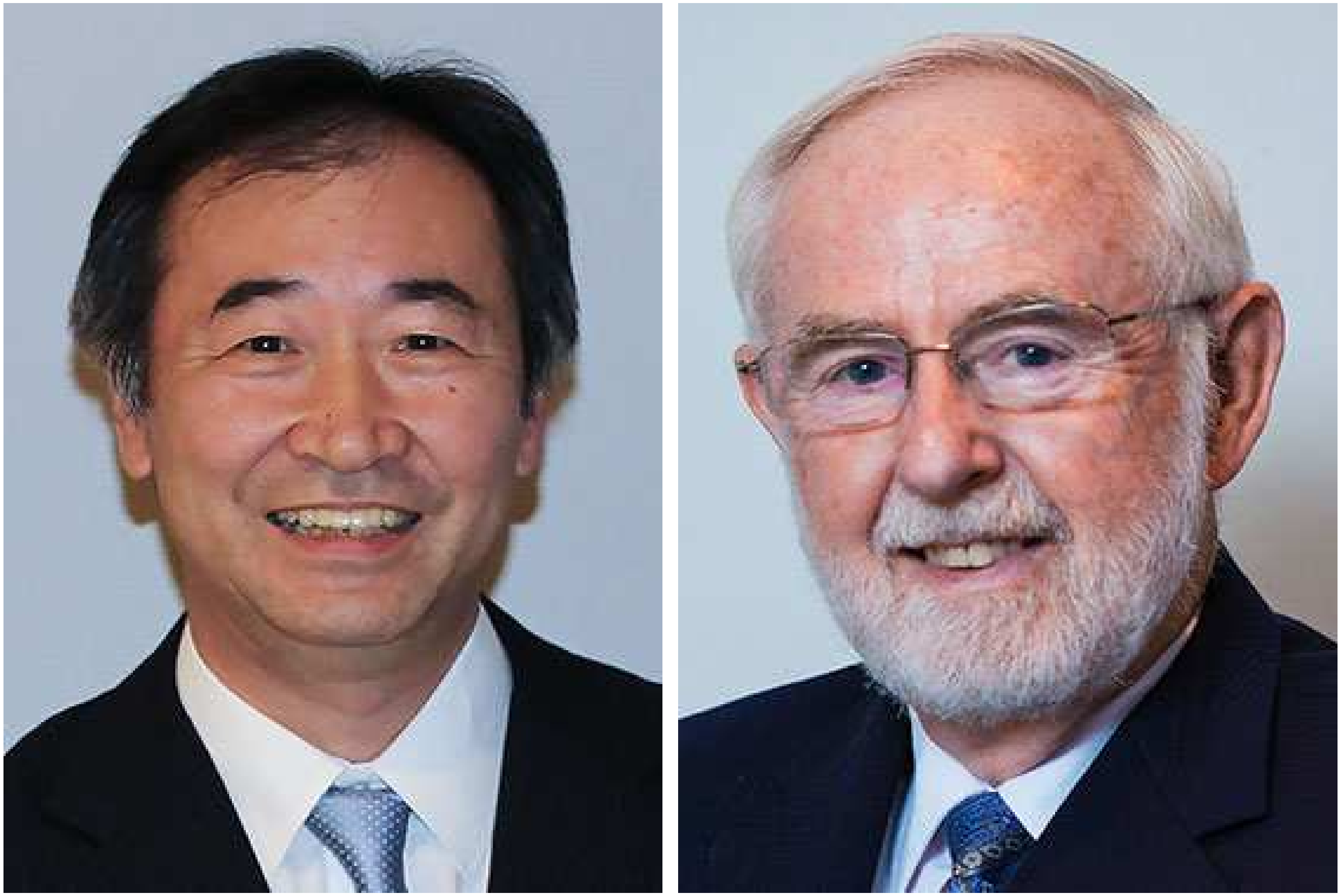
$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\,\nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\; m_j \neq 0\,; \quad l = e,\mu,\tau\,.$$



KamLAND: $L/E-$Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, E = (1.8 - 10) MeV)

$\bar{\nu}_e \rightarrow \bar{\nu}_e$





Dr. T. Kajita, Prof. A. McDonald, Nobel Prize for Physics winners, 2015

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Kam-Biu Luk and the Daya Bay Collaboration


Yifang Wang and the Daya Bay Collaboration

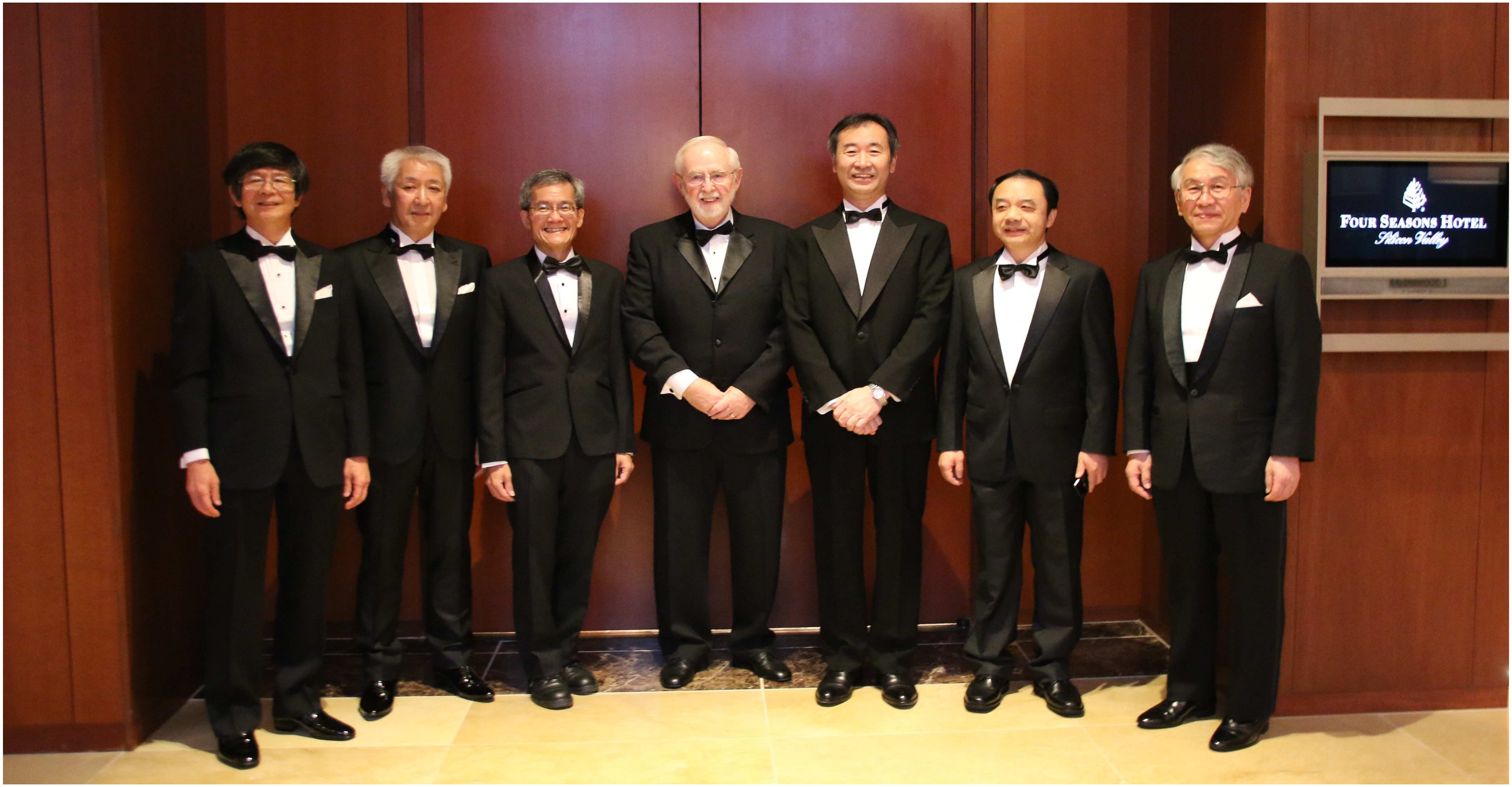

Koichiro Nishikawa and the K2K and T2K Collaboration


Atsuto Suzuki and the KamLAND Collaboration


Arthur B. McDonald and the SNO Collaboration


Takaaki Kajita and the Super K Collaboration


Yoichiro Suzuki and the Super K Collaboration


FOUR SEASONS HOTEL
Silicon Valley

The Pontecorvo Prize for 2016 (24/02/2017): Prof. Yifang Wang (Daya Bay), Prof. Soo-Bong Kim (RENO), Prof. K. Nishikawa (T2K)

"For their outstanding contributions to the study of the neutrino oscillation phenomenon and to the measurement of the Theta13 mixing angle in the Daya Bay, RENO and T2K experiments."

The relatively large value of the "reactor" angle $\theta_{13} \cong 0.15$ measured in the Daya Bay, RENO and Double Chooz experiments, indications for which were obtained first in the T2K experiment, opened up the possibility to search for CP violation effects in neutrino oscillations.

These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q,\ q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

These discoveries suggest the existence of New Physics beyond that of the ST.

The New Physics can manifest itself (can have a variety of different "flavours"):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...).

- In the Majorana nature of massive neutrinos.

- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.

- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j, doubly charged scalars,...

- In the existence of LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.

- In the existence of new (FChNC, FCFNSNC) neutrino interactions.

- In the existence of "unknown unknowns"...

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,...} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"); tests (STEREO, SOX, CeLAND, DANS, ICARUS (at Fermilab), ... under way).

ii) $M_{4,5,...} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
$M_{4,5,...} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

centerline Majorana Neutrinos (Particles)

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x),\quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\mathrm{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)-$Dirac, $\chi(x)-$Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

Majorana propagators:

$$|\Delta L| = 2: \quad (\mathrm{A,Z}) \rightarrow (\mathrm{A,Z}+2) + e^- + e^-, \quad (\beta\beta)_{0\nu} - decay.$$

Leptogenesis scenario of generation of BAU.

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}\,,$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,

- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;

- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.37 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ),

- $|\Delta m^2_{31(32)}| \cong 2.53$ (2.43) [2.56 (2.54)] $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.569) [0.425 (0.589)], NO (IO) ,

- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0214$ (0.0218) [0.0215 (0.0216)], Capozzi et al. NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1601.07777 [arXiv:1703.04471].

- $1\sigma(\Delta m^2_{21}) = (2.6)[2.3]\%$, $1\sigma(\sin^2\theta_{12}) = (5.4)[5.4]\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = (2.6)[1.6]\%$, $1\sigma(\sin^2\theta_{23}) = (9.6)[9.6]\%$;
- $1\sigma(\sin^2\theta_{13}) = (8.5)[4.0]\%$;

- $3\sigma(\Delta m^2_{21}):\ (6.99 - 8.18) \times 10^{-5}\ \text{eV}^2$; $3\sigma(\sin^2\theta_{12}):\ (0.259 - 0.359)$;
$(3\sigma(\Delta m^2_{21}):\ (6.93 - 7.97) \times 10^{-5}\ \text{eV}^2$; $3\sigma(\sin^2\theta_{12}):\ (0.250 - 0.354);)$
$[3\sigma(\Delta m^2_{21}):\ (6.93 - 7.96) \times 10^{-5}\ \text{eV}^2$; $3\sigma(\sin^2\theta_{12}):\ (0.250 - 0.354)]$.

- $3\sigma(|\Delta m^2_{31(23)}|):\ 2.27(2.23) - 2.65(2.60) \times 10^{-3}\ \text{eV}^2$;
$(2.40(2.30) - 2.66(2.57) \times 10^{-3}\ \text{eV}^2)$;
$[2.45(2.42) - 2.69(2.62) \times 10^{-3}\ \text{eV}^2]$;
$3\sigma(\sin^2\theta_{23}):\ 0.374(0.380) - 0.628(0.641)$;
$(3\sigma(\sin^2\theta_{23}):\ 0.379(0.383) - 0.616(0.637))$
$[3\sigma(\sin^2\theta_{23}):\ 0.381(0.384) - 0.615(0.636)]$

- $3\sigma(\sin^2\theta_{13}):\ 0.0176(0.0178) - 0.0296(0.0298)$
$(3\sigma(\sin^2\theta_{13}):\ 0.0185(0.0186) - 0.0246(0.0248))$
$[3\sigma(\sin^2\theta_{13}):\ 0.0190(0.0190) - 0.0240(0.0242)]$

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)
(F. Capozzi et al. (Bari Group), arXiv:1601.07777)
[F. Capozzi et al. (Bari Group), arXiv : 1703.04471]

Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e> \;\; = |\nu_1> \cos\theta + |\nu_2> \sin\theta,$$

$$|\nu_{\mu(\tau)}> \;\; = -|\nu_1> \sin\theta + |\nu_2> \cos\theta; \qquad \nu_{1,2}: \quad m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e>_t \;\; = e^{-iE_1 t}|\nu_1> \cos\theta + e^{-iE_2 t}|\nu_2> \sin\theta, \;\; E_{1,2} = \sqrt{p^2 + m_{1,}^2}$$

$$A(\nu_e \to \nu_\mu; t) = <\nu_\mu|\nu_e>_t = \frac{1}{2}\sin 2\theta \; (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta \; (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \to \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \to \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L$, $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi\frac{L}{L_{osc}^{vac}},\;\; L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta\;\left(1 - \cos 2\pi\frac{L}{L_{osc}^{vac}}\right),\;\; L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$L_{osc}^{vac} \cong 2.48\;m\;\frac{E[MeV]}{\Delta m^2[eV^2]}$$

Effects of oscillations observable if

$\sin^2 2\theta - sufficiently\ large,\quad L \gtrsim L_{osc}^{vac}$

$E \cong 1$ GeV, $\Delta m^2[eV^2] \cong 2.5 \times 10^{-3}$: $L_{osc}^{vac} \cong 1000$ km

$E \cong 3$ MeV, $\Delta m^2[eV^2] \cong 7.5 \times 10^{-5}$: $L_{osc}^{vac} \cong 100$ km

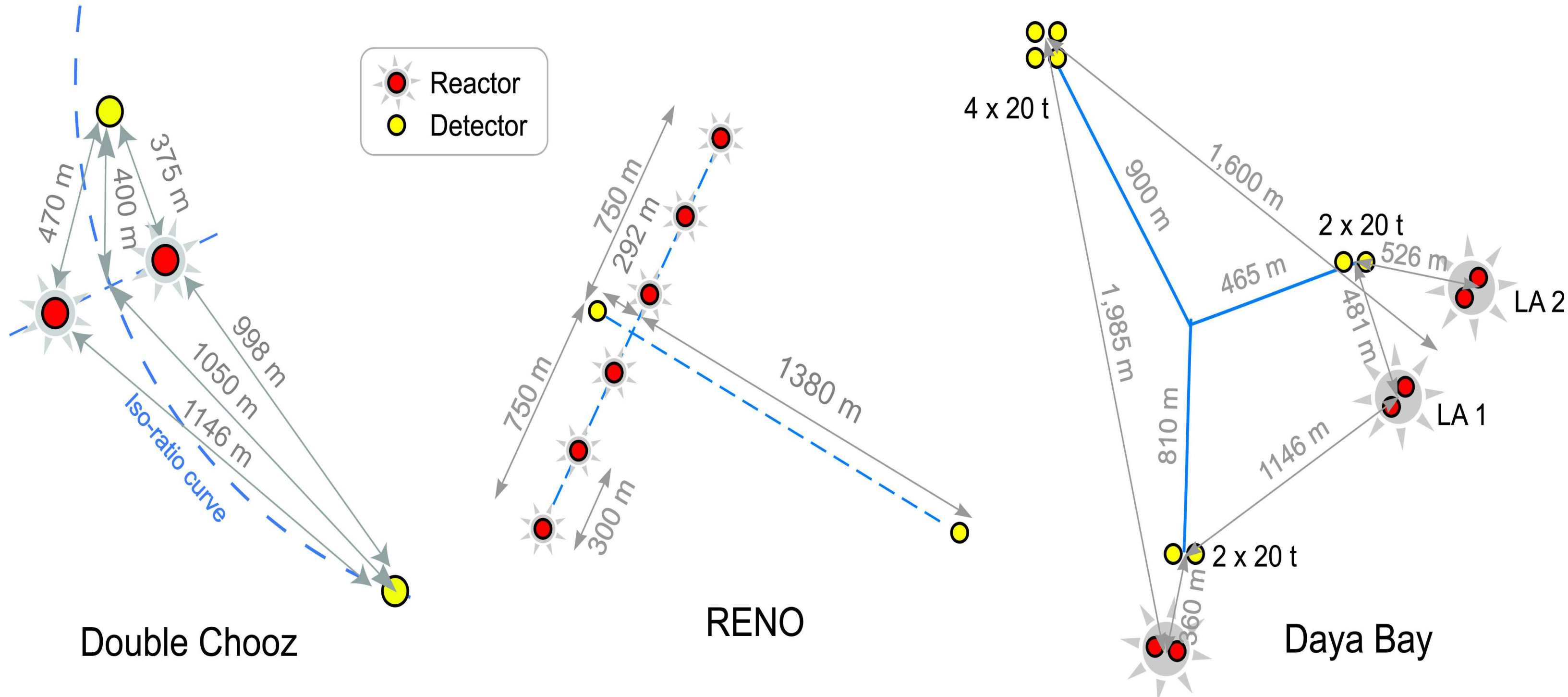
$E \cong 3$ MeV, $\Delta m^2[eV^2] \cong 2.5 \times 10^{-3}$: $L_{osc}^{vac} \cong 3$ km

Two basic parameters: $\sin^2 2\theta$, Δm^2

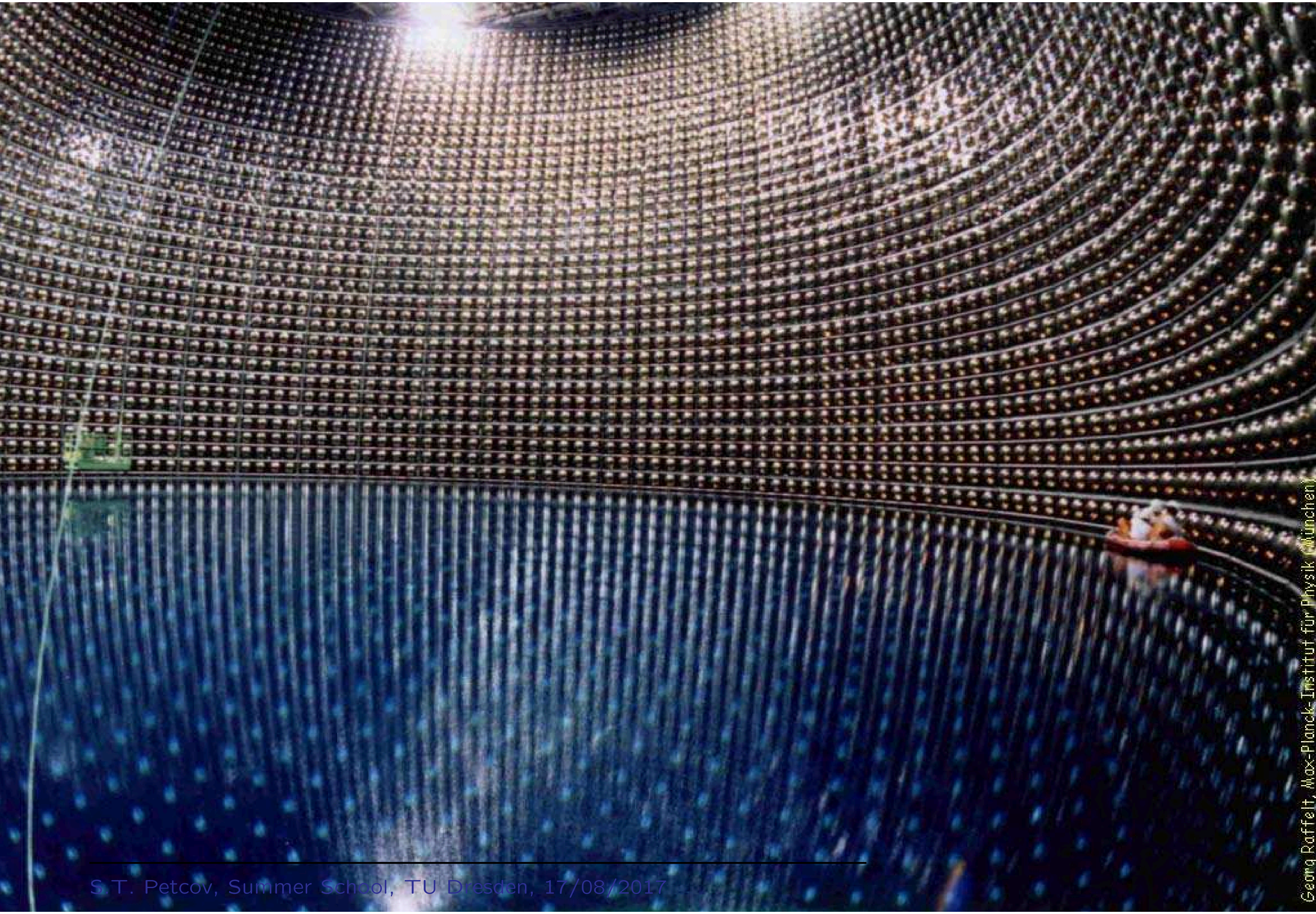
Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K, T2HK


Ontario
NOνA
Soudan
Minnesota
Wisconsin
735 kilometers
Iowa
Fermilab
Illinois



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]


Georg Raffelt, Max-Planck-Institut für Physik (München)

• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $\quad m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = -\,\Delta m^2_{32}(IO), \quad \Delta m^2_{32}(NO) = -\,\Delta m^2_{31}(IO)$$

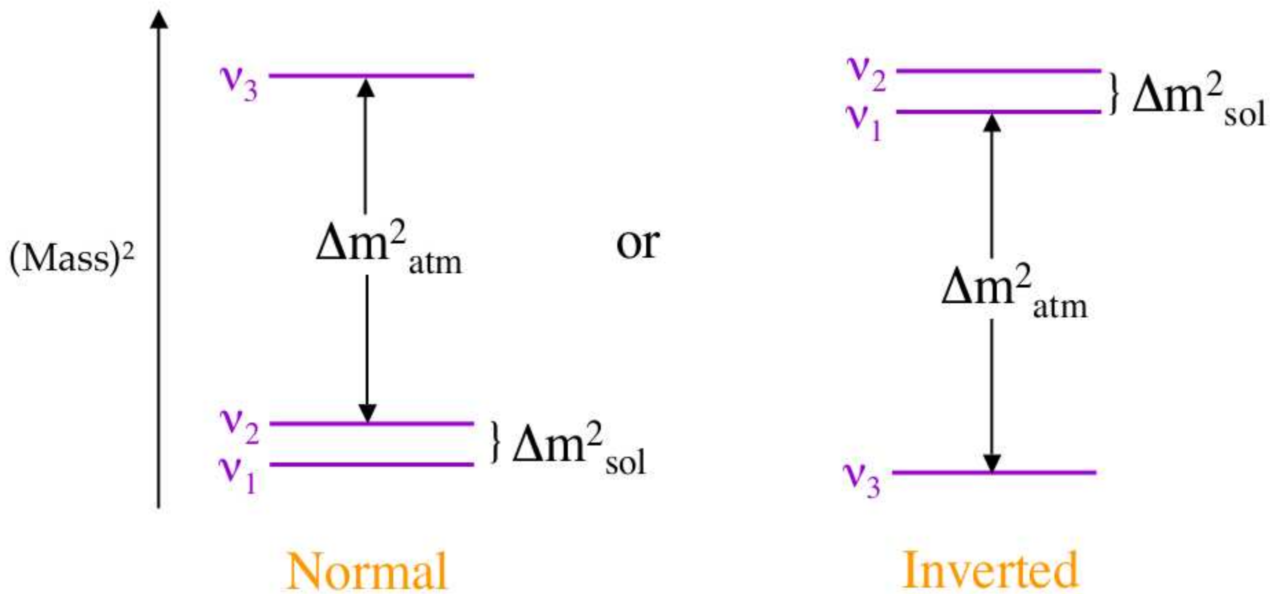
$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> |\Delta m^2_{31(32)}|, \ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

• $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $\ m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;
• $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $\ m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;

The (Mass)^2 Spectrum

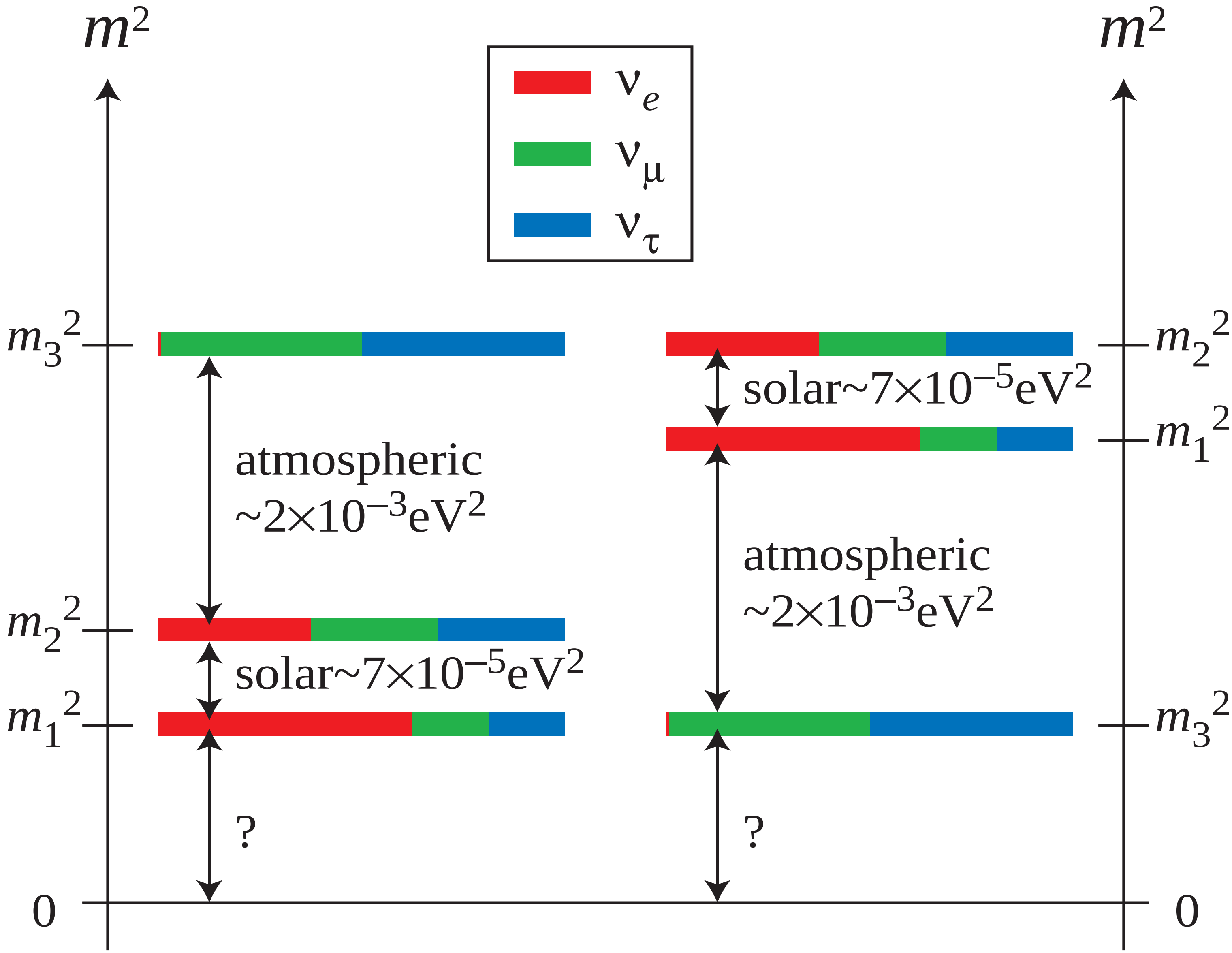


$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

3

Due to B. Kayser



S. King, Ch. Luhn, 2013

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13}\sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

Current data:
$0.026|\sin\delta| \lesssim |J_{CP}| \lesssim 0.036|\sin\delta|$ (3σ; can be relatively large!);
θ_{ij} b.f.v. $+\ \delta \cong 3\pi/2$: $J_{CP} \cong -0.032$.

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

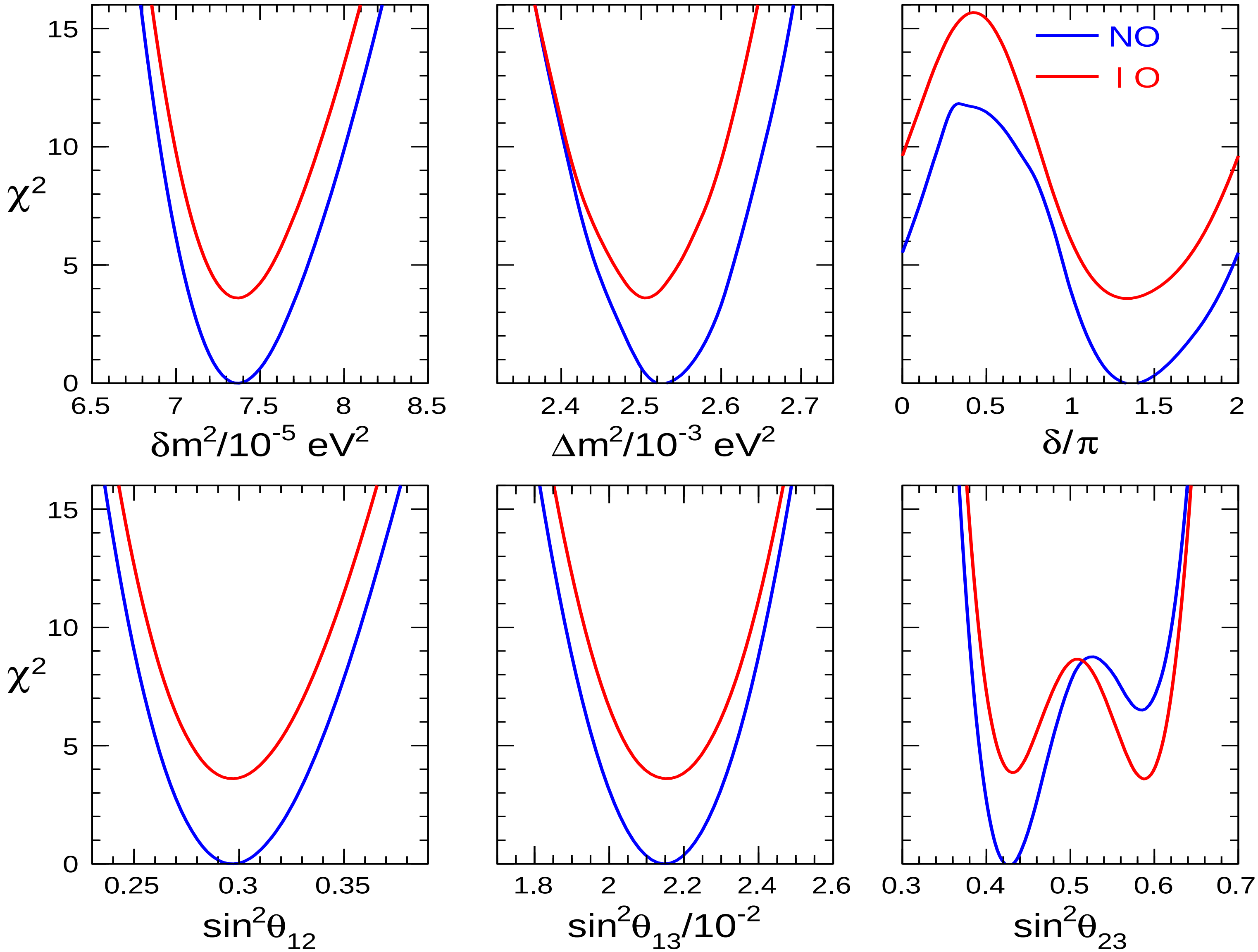
S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left|< m >\right|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

$$\delta \cong 3\pi/2?$$

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left\{U_{e1}\,U_{\mu 2}\,U^{*}_{e2}\,U^{*}_{\mu 1}\right\} \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

Oscillation parameters



F. Capozzi, E. Lisi *et al.*, arXiv:1703.04471

- **Best fit value:** $\delta = 1.38\ (1.31)\pi$;
- $\delta = 0$ **or** 2π **are disfavored at** 2.4σ **(**3.2σ**);**
- $\delta = \pi$ **is disfavored at** 2.0σ **(**2.5σ**);**
- $\delta = \pi/2$ **is strongly disfavored at** 3.4σ **(**3.9σ**).**
- **At** 3σ**:** δ/π **is found to lie in** $(0.00 - 0.17(0.16)) \oplus (0.76(0.69) - 2.00)$**).**

F. Capozzi, E. Lisi *et al.*, arXiv:1703.04471

Large $\sin\theta_{13} \cong 0.15 + \delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.032$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

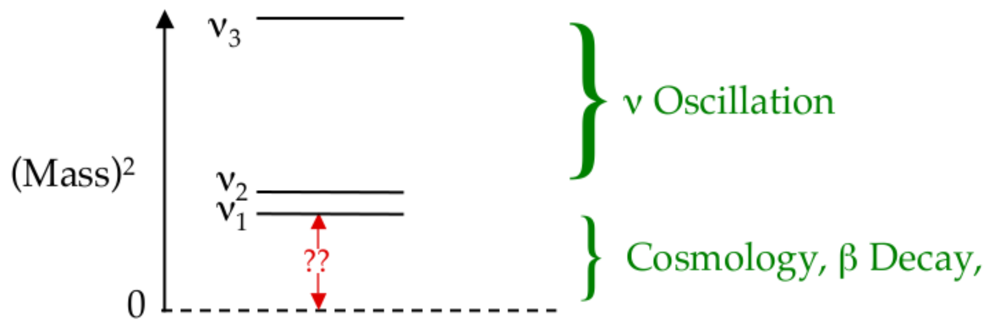
If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU (in FLG with hierarchical N_j) is

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

$$\text{Oscillation Data} \Rightarrow \sqrt{\Delta m^2_{atm}} < \text{Mass[Heaviest } \nu_i]$$

4

Due to B. Kayser

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on ${}^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
Scale 1: 19,500,000
Lambert Conformal Conic Projection,
standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: using their latest (2016) data on CMB T power spectrum anisotropies, polarisation, grav. lensing effects, the low l CMB polarisation spectrum data ("low P" data) and adding data on the baryon acoustic oscillations (BAO) and using ΛCDM (6 parameter) model + assuming 3 light massive neutrinos, the Planck collaboration published in 2016 the following limit:

$$\sum_j m_j \equiv \Sigma < 0.170\ \text{eV} \quad (95\%\ \text{C.L.})$$

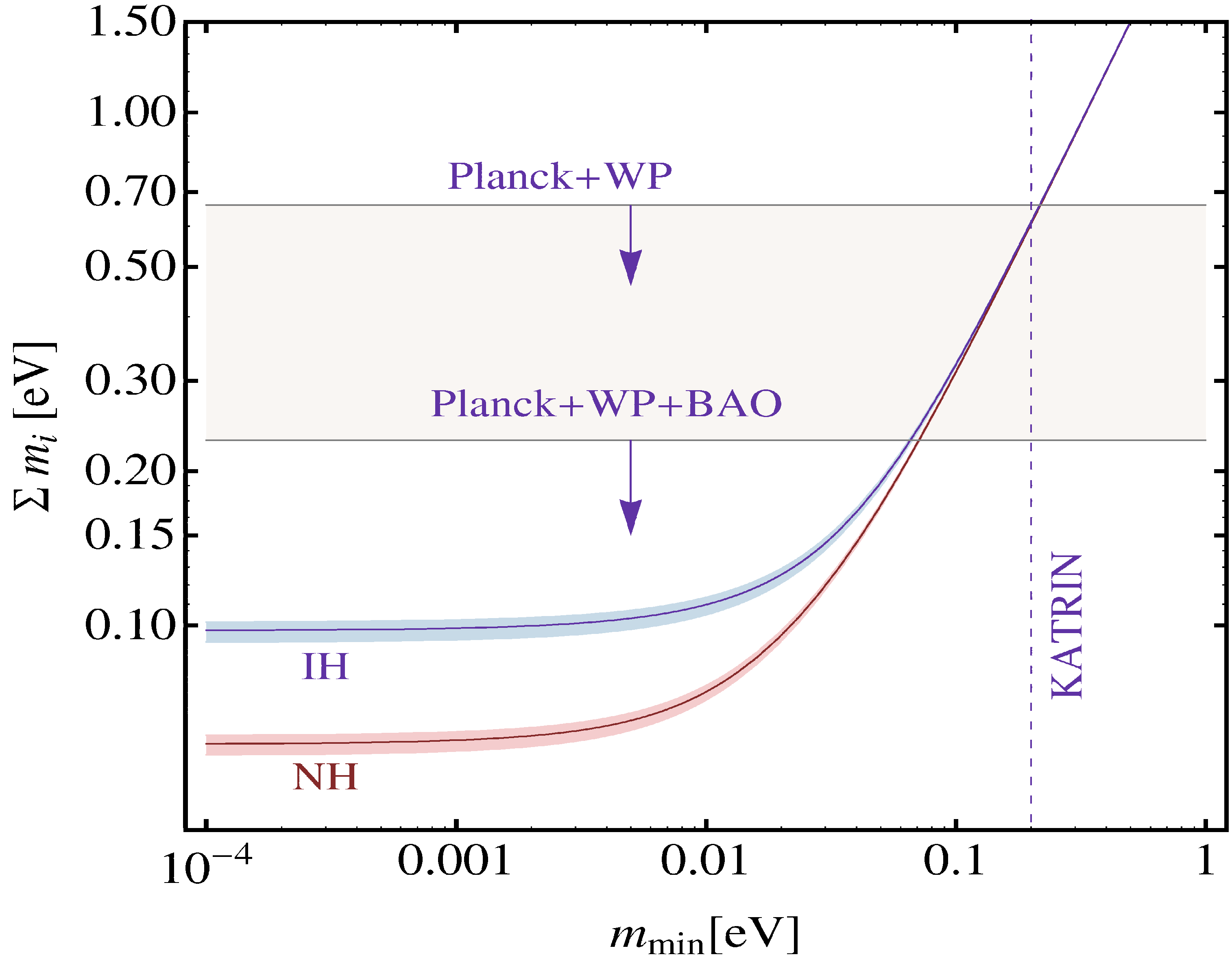
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04)\ \text{eV}.$$

NH: $\sum_j m_j \leq 0.061$ eV (3σ);

IH: $\sum_j m_j \geq 0.098$ eV (3σ).

Mass and Hierarchy from Cosmology



Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of $\text{sgn}(\Delta m^2_{\text{atm}})$, type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or $\min(m_j)$.

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, θ_{12}, Δm^2_{atm}, θ_{23}, θ_{13}

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos ($(\beta\beta)_{0\nu}$-decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+, SuperNEMO, MAJORANA, AMORE,...).

- determination of the status of the CP symmetry in the lepton sector (T2K, NOνA; DUNE, T2HK)

- determination of the neutrino mass ordering (JUNO, RENO50; ORCA, PINGU (IceCube), HK, INO; T2K + NOνA; DUNE (future); + T2HKK (future)) ;

- determination of the absolute neutrino mass scale, or min(m_j) (KATRIN, new ideas; cosmology);

The program of research extends beyond 2030.

The Nature of Massive Neutrinos I:

Majorana versus Dirac Massive Neutrinos

Majorana Fermions?

Electrically neutral particles can be Majorana fermions

• **Light neutrinos** ν_j, $m_j \neq 0$, $j = 1,2,3$

• **Heavy (RH seesaw) neutrinos** N_k, $M_k \gtrsim 100$ **GeV**, $k = 1,2,...$

• **Neutron + anti-neutron cam be linear combinations of two Majorana fermions** $n_{1,2}$ ($n\bar{n}$ **oscillations):**

$$|n> \;= |n_1> \cos\theta^n + |n_2> \sin\theta^n,$$
$$|\bar{n}> \;= -|n_1> \sin\theta^n + |n_2> \cos\theta^n; \quad n_{1,2}:\; \tilde{M}_{1,2} \neq 0$$

• **Minimal SUSY extension of the ST: the superpartners of** γ, $Z-$**boson, neutral Higgses** $H^0_{1,2}$, **i.e.,**

the 4 "neutralinos" $\chi_{1,2,3,4}$, $+$ the super-partners of the 8 gluon fields, i.e., the 8 "gluinos" λ_j, $j = 1, 2, ..., 8$.

Majorana Neutrinos (Fermions)

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ **- 4 component (spin 1/2), complex,** m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$**.**

Implications:

$$U(1): \ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \ \text{impossible}$$

– $\chi_k(x)$ **cannot absorb phases.**
– $Q_{U(1)} = 0: \ Q_{\mathrm{el}} = 0, \ L_l = 0, \ L = 0, \ldots$
– $\chi_k(x)$**: 2 spin states of a spin 1/2 absolutely neutral particle**
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$**–Dirac,** $\chi(x)$**–Majorana**

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0\ ,\quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0\ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta}\ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi\ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP}\ \chi(x)\ U^{-1}_{CP} = \eta_{CP}\ \gamma_0\ \chi(x'),\quad \eta_{CP} = \pm i\ .$$

Special Properties of the Currents of $\chi(x)$−Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0);$$

Has imortant implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \ d_\chi = 0 \ (d_\Psi \neq 0, \ if\ CP\ is\ not\ conserved)$$

$\chi(x)$ **cannot couple to a real photon (field).**

$\chi(x)$ **couples to a virtual photon through an anapole moment:**

$$(g_{\alpha\beta}\, q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5\ F_a(q^2)\,.$$

Properties of Currents Formed by $\chi_1(x)$, $\chi_2(x)$: $\chi_2 \to \chi_1 + \gamma$, $\chi_2 \to \chi_1\chi_1\chi_1$, etc.

$$\bar{\chi_1}(x)\gamma_\alpha(v - a\gamma_5)\chi_2(x) \quad (\bar{\chi_1}(x)\gamma^\alpha(1-\gamma_5)\chi_1(x)\,,...):$$

- **CP is conserved:** $v = 0$ $(a = 0)$ **if** $\eta_{1CP} = \eta_{2CP}$ $(\eta_{1CP} = -\eta_{2CP})$
- **CP is not conserved:** $v \neq 0\,,\ a \neq 0$

(Has imortant implications also, e.g. for SUSY neutralino phenomenology: $e^+ + e^- \to \chi_1 + \chi_2\,,\ \ \chi_2 \to \chi_1 + l^+ + l^-$, etc.)

$$\bar{\chi_1}(x)\sigma_{\alpha\beta}(\mu_{12} - d_{12}\gamma_5)\chi_2(x) \quad (F^{\alpha\beta}(x)):$$

- **CP is conserved:** $\mu_{12} = 0$ $(d_{12} = 0)$ **if** $\eta_{1CP} = \eta_{2CP}$ $(\eta_{1CP} = -\eta_{2CP})$
- **CP is not conserved:** $\mu_{12} \neq 0\,,\ d_{12} \neq 0$

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,, \quad m_1 \neq m_2 > 0\,, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - **Majorana, maximal mixing .**

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L}\cos\theta_C + \Psi_{2L}\sin\theta_C\,,$$

$$\nu_{\mu L}(x) = -\Psi_{1L}\sin\theta_C + \Psi_{2L}\cos\theta_C\,,$$

$\Psi_{1,2}$ - **Dirac (composite),** θ_C- **the Cabbibo angle .**

The Nature of Massive Neutrinos II:

Origins of Dirac and Majorana Massive Neutrinos

• Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L.

• Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}^{\nu}_{m}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}^{\nu}_{m}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}^{\nu}_{m}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}^{\nu}_{m}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires $\nu_R(x)$ - $SU(2)_L$ singlet RH ν fields

$$\mathcal{L}^{\nu}_{D}(x) = -\ \overline{\nu_{l'R}}(x)\ M_{Dl'l}\ \nu_{lL}(x) + h.c.\ ,\ \ M_D - complex$$

- $\mathcal{L}^{\nu}_{D}(x)$ conserves L: $L = const.$

$$M_D = V M_D^{diag} W^{\dagger}\,,\ \ V, U - \text{unitary}\ (bi - unitary\ transformation)\,,\ \ W \equiv U_{\text{PMNS}}$$

- ST + 3 $\nu_R(x)$ - RH ν fields: $n = 3$

$$\begin{aligned} \mathcal{L}_{\mathrm{Y}}(x) &= Y^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^{T}(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,, \\ M_D &= \frac{v}{\sqrt{2}} Y^{\nu}\,,\quad v = 246\ \text{GeV}\,. \end{aligned}$$

No explanation why $m(\nu_j) <<< m_l, m_q$.

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes $\mu^+ \to e^+ + \gamma$ decay, $\mu^- \to e^- + e^+ + e^-$ decay, $\tau^- \to e^- + \gamma$ decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \to e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej}\, U^*_{\mu j}\, \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$

$$M_W \cong 80 \text{ GeV}, \text{ the } W^\pm - \text{mass}$$

S.T.P., 1976

Current limit: $BR(\mu \to e + \gamma) < 5.7 \times 10^{-13}$

"New Physics": $\nu_l \to \nu_{l'}$, $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$ oscillations.

• **Majorana ν_j: Majorana Mass Term for $\nu_{lL}(x)$, $l = e, \mu, \tau$**

Introduce $\nu^C_{lR}(x) \equiv C\ (\overline{\nu_{lL}}(x))^{\mathsf{T}}$, $C^{-1}\gamma_\alpha C = -\gamma^{\mathsf{T}}_\alpha$:
$\overline{\nu^C_{l'R}}(x)\,\nu_{lL}(x) = -\,\nu^{\mathsf{T}}_{l'L}(x)\ C^{-1}\ \nu_{lL}(x)$ - **invariant under proper Lorentz transformations.**

$$\mathcal{L}^{\nu}_M(x) = \frac{1}{2}\,\nu^{\mathsf{T}}_{l'L}(x)\ C^{-1}\ M_{l'l}\ \nu_{lL}(x) + h.c.$$

• If $M_{l'l} \neq 0$, $L_l \neq const.$, $L \neq const.$; $P \neq const.$, $C \neq const.$

• $\nu_{lL}(x)-$fermions: $M = M^{\mathsf{T}}$, complex.

$$M^{diag} = U^{\mathsf{T}} M U\,,\ U - \text{unitary (congruent transformation)};\ U \equiv U_{\text{PMNS}}$$

$$\nu_j \equiv \chi_j(x) = U^{\dagger}_{jl}\nu_{lL}(x) + U^{*}_{jl}\nu^{c}_{lR} = C\ (\overline{\chi_j}(x))^{\mathsf{T}}\,,\quad m_j \neq 0\,,\ j = 1,2,3$$

CP-invariance: $M^* = M$, M - **real, symmetric.**

$M^{diag} = (m'_1, m'_2, m'_3)$: $m'_j = \rho_j m_j$, $m_j \geq 0$, $\rho_j = \pm 1$; $|n_+ - n_-|$-invariant of M.

χ_j: $m_j \geq 0$: $\eta_{CP}(\chi_j) = i\rho_j$

$\mathcal{L}^{\nu}_M(x)$ **not possible in the ST: requires New Physics Beyond the ST**

$(\beta\beta)_{0\nu}$**-decay is allowed; typically also** $BR(\mu \to e + \gamma)$, $BR(\mu \to 3e)$, $CR(\mu^- + \mathcal{N} \to e^- + \mathcal{N})$ **can be "large", i.e., in the range of sensitivity of ongoing (MEG) and future planned (COMET, Mu2e, etc.) experiments.**

• **Majorana ν_j: Dirac+Majorana Mass Term; requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$:**

$\mathcal{L}^{\nu}_{D+M}(x) =$

$$= - \overline{\nu_{l'R}}(x)\, M_{Dl'l}\, \nu_{lL}(x) + \frac{1}{2}\, \nu^{\mathsf{T}}_{l'L}(x)\, C^{-1}\, M^{LL}_{l'l}\, \nu_{lL}(x) + \frac{1}{2}\, \nu^{\mathsf{T}}_{l'R}(x)\, C^{-1}\, (M^{RR})^{\dagger}_{l'l}\, \nu_{lR}(x) + h.c.\,,$$

$$M \;=\; \begin{pmatrix} M^{LL} & M_D \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad ((M^{LL})^T = M^{LL},\ (M^{RR})^T = M^{RR})$$

• If $M_{Dl'l} \neq 0$ and $M^{LL}_{l'l} \neq 0$ and/or $M^{RR}_{l'l} \neq 0$: $L_l \neq\ const.$, $L \neq\ const.$; $n = 6\ (> 3)$

• $M = M^{\mathsf{T}}$, complex.

$$M^{diag} = W^{\mathsf{T}} M W\,,\ W - \text{unitary},\ 6 \times 6\,;\quad W^T \equiv (U^T\ \ V^T)\,;\ U \equiv U_{\text{PMNS}}:\ 3 \times 6\,.$$

$$\nu_{lL}(x) = \sum_{j=1}^{6} U_{lj} \chi_j(x)\,,\quad \chi_j(x) - \text{Majorana } \nu\text{s}\,,\quad m_j \neq 0\,,\quad l = e, \mu, \tau\,;$$

$$\nu^C_{lL}(x) \equiv C\ (\overline{\nu_{lR}}(x))^{\mathsf{T}} = \sum_{j=1}^{6} V_{lj} \chi_j(x)\,,\quad \nu^C_{lL}(x):\ \text{sterile antineutrino}$$

$\mathcal{L}^{\nu}_{D+M}(x)$ **possible in the ST +** ν_{lR}**:** $M^{LL} = 0$

$(\beta\beta)_{0\nu}$**-decay is allowed;**
phenomenology depends on the relative magnitude of M_D **and** M^{RR}**.**

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}^{\nu}_{M}(x) = -\,\frac{1}{2}\,\overline{\nu^{c}_{l'R}}(x)\;M_{l'l}\;\nu_{lL}(x) + h.c.\;,\;\nu^{c}_{l'R} \equiv C\;(\overline{\nu_{l'L}}(x))^{\mathsf{T}}$$

$\mathcal{L}^{\nu}_{M}(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- **Dirac ν, Ψ, is equivalent to two Majorana ν's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":**

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,,\quad m_1 = m_2 = m_D > 0\,, \eta_{jCP} = i\rho_j\,, \rho_1 = -\rho_2\;\;(C\;(\overline{\chi_j})^{\mathsf{T}} = \rho_j \chi_j)$$

Example ZKM ν : $\nu_{eL}(x) = \Psi_L = \dfrac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}\,,\;\nu_{\mu L}(x) = \Psi^{C}_{L} = \dfrac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$

- **Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}^{\nu}_{M}(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$**

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to "leading order" $m_1 = m_2$, but due to "higher order" corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- **Suppose:** $m_1 = m_2$, $\rho_1 = -\rho_2$, **but $\chi_{1,2}$ are not maximally mixed:**

$$\nu_{eL}(x) = \chi_{1L} \cos\phi + \chi_{2L} \sin\phi = \Psi_L \cos\phi' + \Psi^{C}_{L} \sin\phi'$$

All Majorana effects are $\sim m_D \cos\phi' \sin\phi'$

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\tan\theta_{23} = M_{e\tau}/M_{e\mu}$,

$m_3 = 0$ - **spectrum with IH,** $m_1 = m_2$, $\chi_{1,2}$ - **equivalent to one Dirac** ν, Ψ **.**

Adding L'-breaking term, e.g. M_{ee}, $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, **leads to** $m_1 \neq m_2$ **compatible with** $\Delta m_{21}^2 \neq 0$ **.**

Determining the Nature of Massive Neutrinos

Determining the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos is one of the most challenging and pressing problems in present day elementary particle physics.

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,**
$L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved**,
$\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and a new fundamental (approximate) symmetry.

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that the total lepton charge $L = L_e + L_\mu + L_\tau$ is not conserved in particle interactions by observing the $(\beta\beta)_{0\nu}-$decay would be a fundamental discovery (similar to establishing baryon number nonconservation (e.g., by observing proton decay)).

Establishing that ν_j are Majorana particles would be of fundamental importance, as important as the discovery of $\nu-$ oscillations, and would have far reaching implications.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ \ P_j e^{-i(E_j t - p_j x)} P^*_j = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j.

If ν_j- Majorana particles, U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau,$

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on** $\Delta m^2_{jk} = m^2_j - m^2_k$**, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ **processes:**

$$K^+ \to \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \ (\mathrm{A,Z}) \to \mu^+ + \ (\mathrm{A,Z}-2)$$

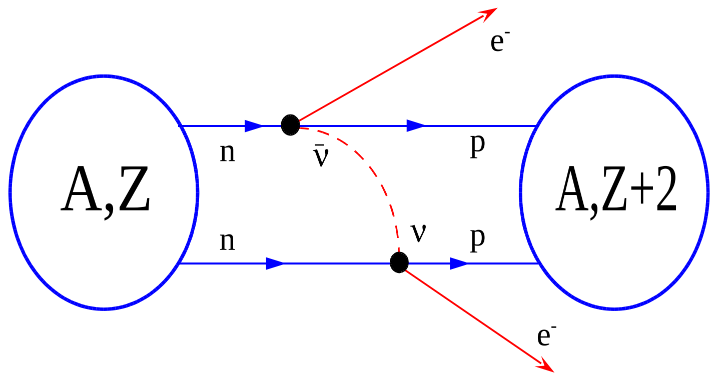
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$**−decay**

$$(\mathrm{A,Z}) \to (\mathrm{A,Z}+2) + e^- + e^-$$

of even-even nuclei, ^{48}**Ca,** ^{76}**Ge,** ^{82}**Se,** ^{100}**Mo,** ^{116}**Cd,** ^{130}**Te,** ^{136}**Xe,** ^{150}**Nd.**

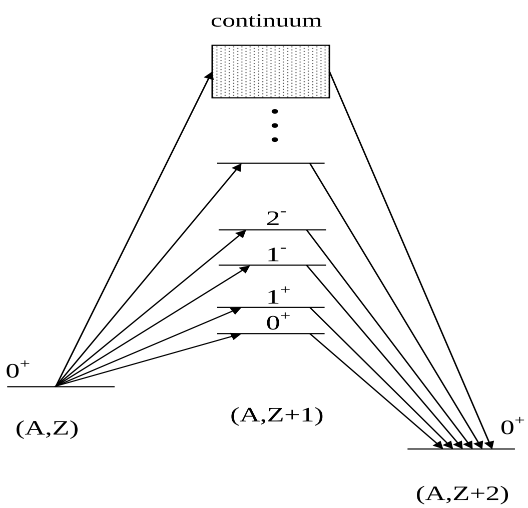
$2n$ **from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.**

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \to uue^-e^-(\overline{\nu}_e\overline{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

Due to V. Rodin

$(\beta\beta)_{0\nu}-$Decay Experiments:

- $L-$nonconservation, Majorana nature of ν_j.
- Type of $\nu-$mass spectrum (NH, IH, QD).
- Absolute neutrino mass scale.

^{3}H β-decay, cosmology: m_ν (QD, IH),

- Majorana CPV phases.

$A(\beta\beta)_{0\nu} \sim < m >$ M(A,Z), M(A,Z) - NME,

$$|< m >| = \left|m_1|U_{\mathrm{e}1}|^2 + m_2|U_{\mathrm{e}2}|^2\ e^{i\alpha_{21}} + m_3|U_{\mathrm{e}3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|,\ \ \theta_{12} \equiv \theta_{\odot},\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} $((\alpha_{31} - 2\delta) \rightarrow \alpha_{31})$ - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$\left|<m>\right| \cong \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta_M}\right|\,,\ m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$\left|<m>\right| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\ m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$\left|<m>\right| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\ m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \ \text{ (QD)},$$

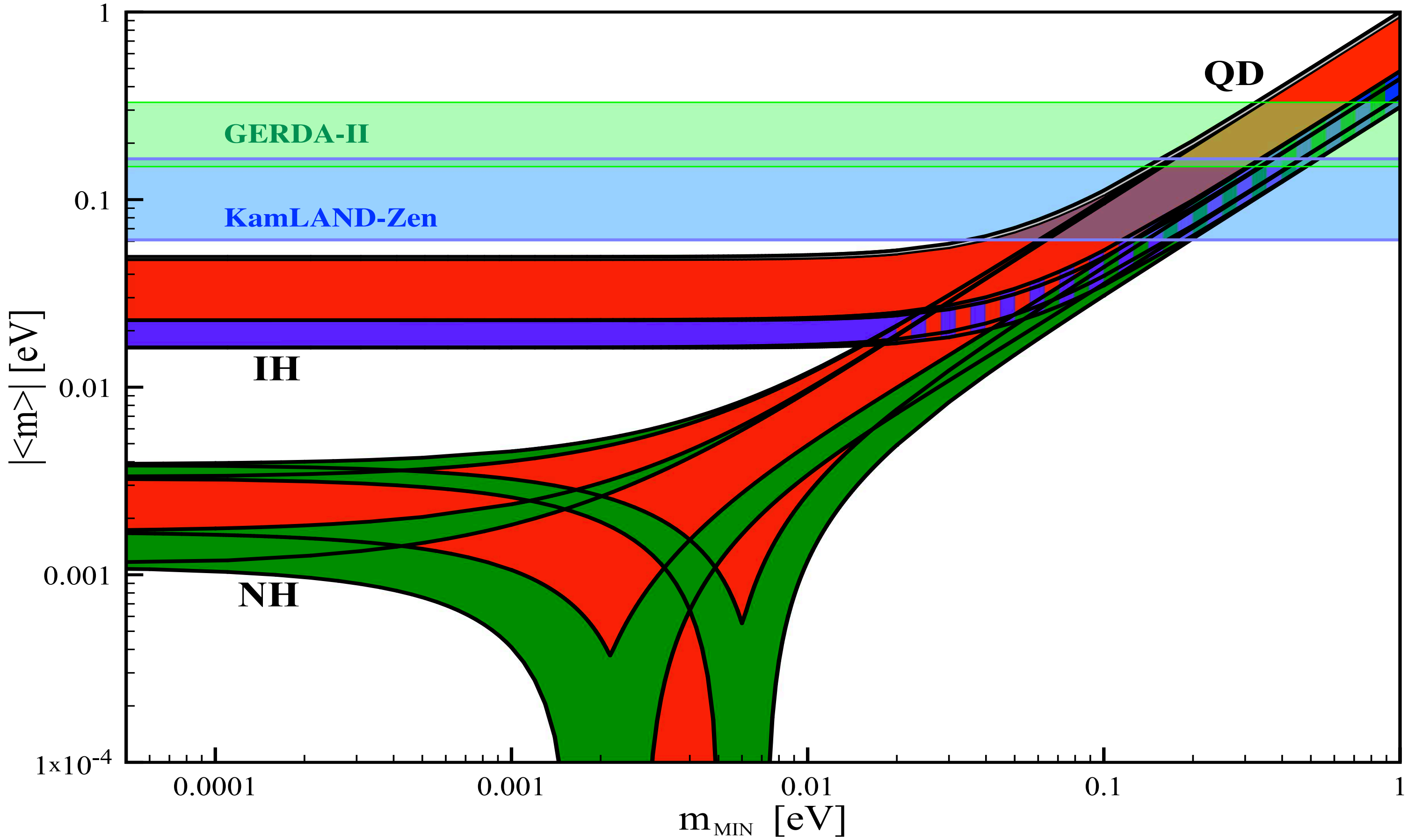
$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$\left|<m>\right| \lesssim 5\times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim \left|<m>\right| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV},\quad \text{IH};$$

$$m\cos 2\theta_{12} \lesssim \left|<m>\right| \lesssim m,\ m \gtrsim 0.10 \text{ eV},\ \text{ QD}\ .$$



S. Pascoli, PDG, 2017

$1\sigma(\Delta m^2_{21}) = 2.3\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$,
$1\sigma(|\Delta m^2_{31(23)}|) = 1.6\%$. $1\sigma(\sin^2\theta_{13}) = 4.0\%$.

From F. Capozzi *et al.*, arXiv:1703.04471

$2\sigma(|<m>|)$ used; $\alpha_{21}, (\alpha_{31} - 2\delta)$ varied in $[0,2\pi]$.

Results from IGEX (^{76}Ge), NEMO3 (^{100}Mo), CUORICINO+CUORE-0 (^{130}Te):

IGEX ^{76}Ge: $|{<}\,m\,{>}| \ < (0.33 - 1.35)$ eV **(90% C.L.).**

Data from NEMO3 (^{100}Mo), CUORICINO+CUORE-0 (^{130}Te):

$T(^{100}Mo) > 1.1 \times 10^{24}$ **yr,** $|{<}\,m\,{>}| \ <$**(0.3–0.6) eV;**
$T(^{130}Te) > 4.0 \times 10^{24}$ **yr.**

Best Sensitivity Results from 2012-2016:

$$\mathrm{T}(^{136}\mathrm{Xe}) > 1.6 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ EXO}$$

$$\mathrm{T}(^{136}\mathrm{Xe}) > 1.07 \times 10^{26}\mathrm{yr\ at\ 90\%\ C.L.,\ KamLAND-Zen}$$

$$|{<}\,m\,{>}| < (0.061 - 0.165)\ \mathrm{eV}\,.$$

$$\mathrm{T}(^{76}\mathrm{Ge}) > 5.2 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ GERDA\ II}$$

$$|{<}\,m\,{>}| < (0.16 - 0.26)\ \mathrm{eV}\,.$$

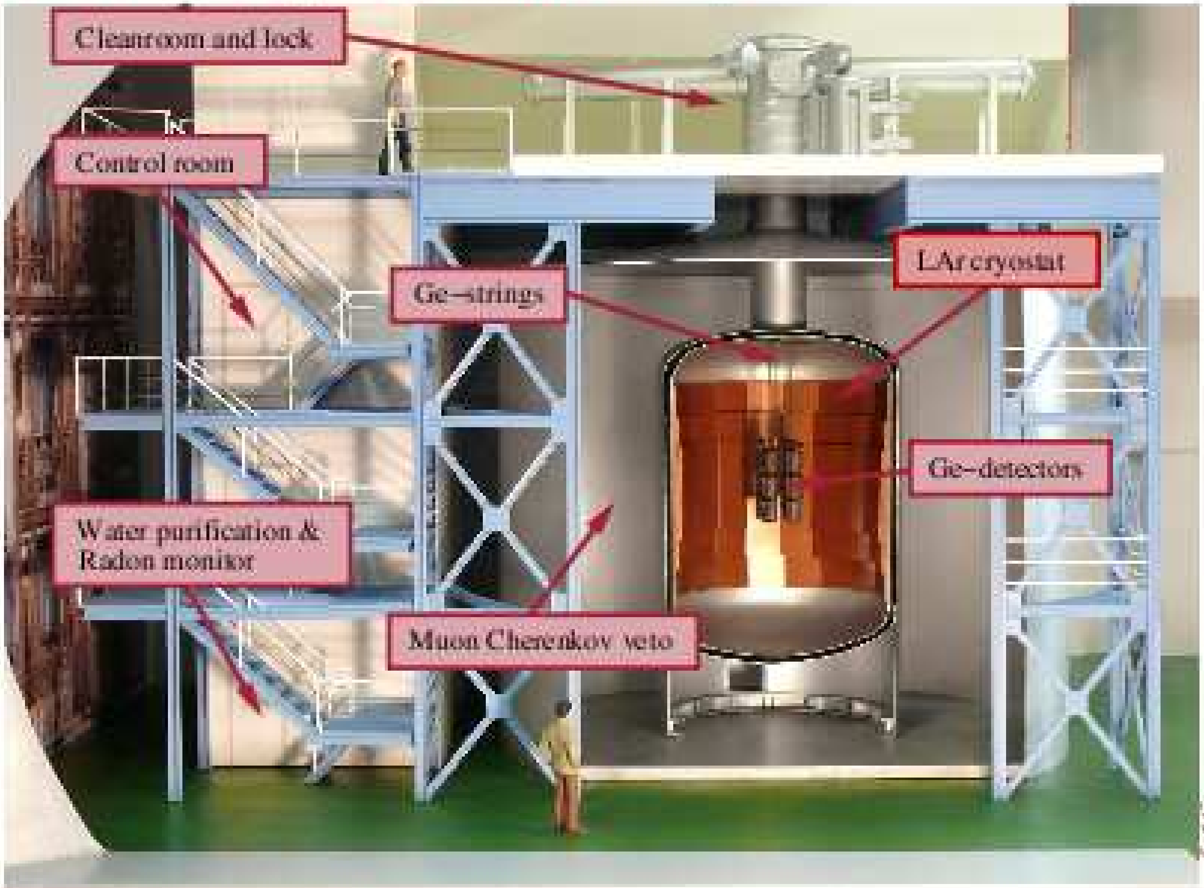
H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$\mathrm{T}(^{76}\mathrm{Ge}) = 2.23^{+0.44}_{0.31} \times 10^{25}\ \mathrm{yr\ at\ 90\%\ C.L.}$$

Large number of experiments: $|<m>|$ $\sim$ (0.01-0.05) eV

CUORE - ^{130}Te;
GERDA-II - ^{76}Ge;
MAJORANA - ^{76}Ge;
KamLAND-ZEN - ^{136}Xe;
(n)EXO - ^{136}Xe;
SNO+ - ^{130}Te;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca;
SuperNEMO - ^{82}Se, ^{150}Nd;
MAJORANA - ^{76}Ge;
NEXT - ^{136}Xe;
DCBA - ^{82}Se, ^{150}Nd;
XMASS - ^{136}Xe;
PANDAX-III - ^{136}Xe;
ZICOS - ^{96}Zr;
MOON - ^{100}Mo;
...

GERDA: Experimental Setup



Majorana CPV Phases and $|{<}m{>}|$

CPV can be established provided

– $|{<}m{>}|$ measured with $\Delta \lesssim 15\%$;

– Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_{\odot} \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

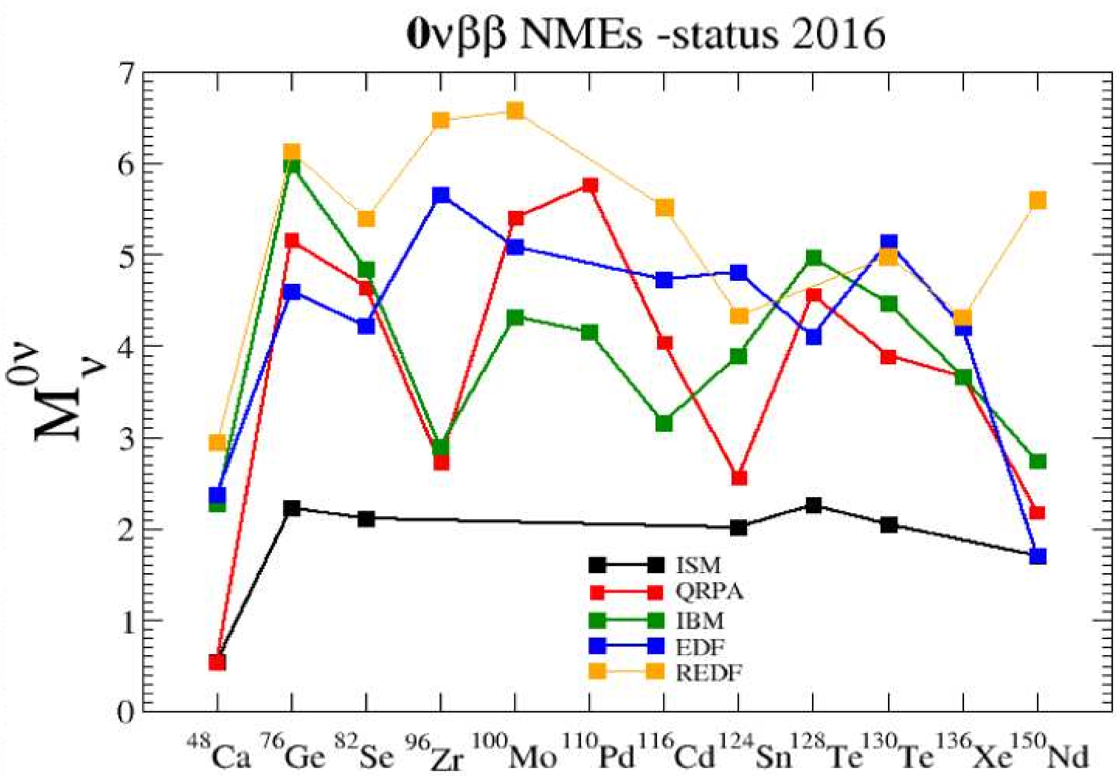
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

NMEs for Light ν Exchange



	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

F. Simkovic, September, 2016

The g_A Quenching Problem

g_A: related to the weak charged axial current which is not conserved and therefore can be and is renormalised, i.e., quenched, by the nuclear medium. Effectively, this implies that g_A is reduced from its current standard value $g_A = 1.269$.

The reduction of g_A can have important implications for the $(\beta\beta)_{0\nu}-$decay searches since $T^{0\nu}_{1/2} \propto (g^{eff}_A)^{-4}$.

The reduction of g_A necessary in various model NME calculations of $T^{2\nu}_{1/2}$ to reproduce the data;

does not imply the same reduction of g_A takes place in the $(\beta\beta)_{0\nu}-$decay NME, there are indications that the reduction is much smaller.

The mechanism of quenching is not understood at present. Thus, the degree of quenching cannot be firmly determined quantitatively and is subject to debates.

New Physics and $(\beta\beta)_{0\nu}$-Decay

Light Sterile Neutrinos and $(\beta\beta)_{0\nu}$-Decay

One Sterile Neutrino: the $3+1$ Model

$$|\langle m \rangle| \; = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha} + m_3 |U_{e3}|^2 e^{i\beta} + m_4 |U_{e4}|^2 e^{i\gamma} \right| .$$

$$U_{e1} = c_{12} c_{13} c_{14}, \quad U_{e2} = e^{i\alpha/2} c_{13} c_{14} s_{12},$$

$$U_{e3} = e^{i\beta/2} c_{14} s_{13}, \quad U_{e4} = e^{i\gamma/2} s_{14}\,,$$

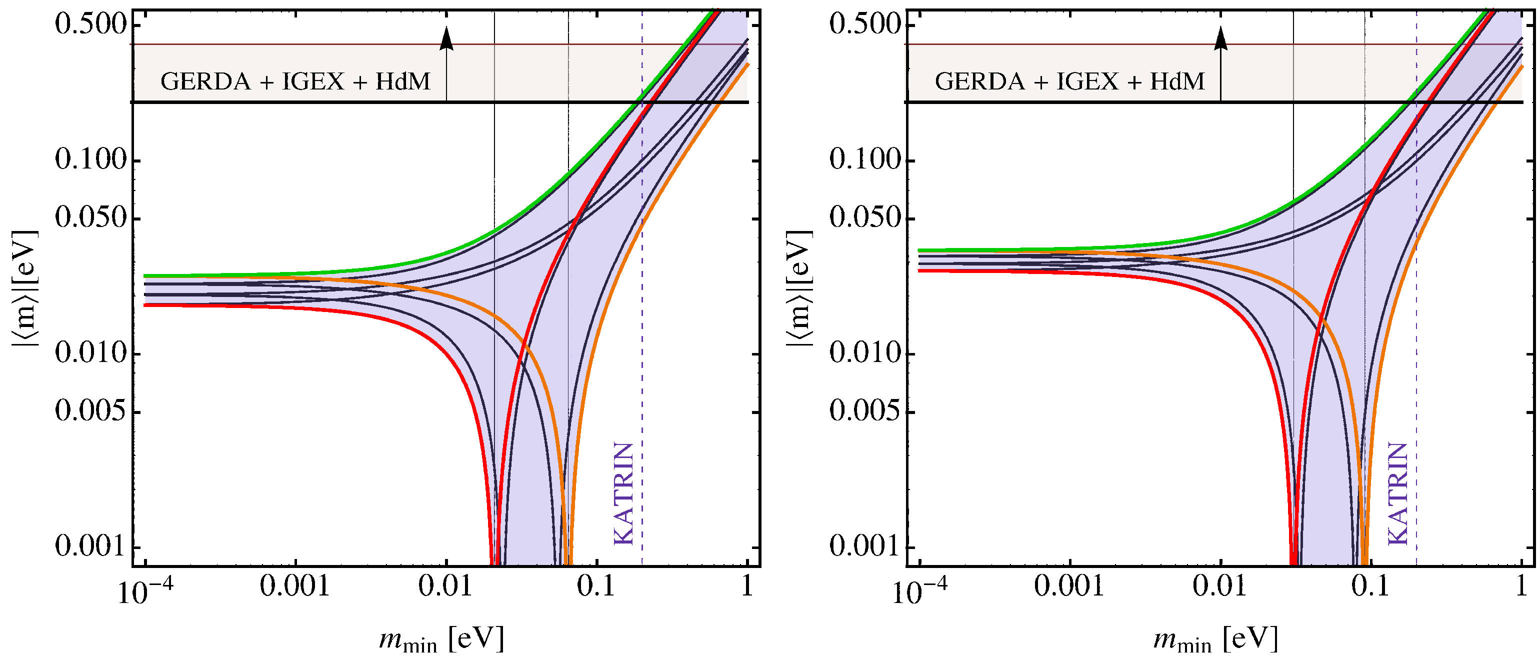
$$\sin^2\theta_{14} = 0.0225\,, \quad \Delta m^2_{41(43)} = 0.93 \text{ eV}^2 \quad (\text{A})\,,$$

J. Kopp et al., 2013

$$\sin^2\theta_{14} = 0.023 \;(0.028)\,, \;\; \Delta m^2_{41(43)} = 1.78 \;(1.60) \text{ eV}^2 \;\; (\text{B})\,.$$

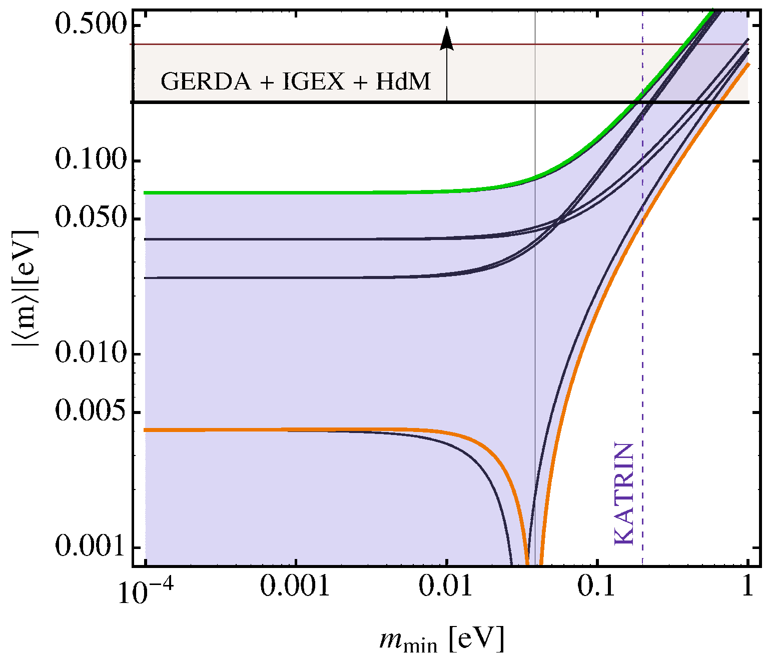
J. Kopp et al., 2013 (ν_e, $\bar{\nu}_e$ disappearnce data);

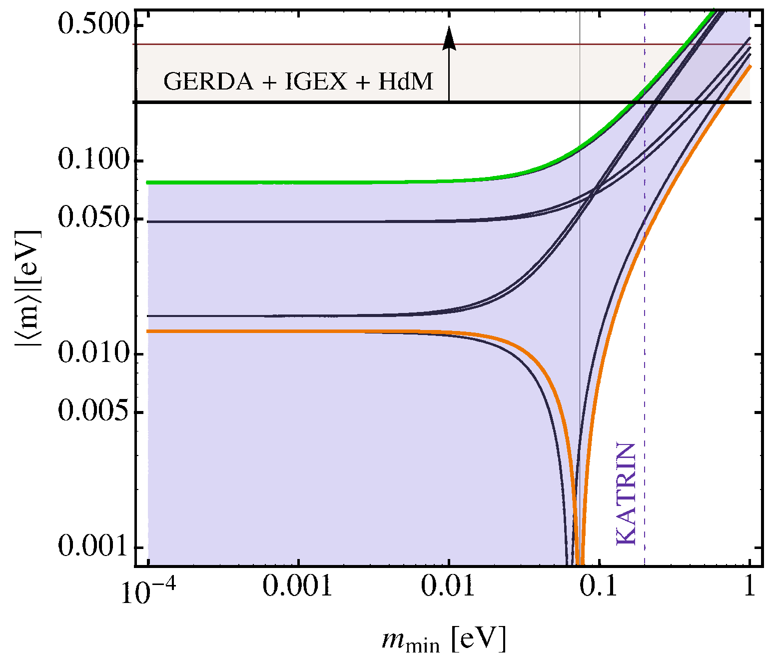
C. Giunti et al., 2013 (global, except for MiniBooNE results at $E_\nu \leq 0.475$ GeV)



I. Girardi A. Meroni, S.T.P., 2013

NO spectrum; green, red and orange lines: $(\alpha, \beta, \gamma) = (0, 0, 0), (0, 0, \pi), (\pi, \pi, \pi)$;
five gray lines: the other five sets of CP conserving values.
Left panel: $\Delta m^2_{41} = 0.93$ eV^2, $\sin\theta_{14} = 0.15$.
Right panel: $\Delta m^2_{41} = 1.78$ eV^2, $\sin\theta_{14} = 0.15$.





I. Girardi A. Meroni, S.T.P., 2013

IO spectrum; green and orange lines: $(\alpha, \beta, \gamma) = (0, 0, 0), (\pi, \pi, \pi)$; six gray lines: the other six sets of CP conserving values.
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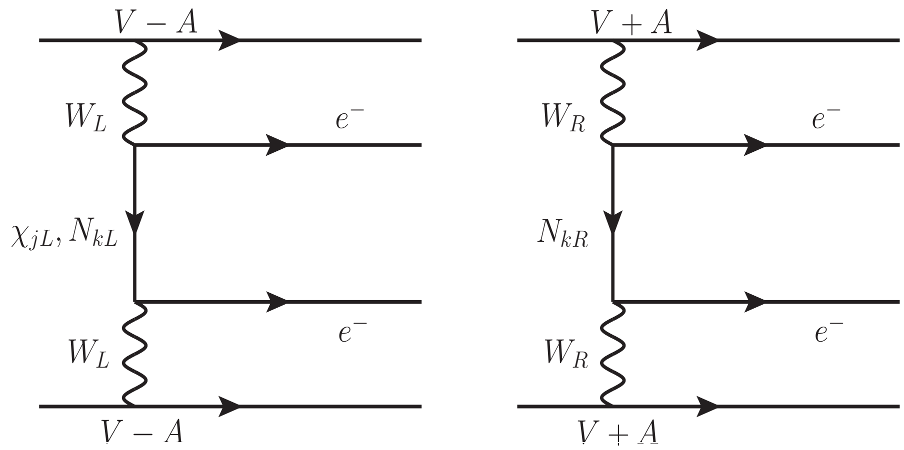
Heavy Majorana Neutrino Exchange Mechanisms



Light Majorana Neutrino Exchange

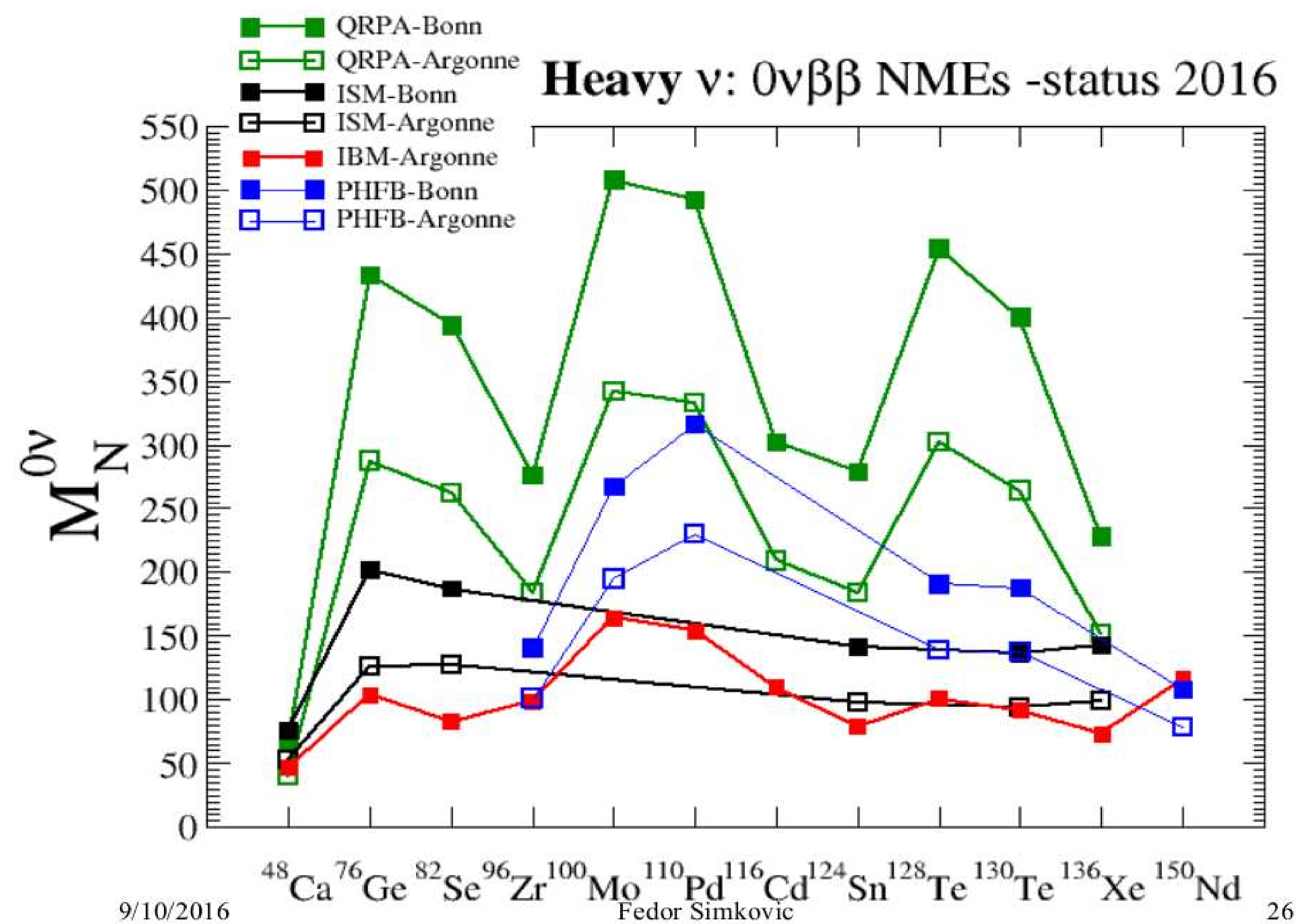
$$\eta_\nu = \frac{<m>}{m_e}\,.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim$ 10 GeV:

$\eta_N^L \;=\; \Sigma_k^{heavy}\; U_{ek}^2 \frac{m_p}{M_k}$, m_p - proton mass , U_{ek} - CPV .

NMEs for Heavy Majorana Neutrino Exchange



F. Simkovic, September, 2016

Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

If the decay $(A,Z) \to (A,Z+2) + e^- + e^-$ ($(\beta\beta)_{0\nu}$-decay) will be observed, the question will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

"Standard Mechanism": light Majorana ν exchange.

Fundamental parameter - the effective Majorana mass:

$< m > = \sum_j^{light} \left(U_{ej}\right)^2 m_j$, all $m_j \geq 0$,

U - the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix, m_j - the light Majorana neutrino masses, $m_j \lesssim 1$ eV.

U - CP violating, in general: $(U_{ej})^2 = |U_{ej}|^2 e^{i\alpha_{j1}}$, $j = 2,3$, α_{21}, α_{31} - Majorana CPV phases.

S.M. Bilenky, J. Hosek, S.T.P.,1980

A number of different mechanisms possible.

For a given mechanism κ we have in the case of $(A,Z) \to (A,Z+2) + e^- + e^-$:

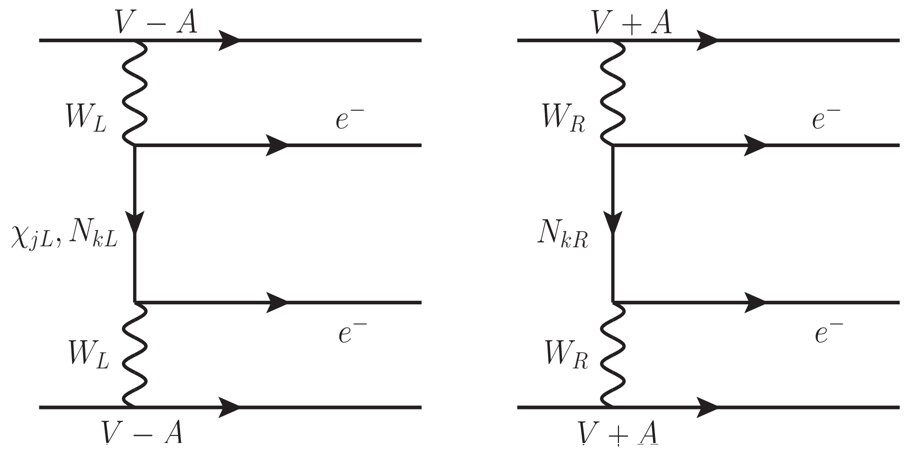
$$\frac{1}{T^{0\nu}_{1/2}} = |\eta^{LNV}_{\kappa}|^2 \; G^{0\nu}(E_0,Z) |M'^{0\nu}_{\kappa}|^2 \,,$$

η^{LNV}_{κ} - the fundamental LNV parameter characterising the mechanism κ,

$G^{0\nu}(E_0,Z)$ - phase-space factor (includes $g_A^4 = (1.25)^4$, as well as $R^{-2}(A)$, $R(A) = r_0 A^{1/3}$ with $r_0 = 1.1 \; fm$),

$M'^{0\nu}_{\kappa} = (g_A/1.25)^2 M^{0\nu}_{\kappa}$ - NME (includes $R(A)$ as a factor).

Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay



Light Majorana Neutrino Exchange

$$\eta_\nu = \frac{<m>}{m_e}.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim 10$ GeV:

$\eta_N^L \;=\; \sum_k^{heavy}\; U_{ek}^2 \frac{m_p}{M_k}$, m_p - proton mass , U_{ek} - CPV .

(V+A) Weak Interaction, RH N_k, $M_k \gtrsim 10$ GeV:

$\eta_N^R \;=\; \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} \; V_{ek}^2 \frac{m_p}{M_k}$; V_{ek}: $N_k - e^-$ in the CC .

$M_W \cong 80$ GeV; $M_{WR} \gtrsim 2.5$ TeV; V_{ek} - CPV, in general.

A comment.

(V-A) CC Weak Interaction:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e_L}\,(e^c)_R\,,\; e^c = C(\bar{e})^T\,,$

C - the charge conjugation matrix.

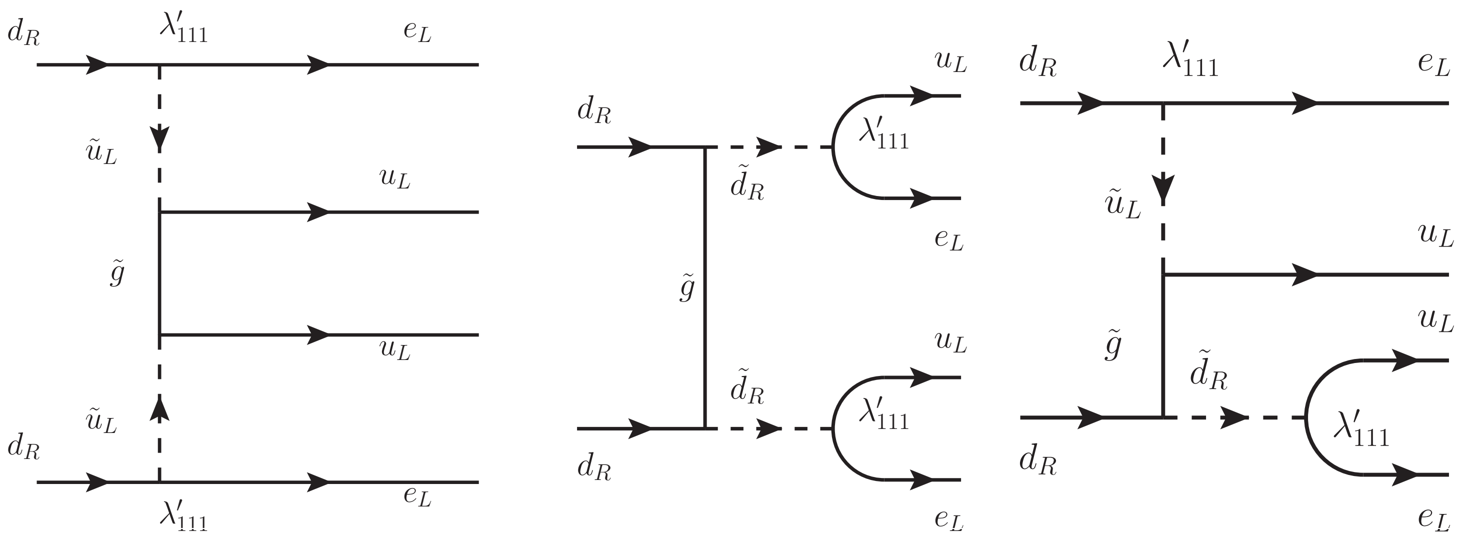
(V+A) CC Weak Interaction:

$\bar{e}(1-\gamma_5)e^c \equiv 2\bar{e_R}\,(e^c)_L\,.$

The interference term: $\propto m_e$, suppressed.

A. Halprin, S.T.P., S.P. Rosen, 1983

SUSY Models with R-Parity Non-conservation



$$\mathcal{L}_{R_p} = \lambda'_{111}\Big[(\bar{u}_L\ \bar{d}_L)\begin{pmatrix} e^c_R \\ -\nu^c_{eR}\end{pmatrix}\tilde{d}_R + (\bar{e}_L\ \bar{\nu}_{eL})d_R\begin{pmatrix}\tilde{u}^*_L \\ -\tilde{d}^*_L\end{pmatrix} + (\bar{u}_L\ \bar{d}_L)d_R\begin{pmatrix}\tilde{e}^*_L \\ -\tilde{\nu}^*_{eL}\end{pmatrix}\Big] + h.c.$$

The Gluino Exchange Dominance Mechanism

$$\eta_{\lambda'} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}}\frac{m_p}{m_{\tilde{g}}}\left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 ,$$

G_F - the Fermi constant, $\alpha_s = g_3^2/(4\pi)$, g_3 - the SU(3)$_c$ gauge coupling constant, $m_{\tilde{u}_L}$, $m_{\tilde{d}_R}$ and $m_{\tilde{g}}$ - the masses of the LH u-squark, RH d-squark and gluino.

The Squark-Neutrino Mechanism

$$\eta_{\tilde{q}} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{2\sqrt{2}G_F}\sin 2\theta^d_{(k)}\left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}}\right) ,$$

$d_{(k)} = d, s, b$; θ^d: $\tilde{d}_{kL} - \tilde{d}_{kR}$ - mixing (3 light Majorana neutrinos assumed).

The $2e^-$ current in both mechanisms:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e_L}\,(e^c)_R$, as in the "standard" mechanism.

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$-decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, "Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nuclei with Largely Different Nuclear Matrix Elements", (arXiv:1212.1331, JHEP **1302** (2013) 025.

Earlier studies include:
A. Halprin, S.T.P., S.P. Rosen, "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335).

The Nature of Massive Neutrinos III:

The Seesaw Mechanisms of Neutrino Mass Generation

• Explain the smallness of $\nu-$masses.

• Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH νs' (heavy).

Type II seesaw mechanism: $\mathbf{H(x)}$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\mathbf{T(x)}$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$-decay, LFV processes, etc.) and New Physics at LHC.

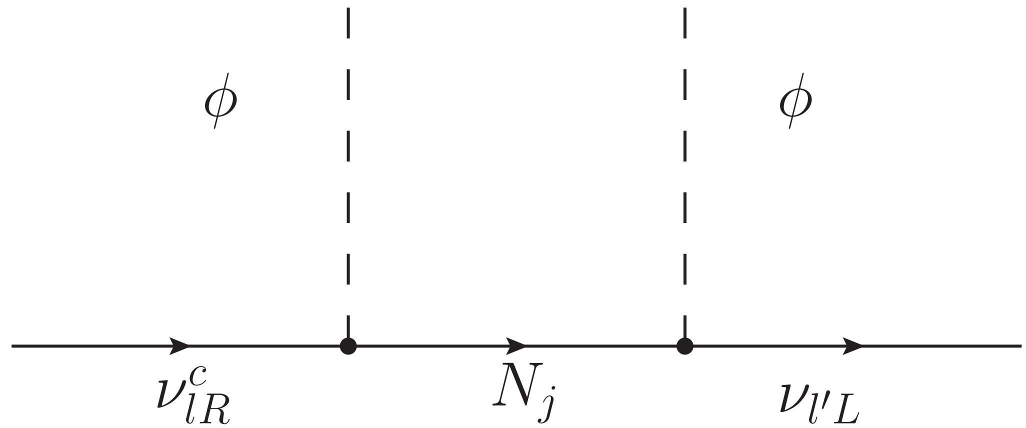
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^{\nu}/\sqrt{2}| << |M^{RR}|$.
- Diagonalising M^{RR}: N_j - heavy Majorana neutrinos, $M_j \sim$TeV; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large M_j, Majorana mass term for $\nu_{lL}(x)$:

$$M_{\nu} \cong v_u^2\,(Y^{\nu})^T\,M_j^{-1}\,Y^{\nu} = U^*_{\text{PMNS}}\,m_{\nu}^{\text{diag}}\,U^{\dagger}_{\text{PMNS}}\,.$$

$v_u\,Y^{\nu} = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_{\nu} \sim 0.1$ eV.



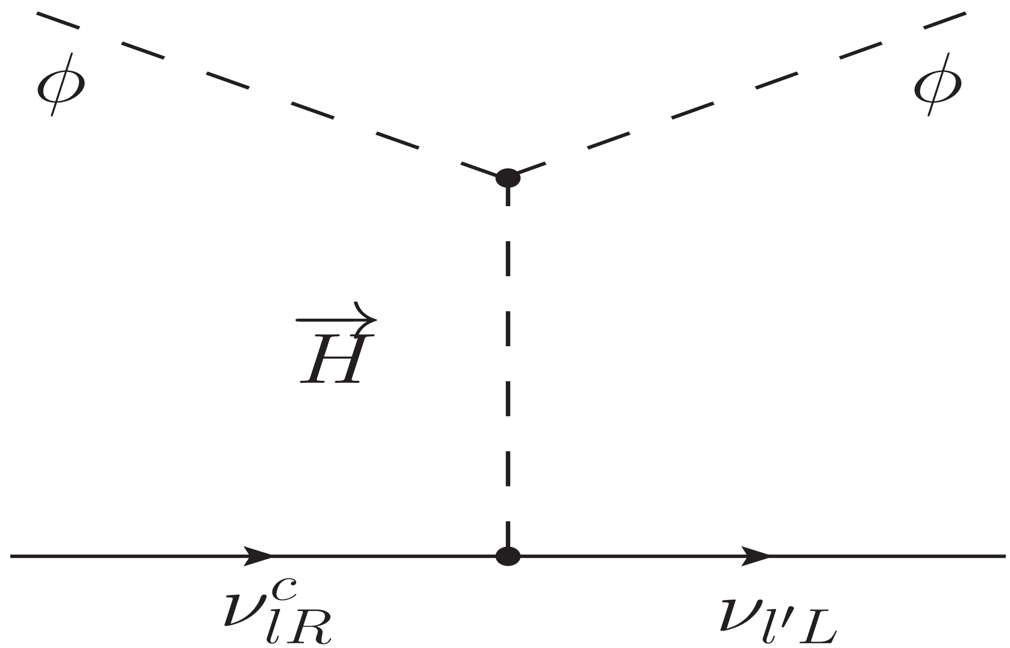
• $\nu_{l'R}(x)$: Majorana mass term at "high scale" (∼TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)

$$\mathcal{L}^{\nu}_{M}(x) = +\,\frac{1}{2}\,\nu^{\mathsf{T}}_{l'R}(x)\;C^{-1}\;(M^{RR})^{\dagger}_{l'l}\;\nu_{lR}(x) + h.c. = -\,\frac{1}{2}\sum_j \bar{N}_j\,M_j\,N_j\ ,$$

• Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:

$$\begin{aligned} \mathcal{L}_{\mathsf{Y}}(x) &= \bar{Y}^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \mathrm{h.c.}\,, \\ &= Y^{\nu}_{jl}\,\overline{N_{jR}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \mathrm{h.c.}\,, \\ M_D &= \frac{v}{\sqrt{2}}\,Y^{\nu}\,, \quad v = 246\ \mathrm{GeV}\,. \end{aligned}$$

Type II Seesaw Mechanism

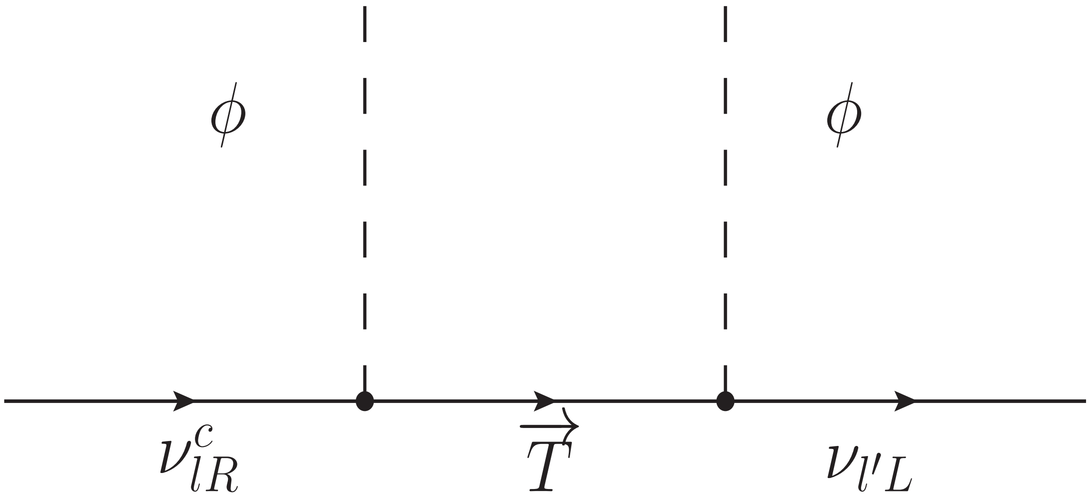


Due to I. Girardi

$$M_\nu \cong h\, v^2\, M_H^{-1} = U^*_{\mathrm{PMNS}}\, m_\nu^{\mathrm{diag}}\, U^\dagger_{\mathrm{PMNS}}\,.$$

$h \sim 10^{-2}$, $v = 246$ GeV, $M_H \sim 10^{12}$ GeV: $M_\nu \sim 0.6$ eV.

Type III Seesaw Mechanism



$$M_\nu \cong v^2 \, (Y_T)^T \, M_T^{-1} \, Y_T = U^*_{\text{PMNS}} \, m_\nu^{\text{diag}} \, U^\dagger_{\text{PMNS}} \, .$$

$v \, Y_T \sim 1$ GeV, $M_T \sim 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.

LEPTOGENESIS

M_ν from type I See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

In GUTs, $M_{1,2,3} < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_{1,2,3} = (10^{11}, 10^{12}, 10^{13})$ GeV, $m_D \sim 1$ GeV.

TeV Scale (Resonant) Leptogenesis:

$M_{1,2,3} \sim (10^2 - 10^3)$ GeV (requires fine-tuning (severe)); observation of N_j at LHC - problematic (low production rates); observable LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- - e^-$ conversion.

Can the CP violation necessary for the generation of the observed value of the Baryon Asymmetry of the Universe (BAU) be provided exclusively by the Dirac and/or Majorana CPV phases in the neutrino PMNS matrix?

Demonstrated in (incomplete list):

S. Pascoli *et al.*, **hep-ph/0609125 and hep-ph/0611338.**
E. Molinaro *et al.*, **arXiv:0808.3534.**
A. Meroni *et al.*, **arXiv:1203.4435.**
C. Hagedorn *et al.*, **arXiv:0908.0240.**
J. Gehrlein *et al.*, **arXiv:1502.00110 and arXiv:1508.07930.**
J. Zhang, Sh. Zhou, arXiv:1505.04858 (FGY 2002 model).
P. Chen *et al.*, **arXiv:1602.03873.**
C. Hegdorn, E. Molinaro, arXiv:1602.04206.
P. Hernandez *et al.*, **arXiv:1606.06719 and 1611.05000.**
M. Drewes *et al.*, **arXiv:1609.09069.**
G. Bambhaniya *et al.*, **arXiv:1611.03827.**

The Seesaw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,,$$

$$\begin{aligned}
\mathcal{L}_{\text{CC}} &= -\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,\\
\mathcal{L}_{\text{Y}}(x) &= \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,\\
\mathcal{L}_{\text{M}}^{\text{N}}(x) &= -\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.
\end{aligned}$$

ψ_{lL} - LH doublet,$\psi_{lL}^{\text{T}} = (\nu_{lL}\ l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\ v = 174\ \text{GeV},\ v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\ \lambda^T\,D_N^{-1}\,\lambda = U_{\text{PMNS}}^*\ D_\nu\ U_{\text{PMNS}}^\dagger\ .$$

$$m_\nu \simeq v^2 \; \lambda^T \, D_N^{-1} \, \lambda = U^*_{\text{PMNS}} \, D_\nu \, U^\dagger_{\text{PMNS}} \,,$$

$$\lambda \equiv Y_\nu$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\text{PMNS}})^\dagger / v_u$, all at M_R;

R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

$D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

Theories, Models:

• R - CP conserving ($SU(5) \times T'$, A. Meroni *et al.*, arxiv:1203.4435; S_4, P. Cheng *et al.*, arXiv:1602.03873; C. Hagedorn, E. Molinaro, arXiv:1602.04206).

• CPV parameters in R determined by the CPV phases in U (e.g., class of A_4 theories).

• Texture zeros in Y_ν: CPV parameters in R determined by the CPV phases in U
(Frampton, Glashow Yanagida (FGY), 2002: $N_{1,2}$, two texture zeros in Y_ν; LG in FGY model: J. Zhang, Sh. Zhou, arXiv:1505.04858).

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j, \;\; C(\overline{N}_k)^T = N_k, \;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$U_{\rm CP}\, N_j(x)\, U^{\dagger}_{\rm CP} \;=\; \eta^{NCP}_j\, \gamma_0\, N_j(x')\,, \quad \eta^{NCP}_j = i\rho^N_j = \pm i\,,$$
$$U_{\rm CP}\, \nu_k(x)\, U^{\dagger}_{\rm CP} \;=\; \eta^{\nu CP}_k\, \gamma_0\, \nu_k(x')\,, \quad \eta^{\nu CP}_k = i\rho^{\nu}_k = \pm i\,.$$

CP-invariance:

$$\lambda^*_{jl} \;=\; \lambda_{jl}\,(\eta^{NCP}_j)^*\,\eta^l\,\eta^{H*}\,, \quad j = 1,2,3,\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i, \;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\lambda^*_{jl} \;=\; \lambda_{jl}\,\rho^N_j\,, \;\; \rho^N_j = \pm 1\,,$$
$$U^*_{lj} \;=\; U_{lj}\,\rho^{\nu}_j\,, \;\; \rho^{\nu}_j = \pm 1\,,$$
$$R^*_{jk} \;=\; R_{jk}\,\rho^N_j\,\rho^{\nu}_k, \quad j,k = 1,2,3,\; l = e,\mu,\tau,$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \;\equiv\; R_{jk}\,R_{jm}\,U^*_{lk}\,U_{lm}\,, \;\; k \neq m\,,$$
$$CP: \quad P^*_{jkml} \;=\; P_{jkml}\,(\rho^N_j)^2\,(\rho^{\nu}_k)^2\,(\rho^{\nu}_m)^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at "high" E due to the interplay of R and U: $\mathrm{Im}(P_{123\tau}) \neq 0$ (!)

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \quad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B **number non-conservation.**
- **Violation of C and CP symmetries.**
- **Deviation from thermal equilibrium.**

Leptogenesis

• **The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.**

• **When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.**

• **If $\Gamma(N_1 \to \Phi^- \ell^+) \neq \Gamma(N_1 \to \Phi^+ \ell^-)$, a lepton asymmetry will be generated.**

• **Wash-out processes, like $\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc. tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final resullt is a net (non-zero) lepton asymmetry.**

• **This lepton asymmetry is then converted into a baryon asymmetry by $(B+L)$ violating but $(B-L)$ conserving sphaleron processes which exist within the SM (at $T \gtrsim M_{\rm EWSB}$).**

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B:

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

2. solve the Boltzmann (or similar) equation to account for the wash-out of the asymmetry:

$$Y_{\mathsf{L}} = \kappa\, \varepsilon$$

where $\kappa = \kappa(\widetilde{m})$ is the "efficiency factor", $\widetilde{m}$ is the "the wash-out mass parameter" - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_{\mathsf{B}} = -\frac{c_s}{g_*}\kappa\varepsilon, \quad c_s \cong 1/3, \quad g_* = 215/2$$

Baryon number violation in the SM

Instanton and Sphaleron processes

SU(2) instantons lead to (leading order) to effective 12 fermion $(B+L)$ nonconserving, but $(B-L)$ conserving, interactions:

$O(B+L) = \Pi_i q_{Li} q_{Li} q_{Li} l_{Li}$

These would induce $\Delta B = \Delta L = 3$ processes:

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \to \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at T = 0 the probability of such processes is $\Gamma/V \sim e^{-4\pi/\alpha} \sim 10^{-165}$.

't Hooft, 1976

At finite T, the transitions proceed via thermal fluctuations (over the barrier) with an unsuppressed probability (due to sphaleron (static) configurations - saddle "points" of the field energy of the $SU(2)$ gauge - Higgs field system):

$$\Gamma/V \sim \alpha^4 T^4.$$

Kuzmin, Rubakov, Shaposhnikov, 1985;
Arnold et al., 1987 and 1997.

Sphaleron processes are efficient (in the case of interest) at

$$T_{\rm EW} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Can generate $B \neq 0$, $L \neq 0$ at $T_{\rm EW} < T (< 10^{12}$ GeV) from $(B-L)_0 \neq 0$ (with $(B-L) = const.$).

Harvey, Turner, 1990

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma:\ \sim 6.1 \times 10^{-10})$$

$$Y_B \cong -3 \times 10^{-3}\ \varepsilon\,\kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

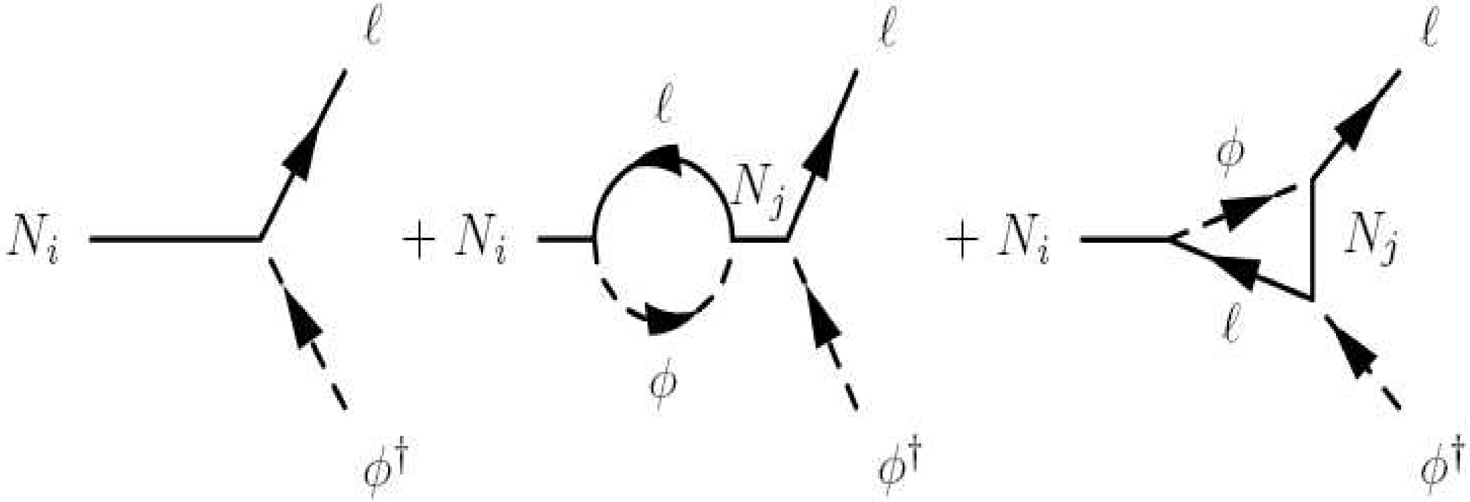
$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\quad \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004



N_i
ℓ
$\phi^\dagger$
$+ N_i$
ℓ
N_j
ℓ
ϕ
$\phi^\dagger$
$+ N_i$
ϕ
ℓ
N_j
ℓ
$\phi^\dagger$

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} = \left| \sum_k R_{1k} \, m_k^{1/2} \, U_{lk}^* \right|^2, \quad l = e, \mu, \tau \, .$$

The "one-flavor" approximation - $\mathbf{Y}_{\mathbf{e},\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l \; H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \, |R_{1k}|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \, |R_{1k}|^2 \, .$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \to (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \to N_1$;

$\tau_L^- + \Phi^0 \to \tau_R^-$, $\quad \tau_L^- + \tau_L^+ \to N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1\ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{lj} U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j\,|R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\,v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U^*_{lk}\right|^2, \quad l = e, \mu, \tau\,.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*}\left(\epsilon_2\,\eta\left(\frac{417}{589}\,\widetilde{m_2}\right) + \epsilon_\tau\,\eta\left(\frac{390}{589}\,\widetilde{m_\tau}\right)\right),$$

$$\eta\,(\widetilde{m_l}) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1}.$$

$Y_{\mathcal{B}} = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$
\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}\,, R_{1j}R_{1k} = \pm \left|R_{1j}R_{1k}\right|, \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j) \left|R_{1j} R_{1k}\right| \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}\,, R_{1j}R_{1k} = \pm i \left|R_{1j}R_{1k}\right|
\end{aligned}
$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\quad \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0, \quad \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0;$

$$
Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589} \widetilde{m_\tau}\right) - \eta\left(\frac{417}{589} \widetilde{m_2}\right) \right)
$$

$m_1 \ll m_2 \ll m_3$, $M_1 \ll M_{2,3}$; $R_{12}R_{13}$ – real; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}}|R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \text{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\text{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi$, $\delta = 0$: Re$(U^*_{\tau 2}U_{\tau 3}) = 0$, CPV due to the interplay of R and U.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \mathrm{arg}(R_{12}R_{13})$.

$|R_{12}| \cong 0.86$, $|R_{13}|^2 = 1 - |R_{12}|^2$, $|R_{13}| \cong 0.51$ - maximise $|Y_B|$:

$$|Y_B| \ \cong 2.1 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.15}\right) \left(\frac{M_1}{10^9\ \mathrm{GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \cong 0.15\,.$$

The lower limit corresponds to

$$|J_{\mathrm{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \cong 0.15\,; \quad |J_{\mathrm{CP}}| \gtrsim 2.0 \times 10^{-2}$$

Realised in a theory based on the S_4 symmetry: P. Cheng *et al.*, arXiv:1602.03873.

The requirement $\sin\theta_{13} \gtrsim 0.09$ (0.11) - compatible with the Daya Bay, RENO, Double Chooz results: $\sin\theta_{13} \cong 0.15$.

$|\sin\theta_{13}\sin\delta| \gtrsim 0.11$ implies $|\sin\delta| \gtrsim 0.7$ - compatible with $\delta \cong 3\pi/2$.

$\sin\theta_{13} \cong 0.15$ and $\delta \cong 3\pi/2$ imply relatively large (observable) CPV effects in neutrino oscillations: $J_{\mathrm{CP}} \cong -3.5 \times 10^{-2}$.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $\quad |R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \;\cong 2.2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05\ \text{eV}}\right)\left(\frac{M_1}{10^9\ \text{GeV}}\right)\frac{|\sin(\alpha_{32}/2)|}{\sin \pi/4}\,.$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

The see-saw mechanism provides a link between the ν-mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos ($(\beta\beta)_{0\nu}$-decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+, SuperNEMO, MAJORANA, AMORE,...).

- determination of the status of the CP symmetry in the lepton sector (T2K, NOνA; DUNE, T2HK)

- determination of the neutrino mass ordering (JUNO, RENO50; ORCA, PINGU (IceCube), HK, INO; T2K + NOνA; DUNE (future); + T2HKK (future)) ;

- determination of the absolute neutrino mass scale, or min(m_j) (KATRIN, new ideas; cosmology);

The program of research extends beyond 2030.

JUNO (20 kton LS, $L \cong 50$ km, China)

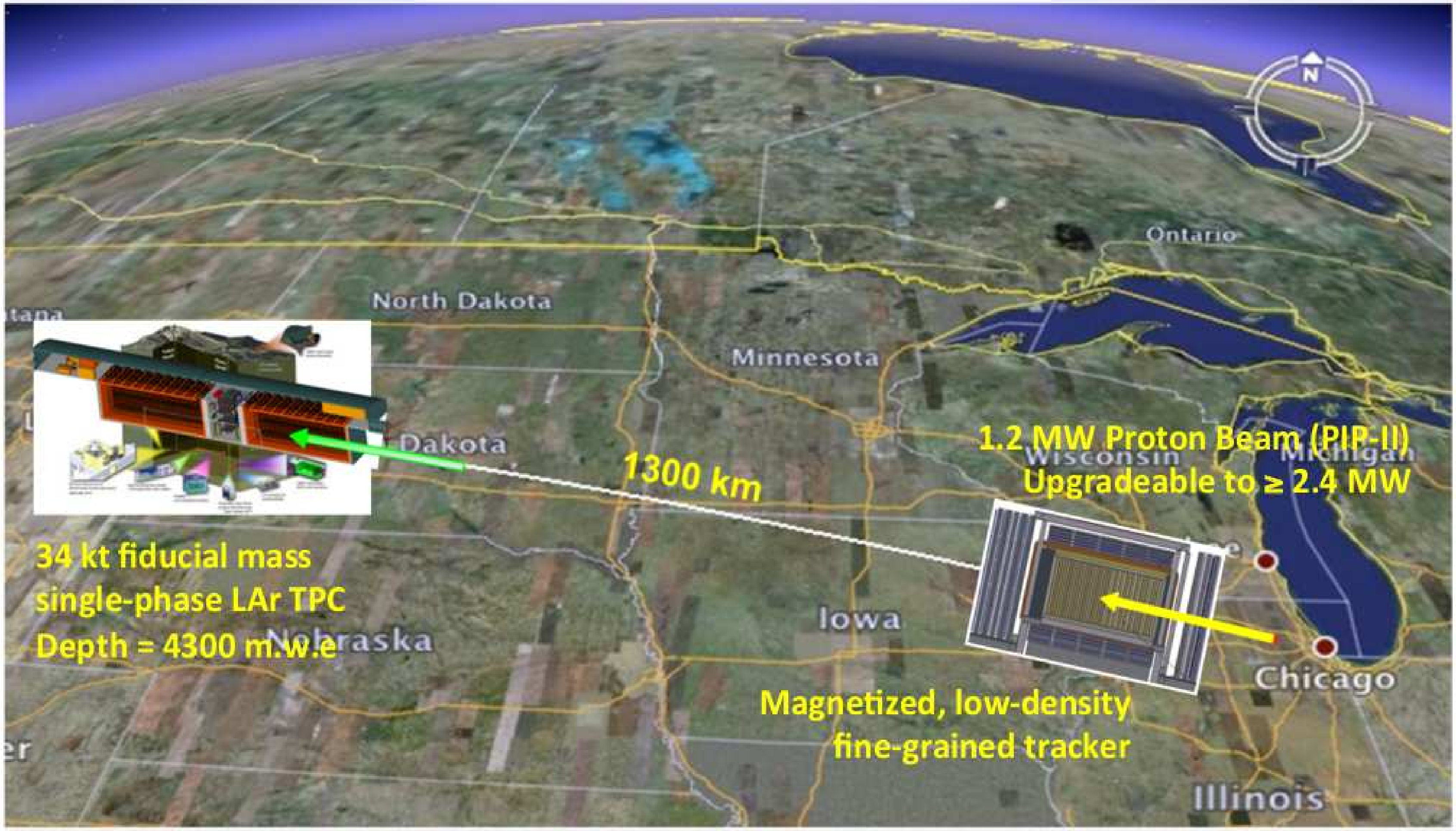
LBL Oscillation Experiments T2K, NOνA, DUNE, T2HK (HK=Hyper-Kamiokande: water-Cherenkov, $\sim$ 0.5 Mton, fiducial $\sim$ 0.25 Mton).

NOνA: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

T2HK: $L = 295$ km, 2.5° off-axis (narrow band) ν_μ beam (from 750 kW proton) beam, maximum at $E \cong 0.6$ GeV (the first osc. maximum).

DUNE: Fermilab-DUSEL, $L = 1290$ km, 1.2 MW (2.3 MW) proton beam, wide band ν beam (first and second osc. maxima at $E = 2.4$ GeV and 0.8 GeV); 34 kt fiducial volume LAr detecors; plans to run 5 years with ν_μ and 5 years with $\bar{\nu}_\mu$; 2025 (?)

LBNE Design



North Dakota
Dakota
Minnesota
Ontario
Wisconsin
Michigan
Iowa
Nebraska
Chicago
Illinois
1300 km
1.2 MW Proton Beam (PIP-II)
Upgradeable to ≥ 2.4 MW
34 kt fiducial mass
single-phase LAr TPC
Depth = 4300 m.w.e
Magnetized, low-density
fine-grained tracker

Atmospheric ν experiments

vspace-0.8cm HyperKamiokande (10SK), IceCube-PINGU, KM3Net-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-, no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

KM3Net in Mediteranian sea: ORCA

vspace-0.8cm HyperKamiokande (10SK), IceCube-PINGU, KM3Net-ORCA;

Iron Magnetised detector: INO

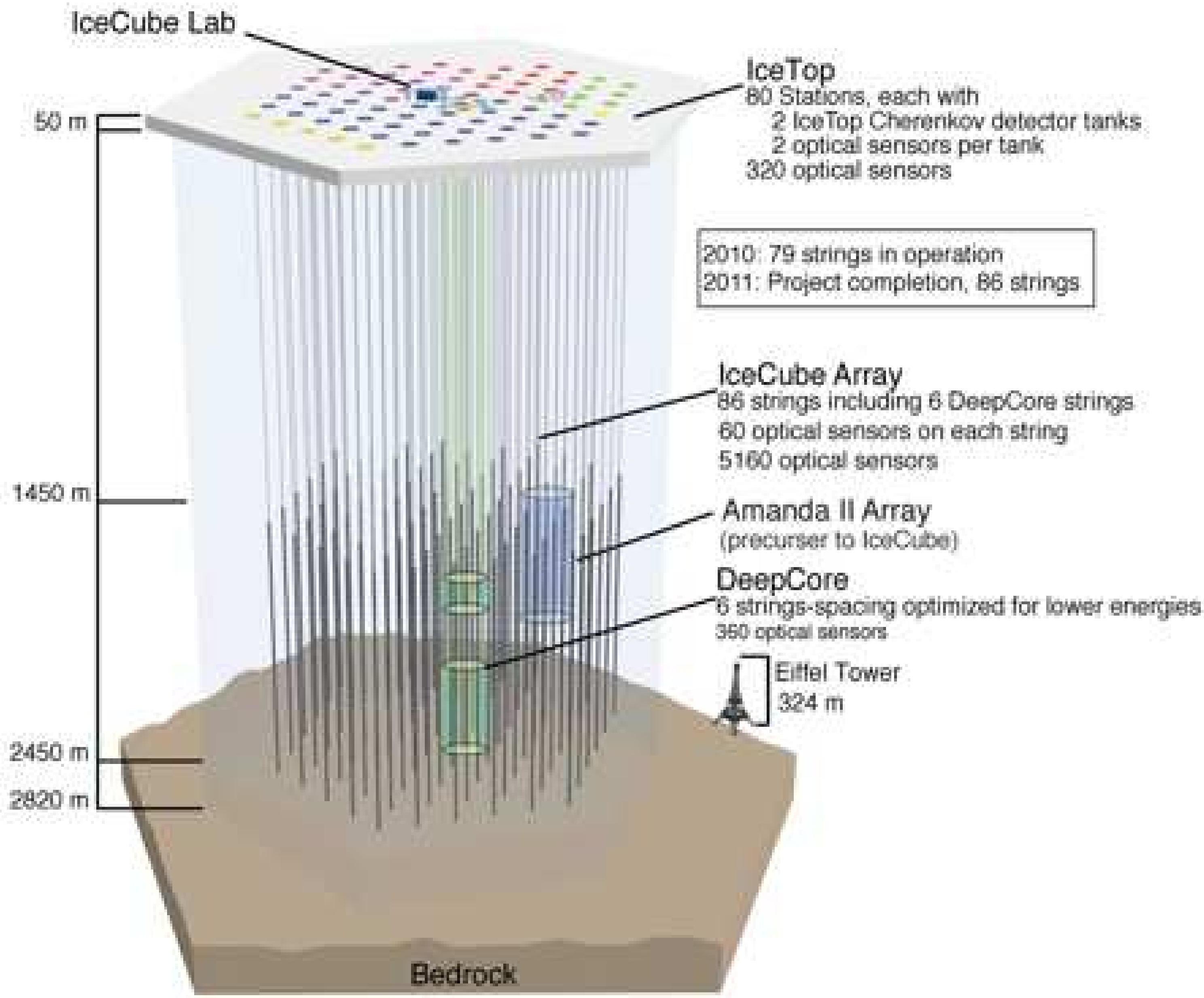
INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

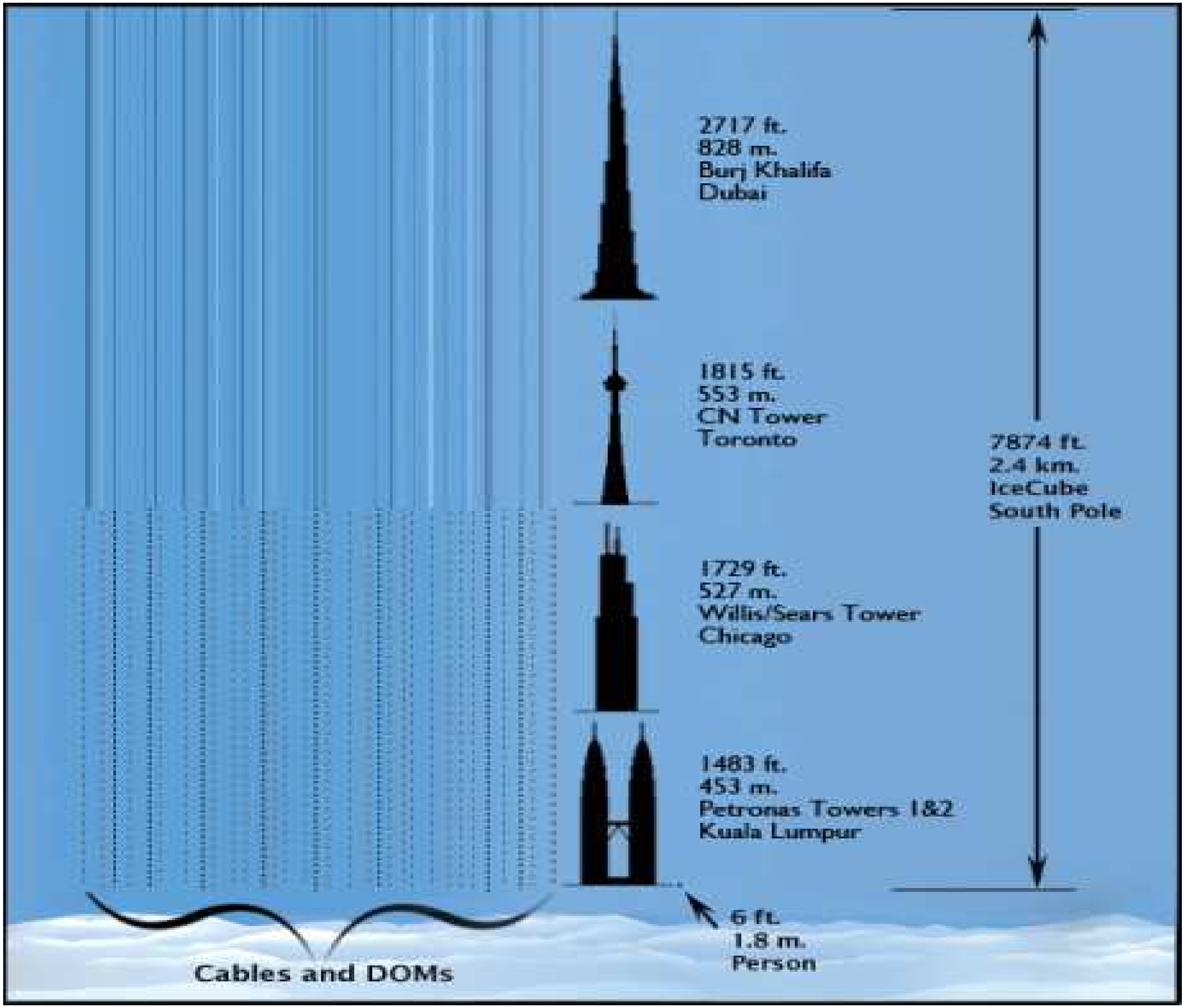
IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-, no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

KM3Net in Mediteranian sea: ORCA


IceCube at the South Pole
Dark sector
Skiway
South Pole
Dome
IceCube
Outline


IceCube Lab
IceTop
80 Stations, each with
2 IceTop Cherenkov detector tanks
2 optical sensors per tank
320 optical sensors
2010: 79 strings in operation
2011: Project completion, 86 strings
IceCube Array
86 strings including 6 DeepCore strings
60 optical sensors on each string
5160 optical sensors
Amanda II Array
(precurser to IceCube)
DeepCore
6 strings-spacing optimized for lower energies
360 optical sensors
Eiffel Tower
324 m
50 m
1450 m
2450 m
2820 m
Bedrock


2717 ft.
828 m.
Burj Khalifa
Dubai
1815 ft.
553 m.
CN Tower
Toronto
7874 ft.
2.4 km.
IceCube
South Pole
1729 ft.
527 m.
Willis/Sears Tower
Chicago
1483 ft.
453 m.
Petronas Towers 1&2
Kuala Lumpur
6 ft.
1.8 m.
Person
Cables and DOMs

Conclusions

There are a number of real (neutrinos, neutron + anti-neutron) and hypothetical (neutralinos, gluinos in minimal SUSY extension of ST) particles which can be Majorana fermions.

Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L.

Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^{\nu}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}_m^{\nu}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The $(\beta\beta)_{0\nu}$-decay experiments:

– Are testing the status of L conservation, can establish the Majorana nature of ν_j;

– Can provide unique information on the ν mass spectrum;

– Can provide unique information on the absolute scale of ν masses;

– Can provide information on the Majorana CPV phases;

– Provide crtitical tests of neutrino-related BSM theoretical ideas.
$T^{0\nu}_{1/2} = 10^{25}$ yr probes $|<m>| \sim 0.1$ eV;
$T^{0\nu}_{1/2} = 10^{25}$ yr probes $\Lambda_{LNV} \sim 1$ TeV.

– Synergy with searches of BSM physics at LHC.

Program of Research - Challenging Problems:

- determination of the nature - Dirac or Majorana, of massive neutrinos ($(\beta\beta)_{0\nu}$-decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+, SuperNEMO, MAJORANA, AMORE,...).

- determination of the status of the CP symmetry in the lepton sector (T2K, NOνA; DUNE, T2HK)

- determination of the neutrino mass ordering (JUNO, RENO50; ORCA, PINGU (IceCube), HK, INO; T2K + NOνA; DUNE (future); + T2HKK (future)) ;

- determination of the absolute neutrino mass scale, or min(m_j) (KATRIN, new ideas; cosmology);

The program of research extends beyond 2030.

We are at the beginning of the Road...

The future of neutrino physics is bright.

Supporting Slides

Understanding the Neutrino Mass and Mixing Patterns
(The Quest for Nature's Message)

Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics. It is part of the more general fundamental problem in particle physics of understanding the origins of flavour in the quark and lepton sectors, i.e., of the patterns of quark masses and mixing, and of the charged lepton and neutrino masses and of neutrino mixing.

With the observed pattern of neutrino mixing Nature is sending us a Message. The Message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The Message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

Understanding the Pattern of Neutrino Mixing: Symmetry Approach.

Examples of Predictions and Correlations.

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}((1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13}))$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545$ **(small uncert.)**.

- $\sin^2\theta_{12} \cong \frac{1}{3}((1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13})) \cong 0.340$.
- $\delta = 0$ **or** π; $\delta = \pi/2$ **or** $3\pi/2$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

Improvements of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, together with the measurement of the Dirac phase in the PMNS mixing matrix, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Understanding the Pattern of Neutrino Mixing: Predictions for the CPV Phase δ.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\mathrm{atm}} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\mathrm{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$,

$\theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 \mp 0.10.$

- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^{\dagger}(\theta_{ij}^{\ell}, \delta^{\ell})\ Q(\psi, \omega) U_{\text{TBM,BM,LC,...}}\ \bar{P}(\xi_1, \xi_2)\,,$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

• $U_{\text{lep}}^{\dagger}(\theta_{ij}^{\ell}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;

• $U_{\text{TBM,BM,LC,...}}\ \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;

• $Q(\psi, \omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix}; \quad \mu-\tau \ \text{symmetry}: \ \theta^{\nu}_{23} = \mp\pi/4\,;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta^{\nu}_{12} = \pi/6\,.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

• U_{TBM}: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected .

• U_{BM}: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

$U_{\text{TBM(BM)}}$: Groups A_4, T' (S_4), ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• U_{GRA}: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• U_{LC}: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

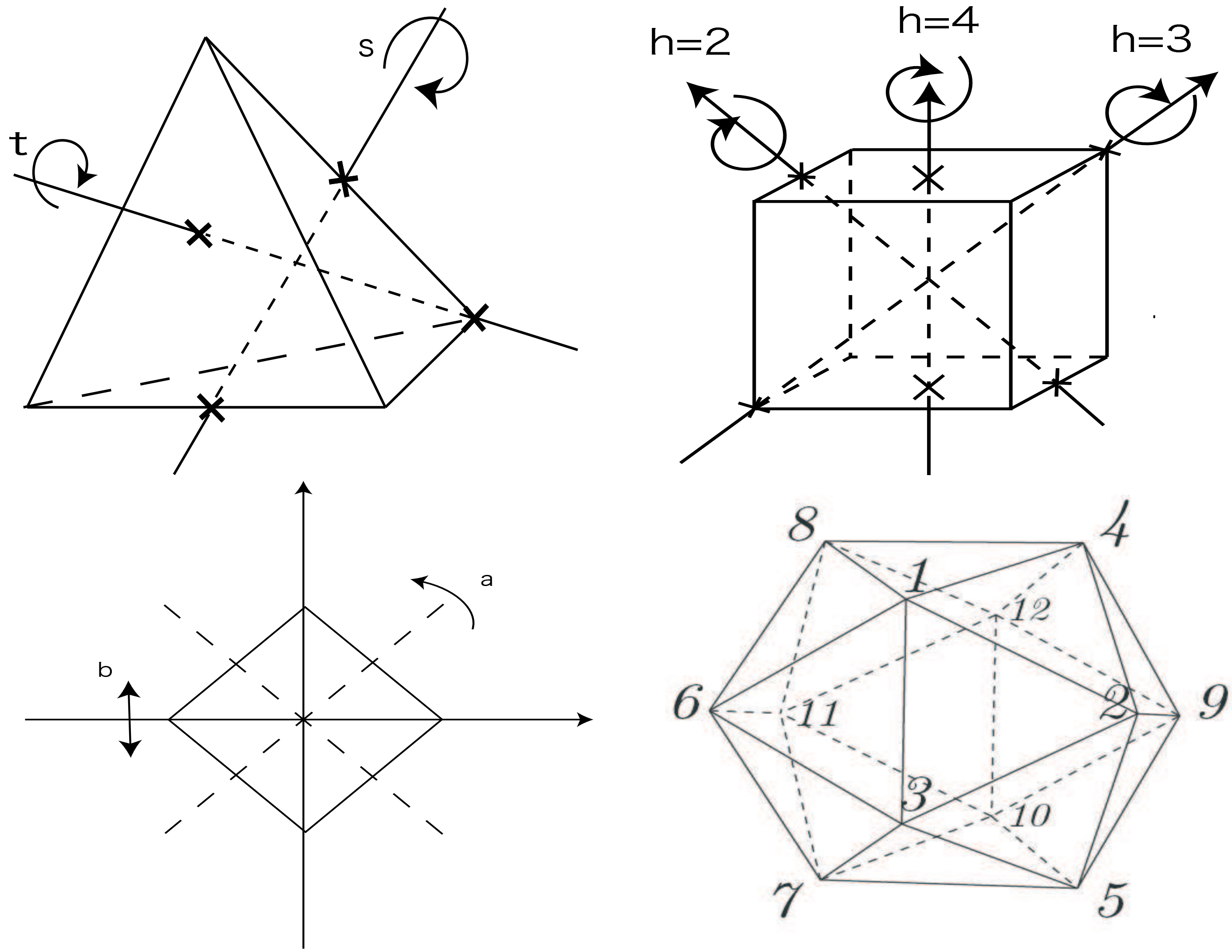
S.T.P., 1982

• U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^{ν}_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta^{\nu}_{13} = 0$, $\theta^{\nu}_{23} = \mp\pi/4$.
They differ by the value of θ^{ν}_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^{\nu}_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	$S,\ T,\ U$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{2},\ \mathbf{3},\ \mathbf{3}'$
A_4	12	$S,\ T$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}'',\ \mathbf{3}$
T'	24	$S,\ T,\ R$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}'',\ \mathbf{2},\ \mathbf{2}',\ \mathbf{2}'',\ \mathbf{3}$
A_5	60	$S,\ T$	$\mathbf{1},\ \mathbf{3},\ \mathbf{3}',\ \mathbf{4},\ \mathbf{5}$
D_{10}	20	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
D_{12}	24	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4,\ \mathbf{2}_5$

Number of elements, generators and irreducible representations of some discrete groups.

In all cases TBM, BM (LC), GRA, GRB, HG:

- New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

S.T.P., arXiv:1405.6006

For arbitrary fixed θ^{ν}_{12} and any θ_{23}
("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right].$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\,\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^\circ$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.

- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^\circ$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.

- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^\circ$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.

- HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^\circ$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.

- BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ one can distiguish between different symmetry forms of $\tilde{U}_{\nu}$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340; P. Coloma *et al.*, arXiv:1203.5651; R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984.

Theoretical Model Predictions

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.(The pre

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.

- ν_j - Majorana particles.

- Diagonalisation of M_ν: $U_{\text{TBM}}\Phi$, $\Phi = \text{diag}(1, 1, 1(i))$

- U_{TBM} "corrected" by

$$U^\dagger_{\text{lep}}\, Q = R_{12}(\theta^\ell_{12})\, R_{23}(\theta^\ell_{23}) Q\,, \quad Q = \text{diag}(1, e^{i\phi}, 1)$$

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

• T': double covering of A_4 (tetrahedral symmetry group).

• T': $\mathbf{1}$, $1'$, $1''$; $\mathbf{2}$, $2'$, $2''$; $\mathbf{3}$.

• T' model: $\psi_{eL}(x)$,$\psi_{\mu L}(x)$,$\psi_{\tau L}(x)$ - triplet of T';
$e_R(x)$,$\mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T';
$\nu_{eR}(x)$,$\nu_{\mu R}(x)$,$\nu_{\tau R}(x)$ - a triplet of T';
the Higgs doublets $H_u(x)$, $H_d(x)$ - singlets of T'.

• The discrete symmetries of the model are $T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2:

$$\text{NO spectrum A}: \quad (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3}$$

$$\text{NO spectrum B}: \quad (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3}$$

$$\text{IO spectrum}: \quad (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3}$$

$$\text{NO A}: \quad \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2}\ \text{eV}\,,$$

$$\text{NO B}: \quad \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2}\ \text{eV}\,,$$

$$\text{IO}: \quad \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2}\ \text{eV}\,,$$

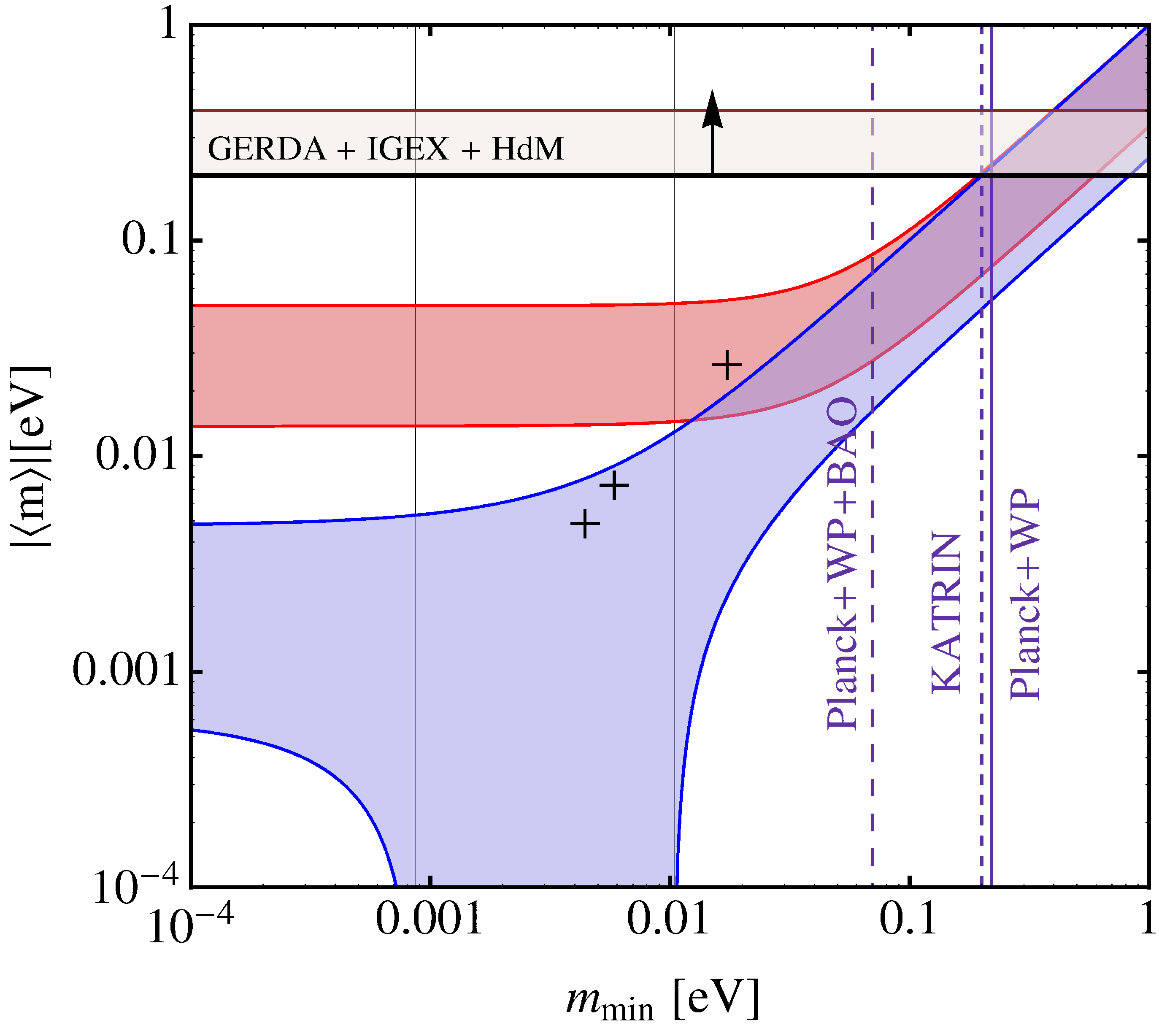
• $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

Given the values of $\theta_{12}, \theta_{23}, \theta_{13}$, $\delta, \alpha_{21}, \alpha_{31}$ are predicted:

$\delta \cong 3\pi/2$ (266°) (or $\pi/2$ (94°));

NO A: $\alpha_{21} \cong +47.0^\circ$ (or -47.0°) $(+2\pi)$,

$\alpha_{31} \cong -23.8^\circ$ (or $+23.8^\circ$) $(+2\pi)$.



1
0.1
0.01
0.001
10^{-4}
$|\langle m \rangle|$ [eV]
10^{-4}
0.001
0.01
0.1
1
$m_{\rm min}$ [eV]
GERDA + IGEX + HdM
Planck+WP+BAO
KATRIN
Planck+WP