



Theory of branched flows and the statistics of extreme waves in random media

Ragnar Fleischmann

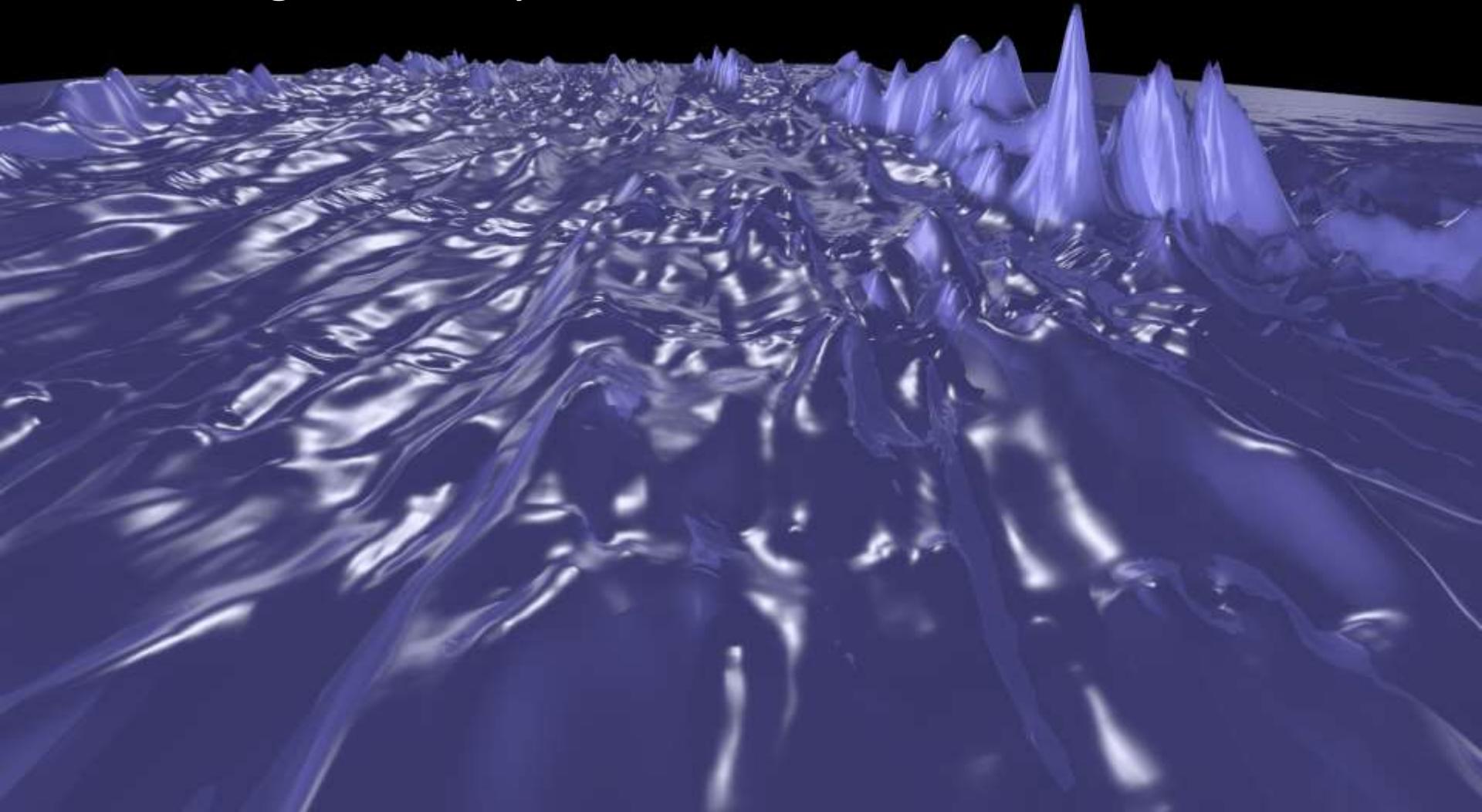
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Branched flow

very general physical mechanism

large intensity fluctuations & extreme events



Overview

Part I

- The phenomenon of branching
- In which systems can it be observed?
- Statistics of random caustics
- Characteristic transport length scale
- Wave intensity/height statistics

Part II

- Catastrophes, Caustics and the statistics of extreme waves
- Branching of Tsunami waves
- Teasers/outlook

Overview

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Literature

Catastrophes and Caustics

- Berry, M.V., Klein, S., 1996. Colored diffraction catastrophes. PNAS 93, 2614–2619.
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Branched Flow (General, Semiconductors, Ocean Waves, Microwaves)

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- Ying, L.H., Zhuang, Z., Heller, E.J., Kaplan, L., 2011. Linear and nonlinear rogue wave statistics in the presence of random currents. Nonlinearity 24, R67–R87. doi:10.1088/0951-7715/24/11/R01

Literature

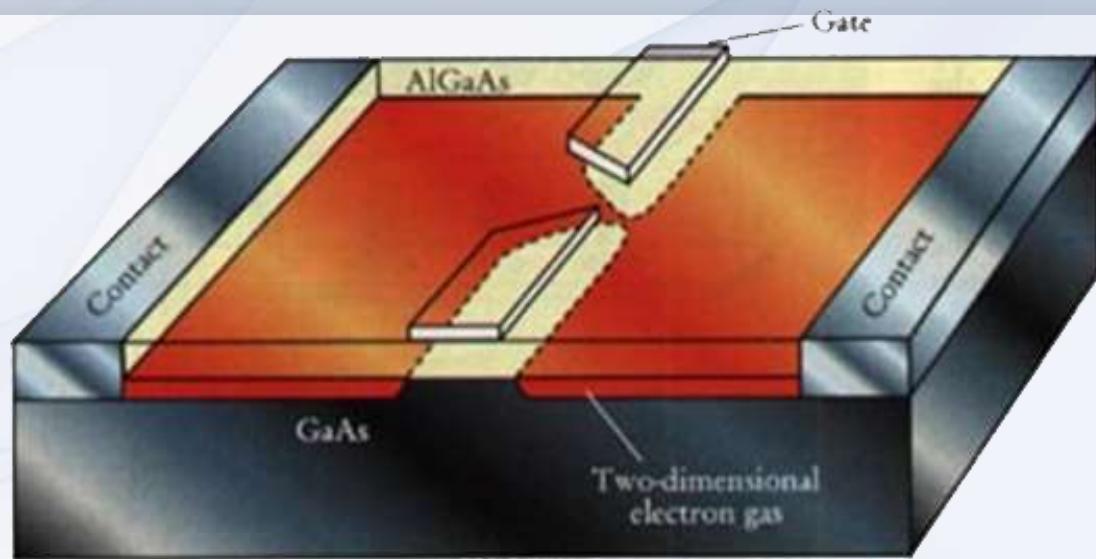
Random systems

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- Schomerus, H., Titov, M., 2002. Statistics of finite-time Lyapunov exponents in a random time-dependent potential. Phys. Rev. E 66. doi:10.1103/PhysRevE.66.066207

Theses

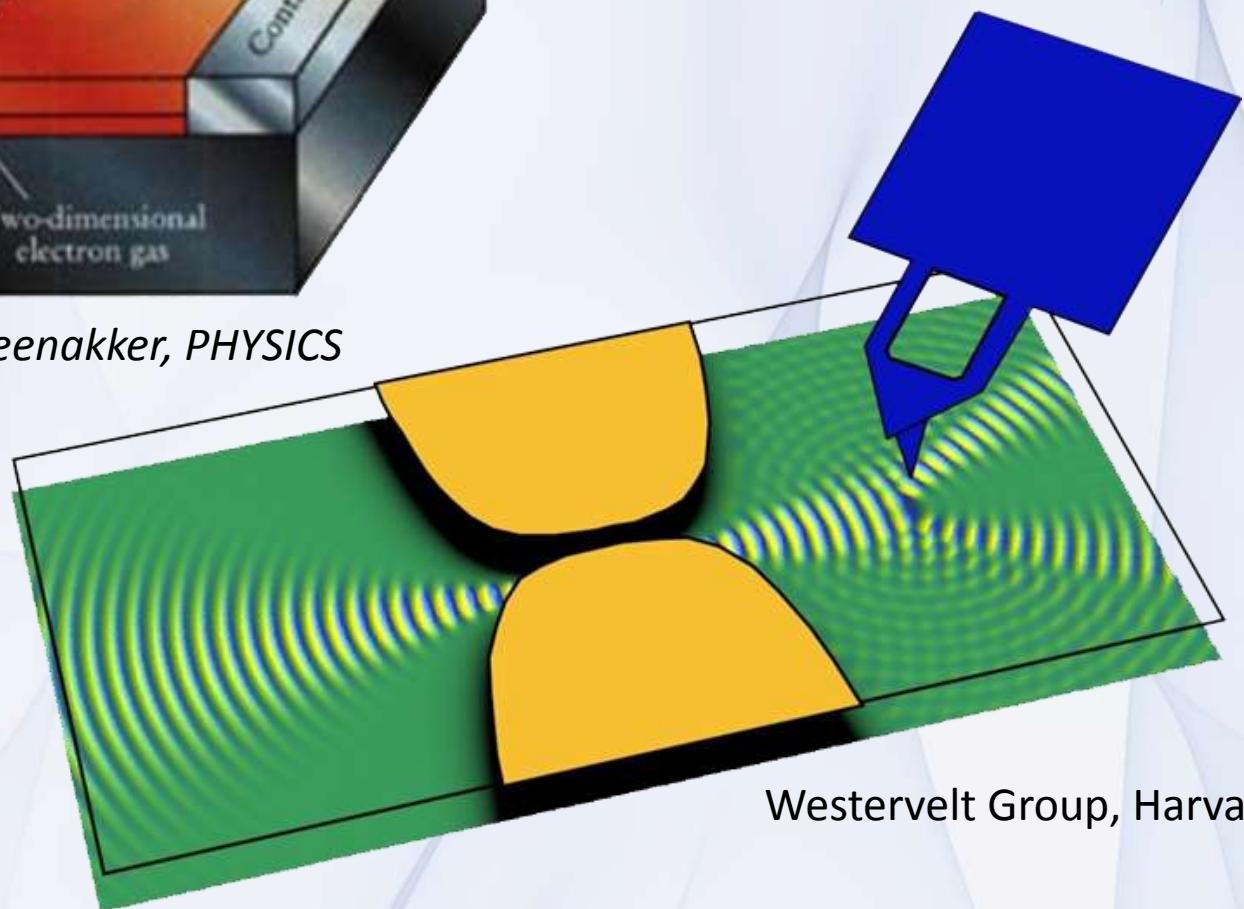
- Scot Shaw, *Propagation in Smooth Random Potentials*, 2002
- Ruven Höhmann, *Experimental tests of random wave models with chaotic microwave billiards*, 2008
- Jakob Metzger, *Branched Flow and Caustics in Two-Dimensional Random Potentials and Magnetic Fields*, 2010
- Sonja Barkhofen, *Microwave Measurements on n-Disk Systems and Investigation of Branching in correlated Potentials and turbulent Flows*, 2013

Ballistic electron transport in the two-dimensional electron gas



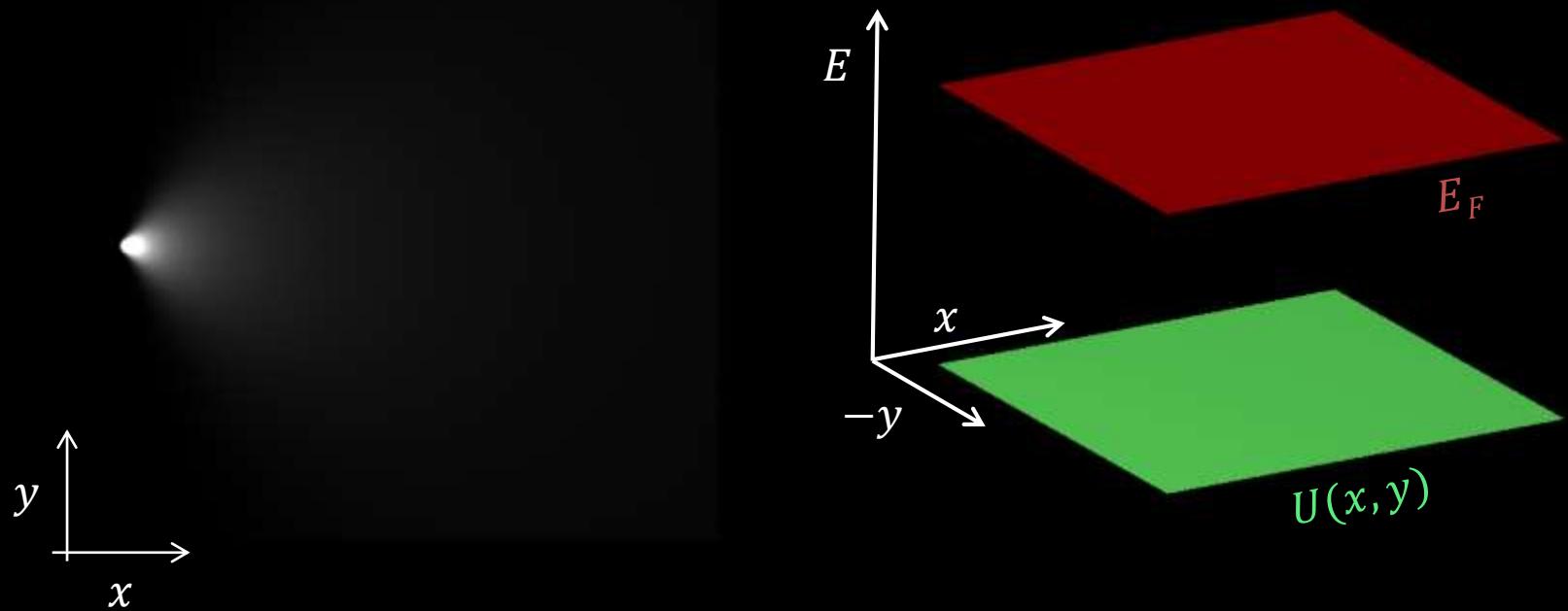
Ballistic quantum point contact

Henk van Houten and Carlo Beenakker, PHYSICS
TODAY, July 1996

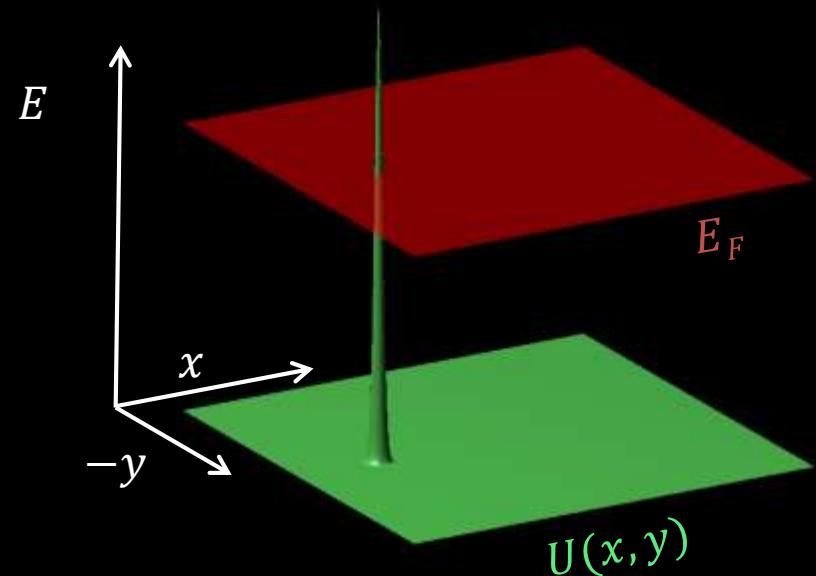
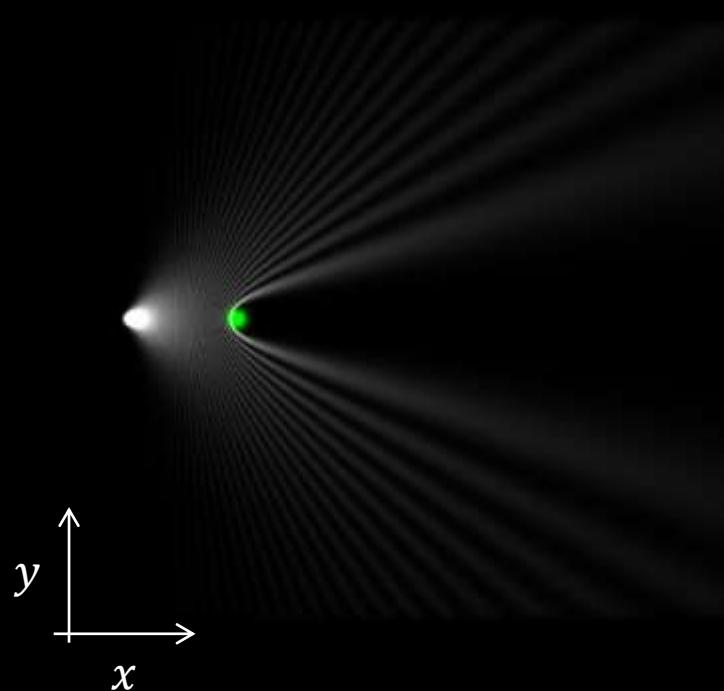


Westervelt Group, Harvard

Wave spreading and...

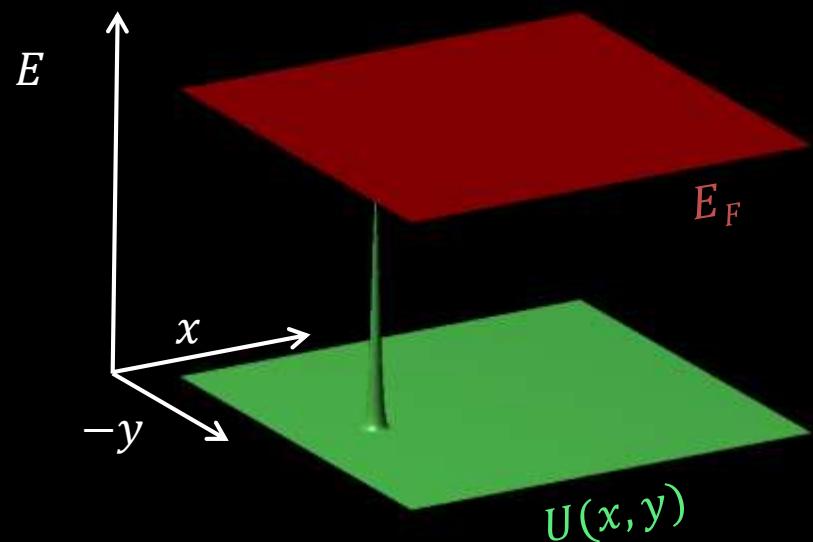
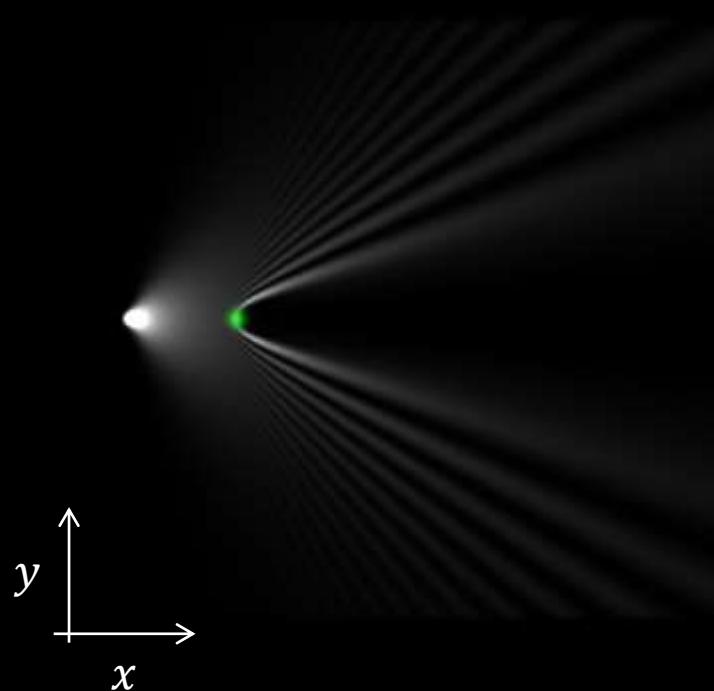


...scattering by impurities



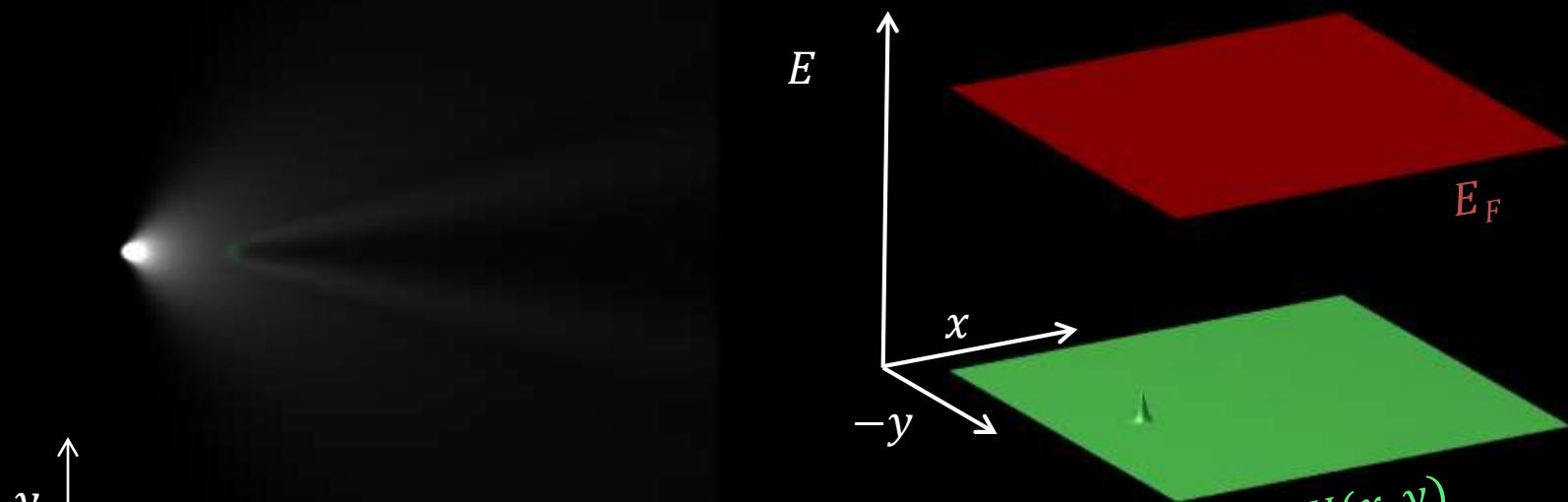
$$\frac{U_0}{E_F} = 160\%$$

...scattering by impurities



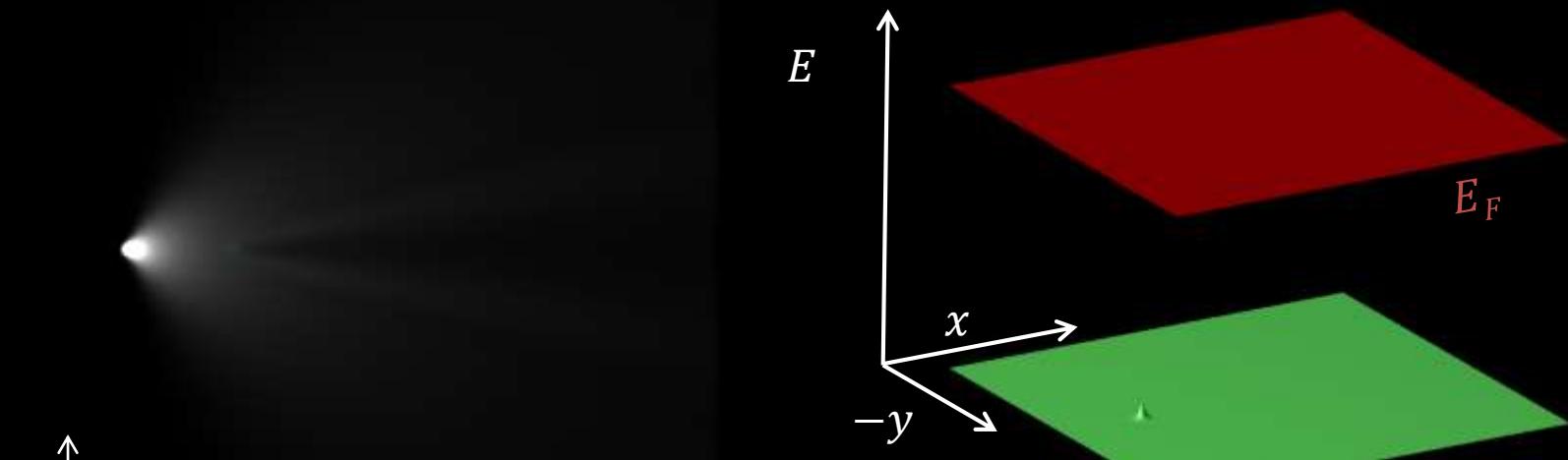
$$\frac{U_0}{E_F} = 80\%$$

...scattering by impurities

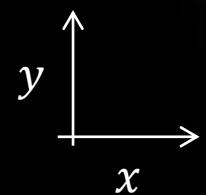


$$\frac{U_0}{E_F} = 10\%$$

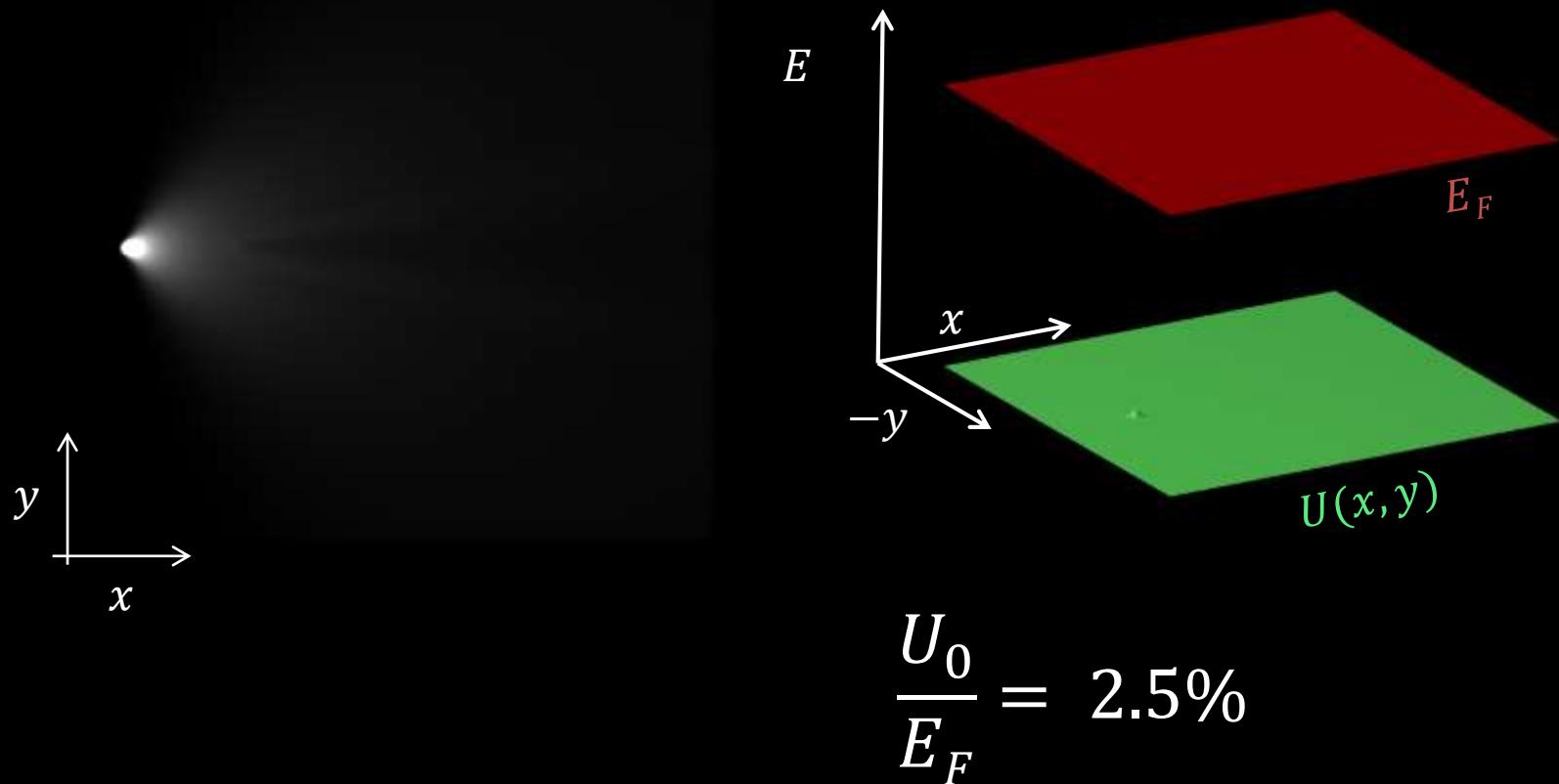
...scattering by impurities



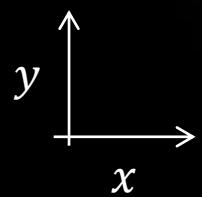
$$\frac{U_0}{E_F} = 5\%$$



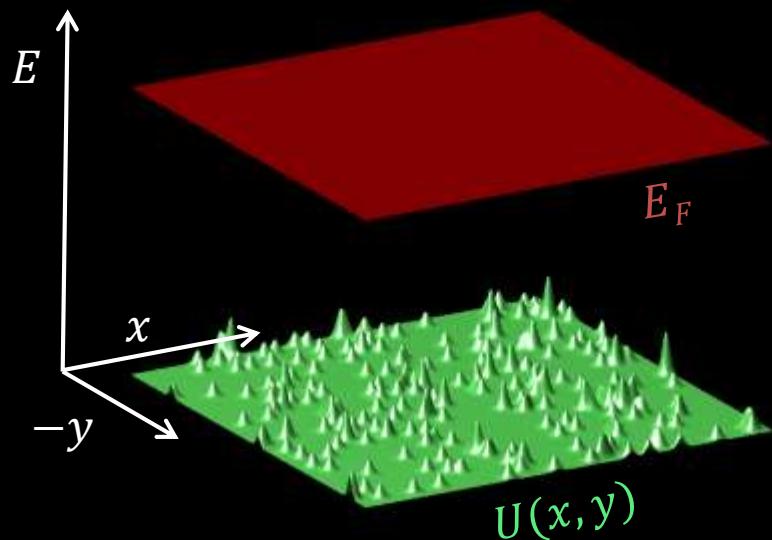
From impurity scattering...



...scattering by impurities

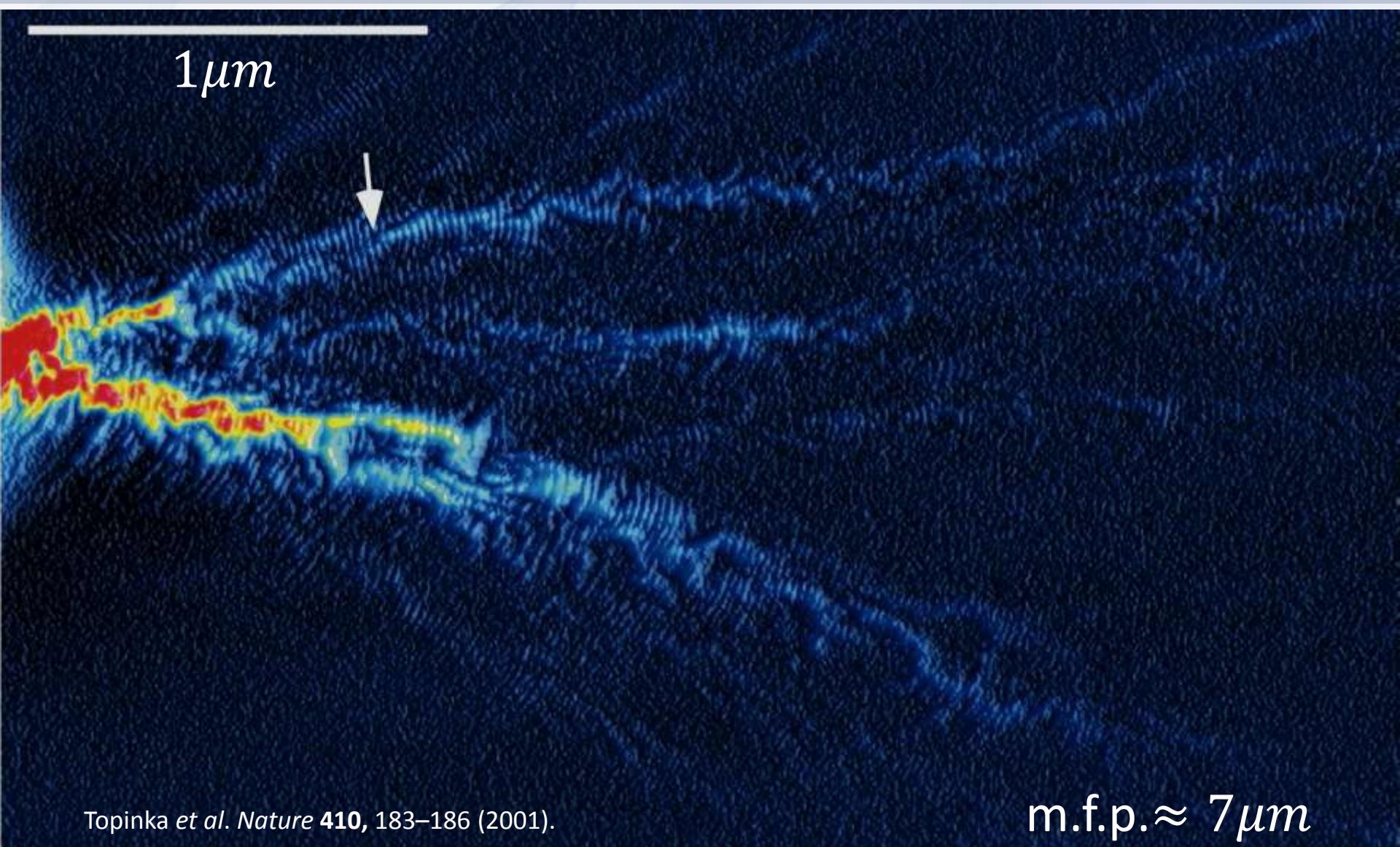


1. Order



$$\frac{U_0}{\langle E \rangle} = 5\%$$

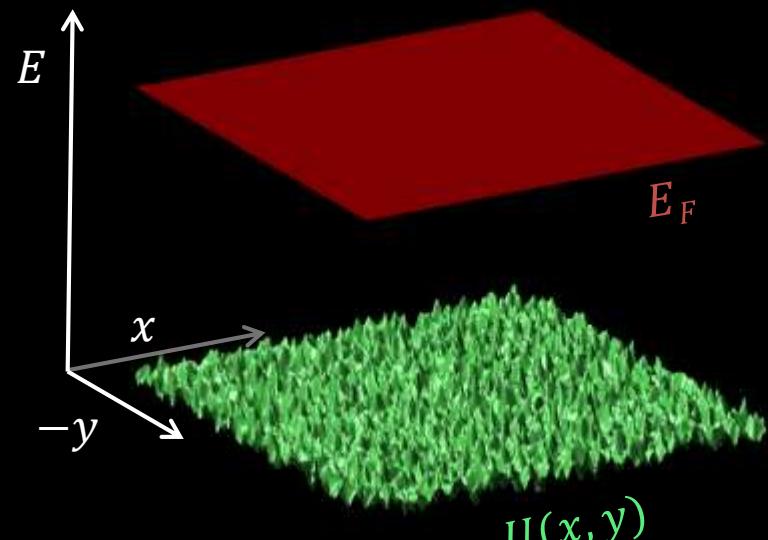
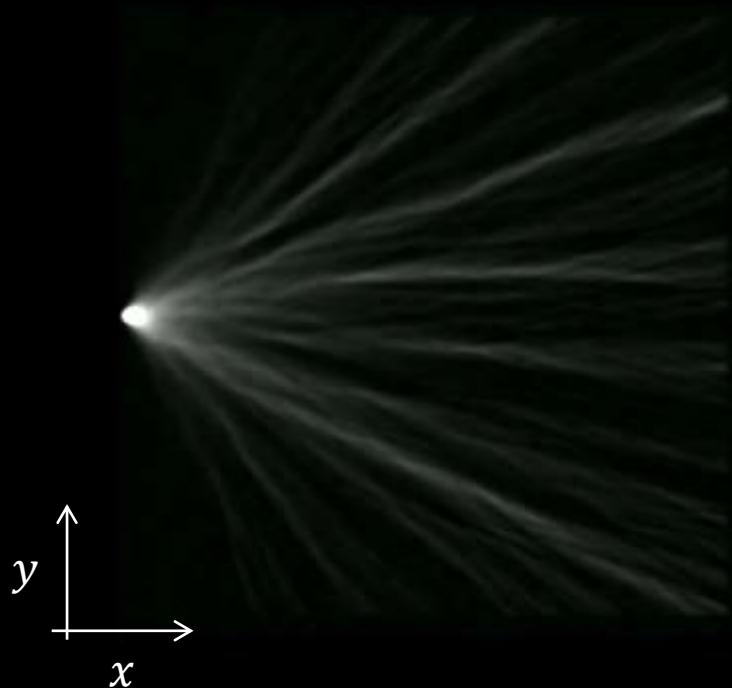
Branched electron flow



Topinka *et al.* *Nature* **410**, 183–186 (2001).

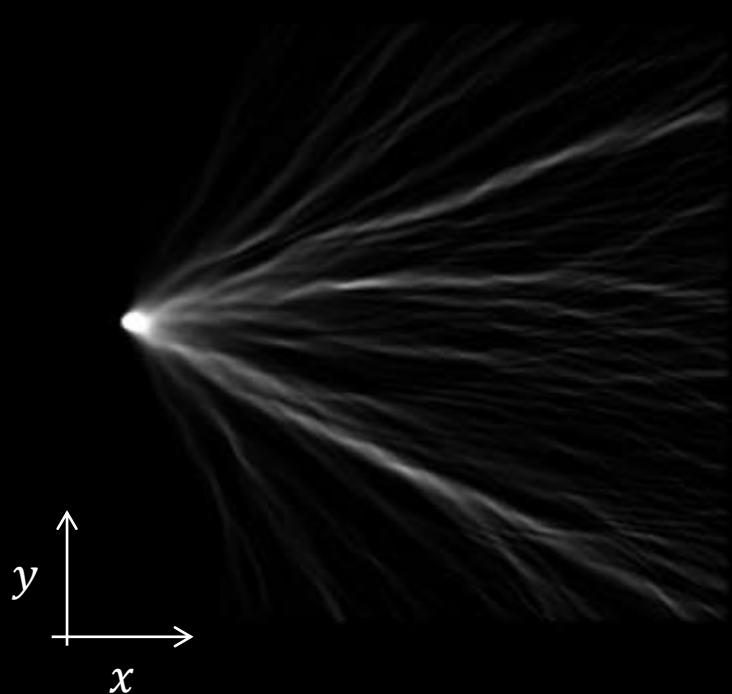
m.f.p. $\approx 7\mu m$

Branched electron flow

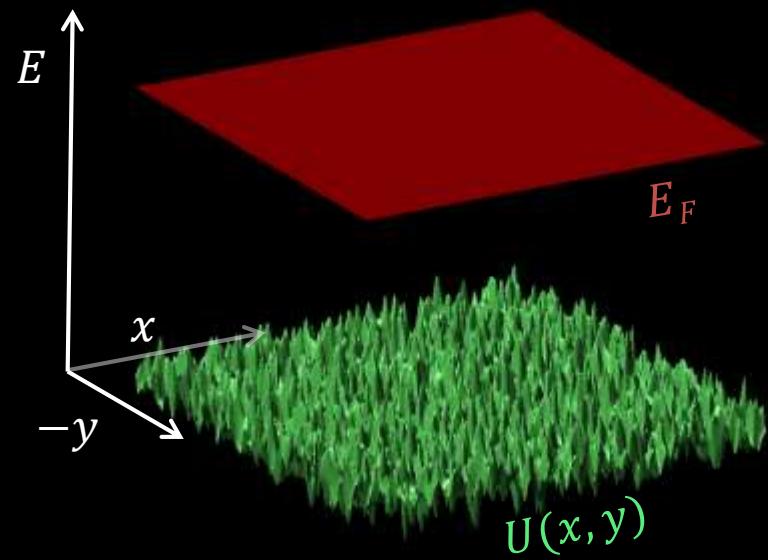


$$\frac{\sqrt{\langle U^2 \rangle}}{E_F} = 2.5\%$$

Branched electron flow



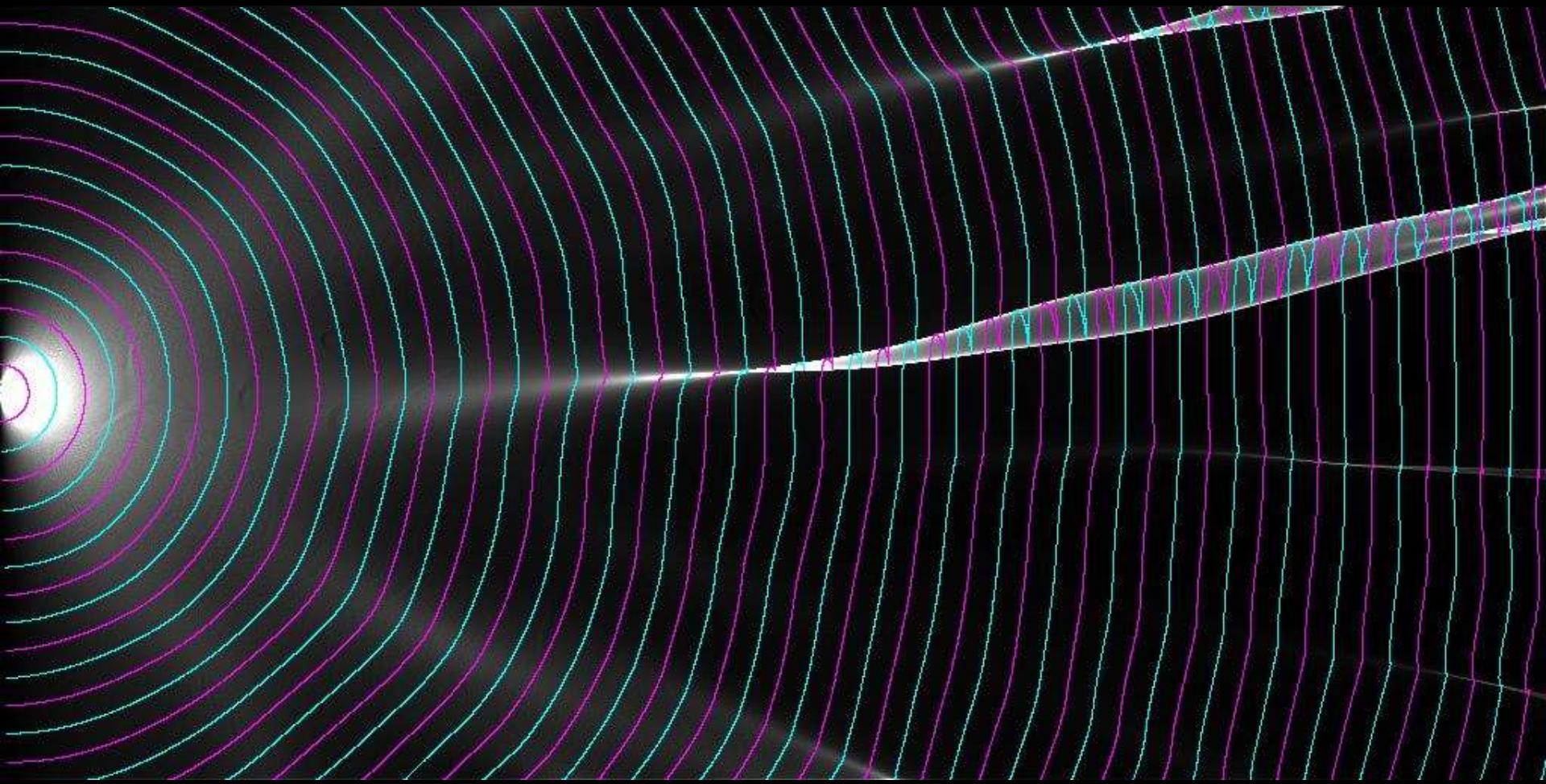
x
 y



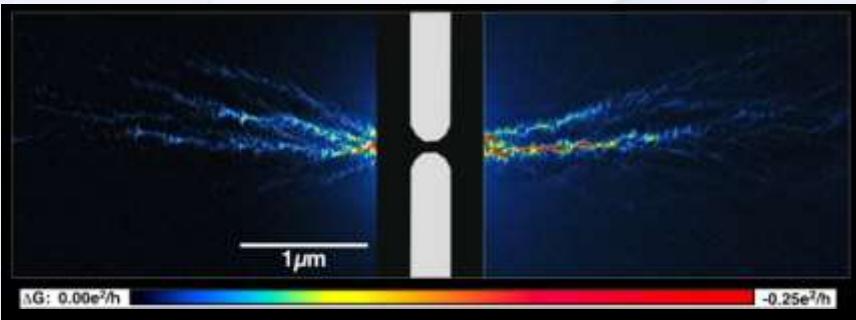
$$\frac{\sqrt{\langle U^2 \rangle}}{E_F} = 5\%$$

The origin of branching

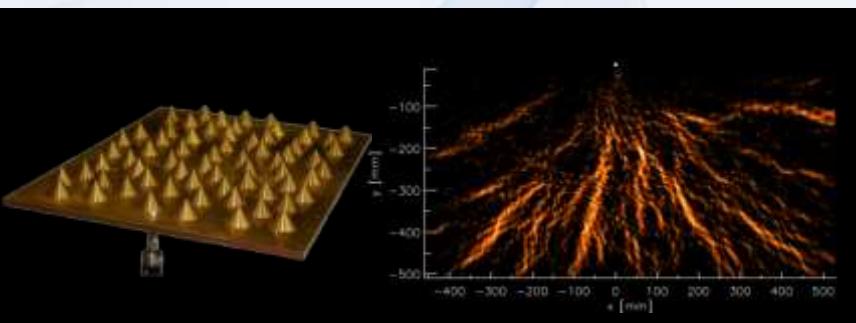
The origin of branching



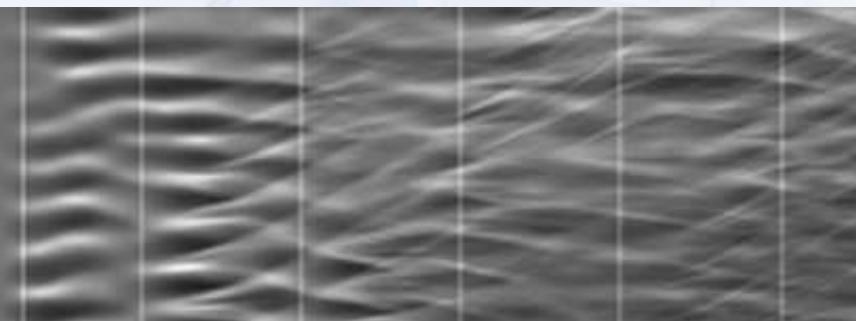
Branched flows and linear rogue waves e.g. in...



...the electron flow
in semiconductors

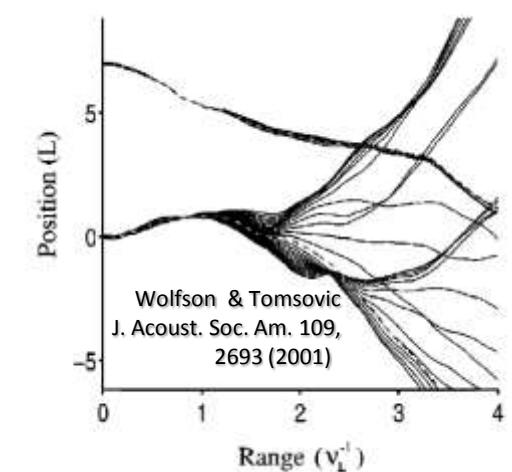


...the transmission
of microwaves



... dynamics of
ocean waves
scattered by
water currents

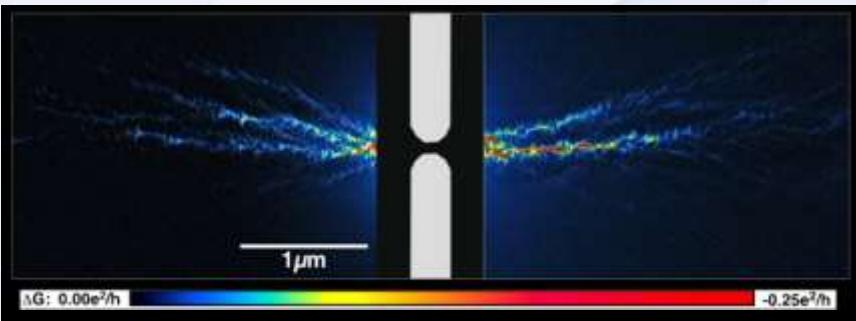
...sound waves in
the ocean



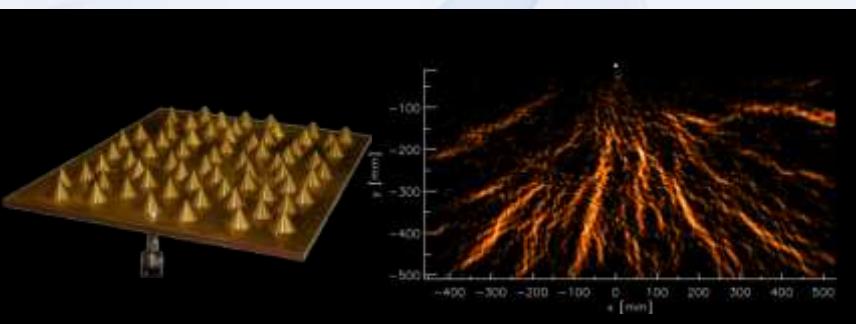
E. J. Heller et al., J. Geophys. Res., 113 C09023 (2008)

Heller europhys.news 2005

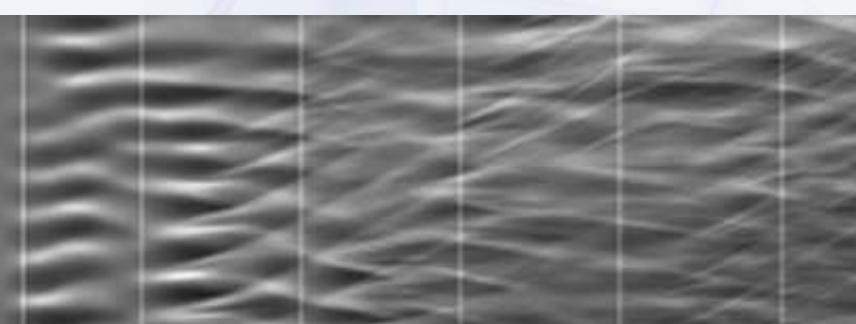
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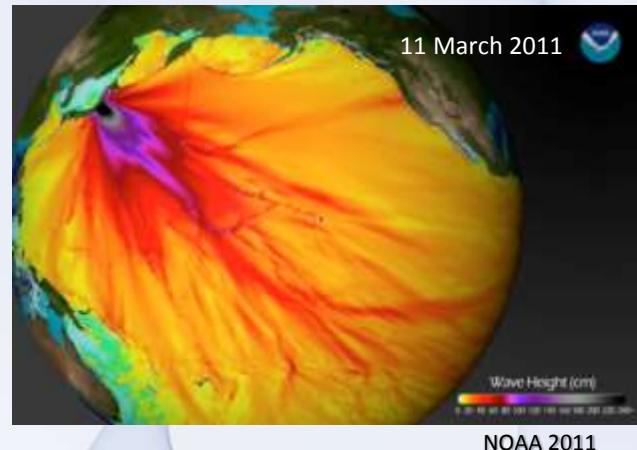
...the electron flow
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...the transmission
of micro waves



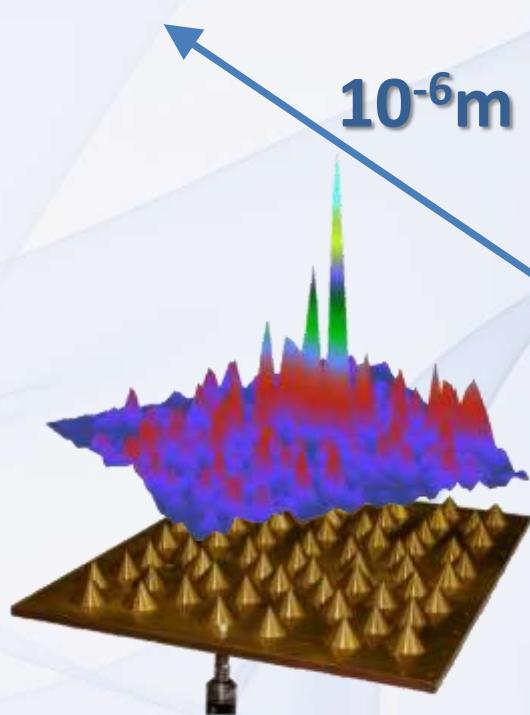
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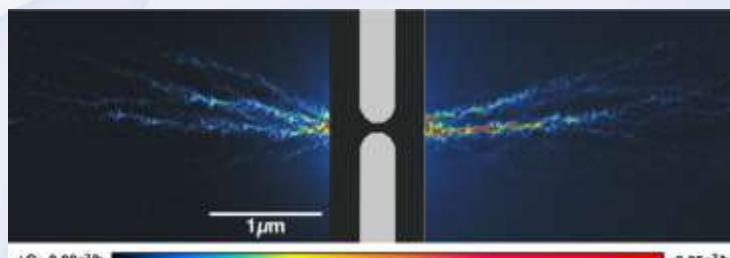
E. J. Heller et al., J. Geophys. Res., 113 C09023 (2008)

...tsunami
propagation?

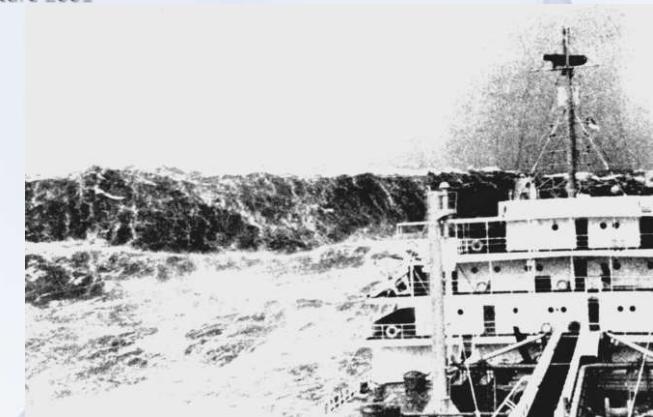
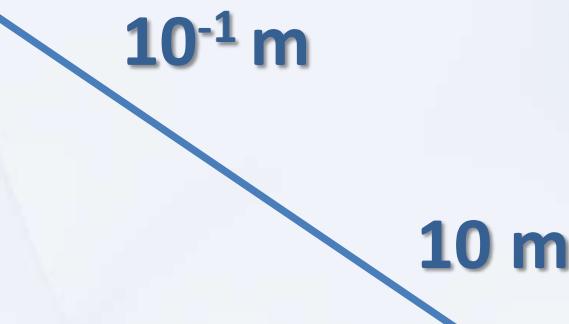
Branching and rogue waves on many scales



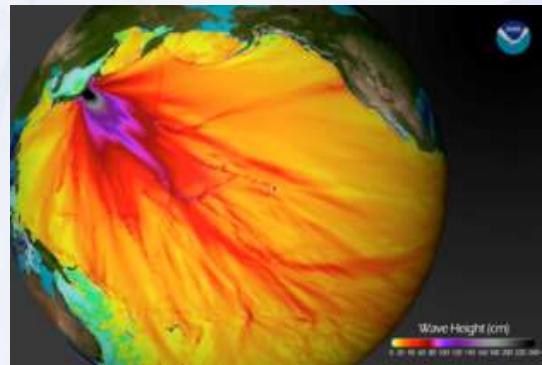
Höhmann et al, PRL 2010



Tokink et al., Nature 2001



Heller europhys.news 2005



NOAA 2011

Random caustics in correlated weakly scattering media

Energy

Kinetic energy of the particles

E

$V(r)$

Random potential

ℓ_c

$$\varepsilon = \frac{\sqrt{\langle V(r)^2 \rangle - \langle V(r) \rangle^2}}{E} \ll 1$$

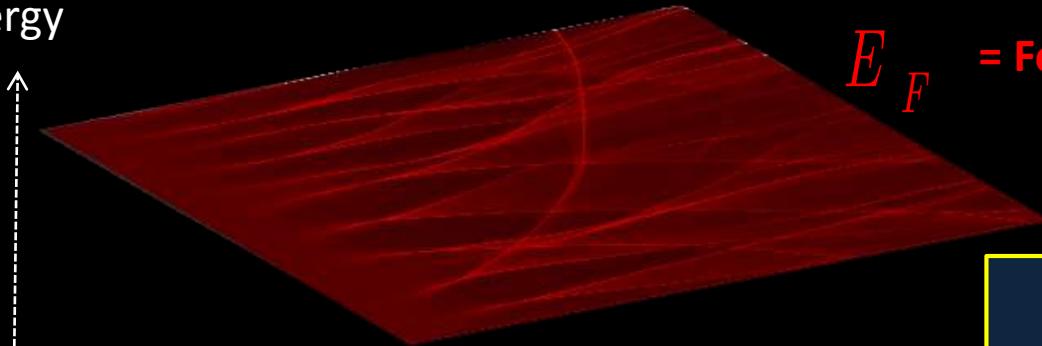
$\lambda \leq \ell_c$

ℓ_c = correlation length,
i.e. typical length scale

Random caustics in correlated weakly scattering media

E.g. electron flow in the two dimensional electron gas of high mobility semiconductors

Energy



$$E_F$$

= Fermi Energy of the electrons

$$\varepsilon = \frac{\sqrt{\langle V(r)^2 \rangle - \langle V(r) \rangle^2}}{E_F} \ll 1$$

$$\lambda_F \leq \ell_c$$

$$V(r)$$

Random Impurities

$$\ell_c$$

ℓ_c = correlation length,
i.e. typical length scale

Waves in weakly scattering complex/random media everywhere

- Light in biological tissue
- Light & sound in the turbulent atmosphere (e.g. twinkling of stars)
- Gravitational lensing of electromagnetic radiation in the universe
- Seismic waves in the inhomogeneous earth
 -
 -
 -

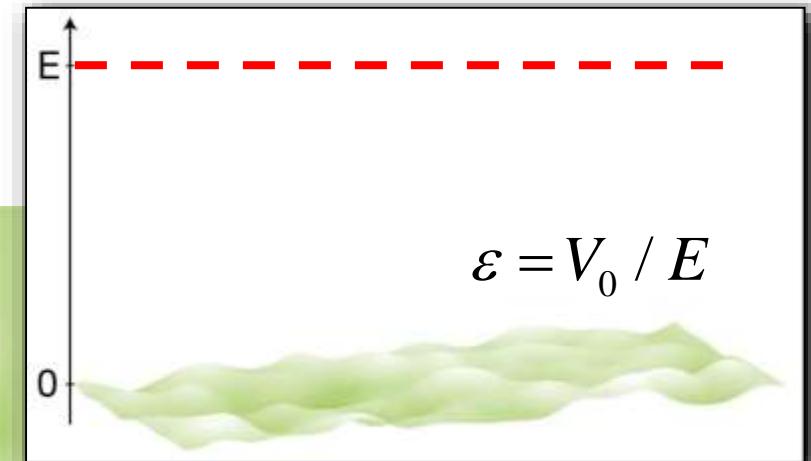
Model: Potential



Gaußian random field

$$\langle V(\vec{r}) \rangle = 0$$

$$\langle V(\vec{r})V(\vec{r}') \rangle = C(|\vec{r} - \vec{r}'|)$$



$V(\vec{r})$: Potential

$$P(V) = \frac{1}{\sqrt{2\pi V_0^2}} e^{-\frac{V^2}{2V_0^2}}$$

$$C(0) = V_0^2$$

Model: Equations of motion



Equations of motion

$$\dot{x} = v_x \quad \dot{v}_x = -\frac{\partial V_x}{\partial x}$$

$$\dot{y} = v_y \quad \dot{v}_y = -\frac{\partial V_y}{\partial y}$$

$$m = 1$$

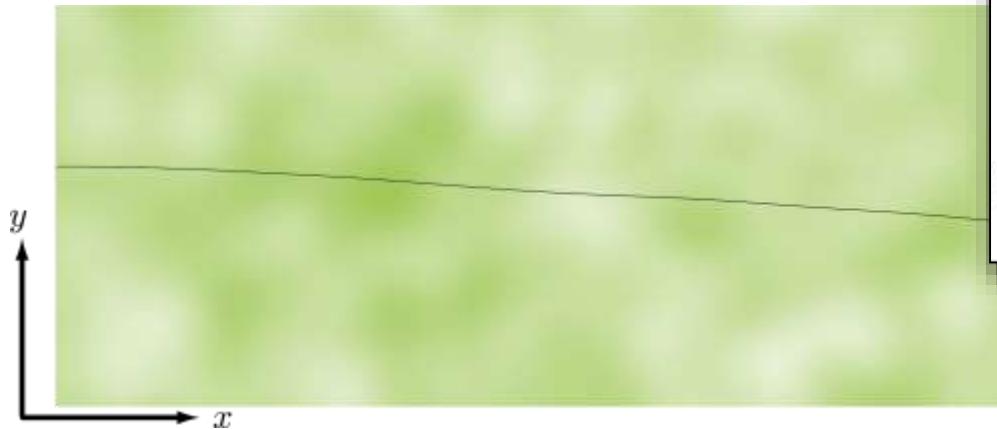
Initial conditions

Plane wave: $\vec{v}(0) = \begin{pmatrix} v_0 \\ 0 \end{pmatrix}$

$$v_0 = \sqrt{2E}$$

Point source: $\vec{r}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Model: Paraxial approximation



Equations of motion

$$\dot{x} = 1$$

$$\dot{v}_x = 0$$

$$\dot{y} = v_y$$

$$\dot{v}_y = -\frac{\partial V}{\partial y}$$

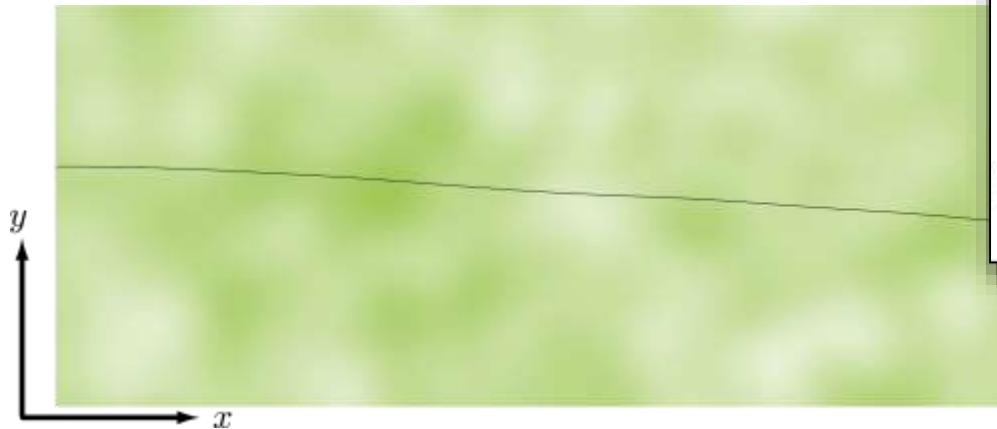
$$m = 1$$

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$$v_0 = \sqrt{2E}$$

Model: Paraxial approximation



Equations of motion

$$x = t$$

$$\dot{y} = v_y \quad \dot{v}_y = -\frac{\partial V}{\partial y}$$

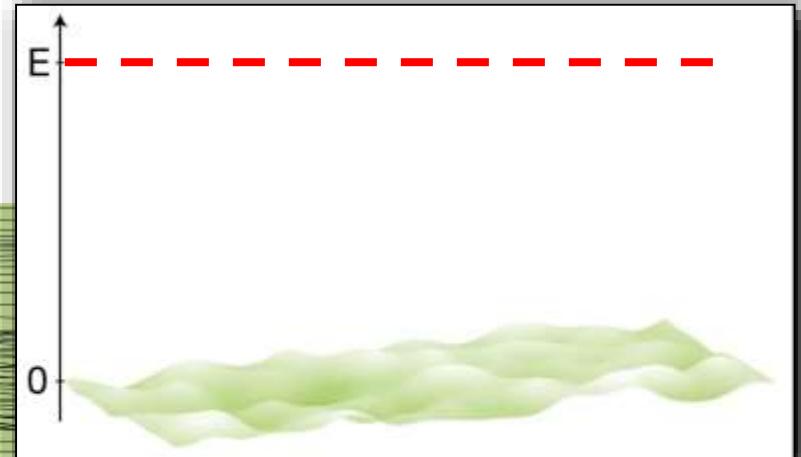
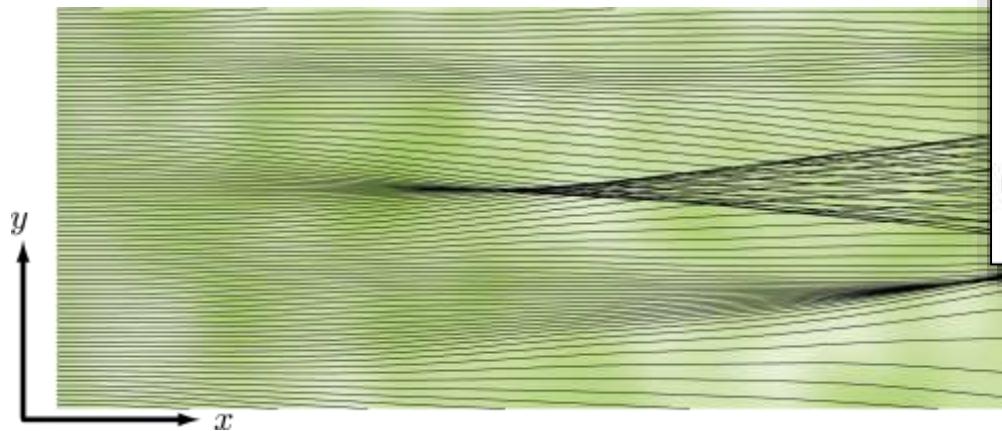
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Formation of Random Caustics



Initial conditions

$$x = t$$

Plane wave: $v_y(0) = 0$

$$\dot{y} = v_y \quad \dot{v}_y = -\frac{\partial V}{\partial y}$$

Formation of Random Caustics

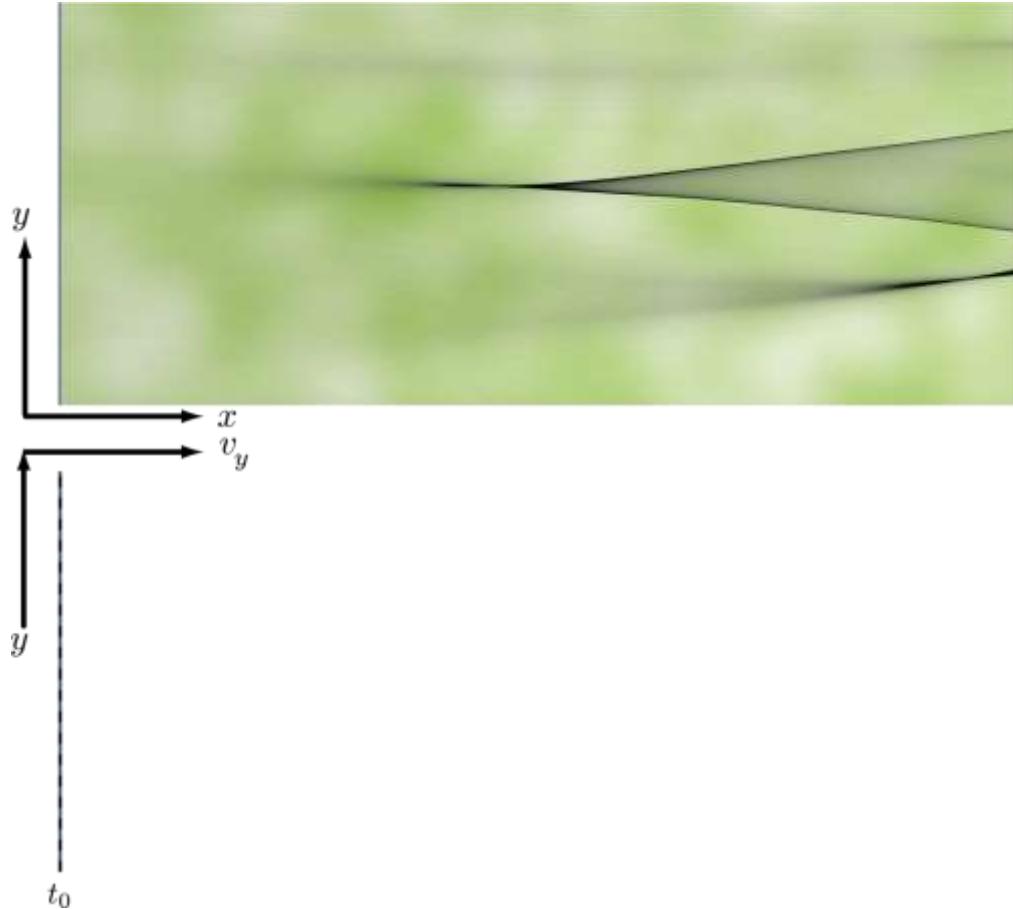


Initial conditions

Plane wave: $v_y(0) = 0$

$x = t$

Formation of Random Caustics

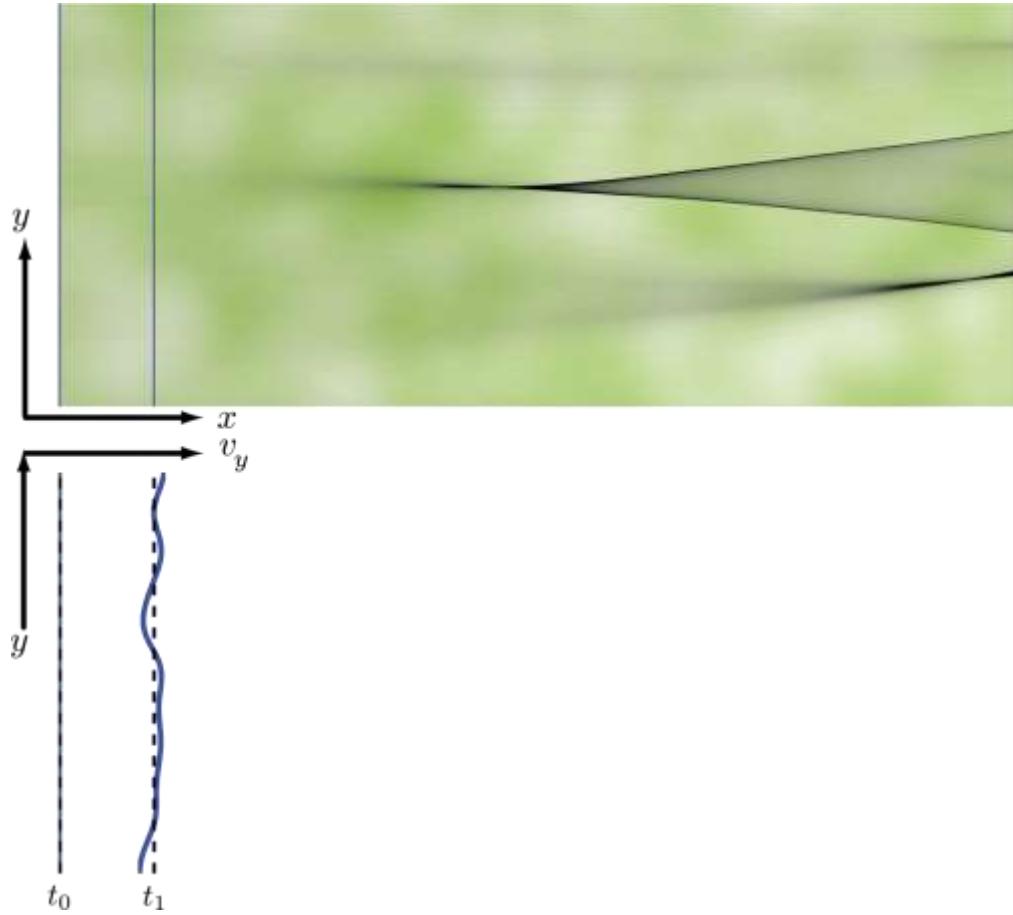


Initial conditions

Plane wave: $v_y(0) = 0$

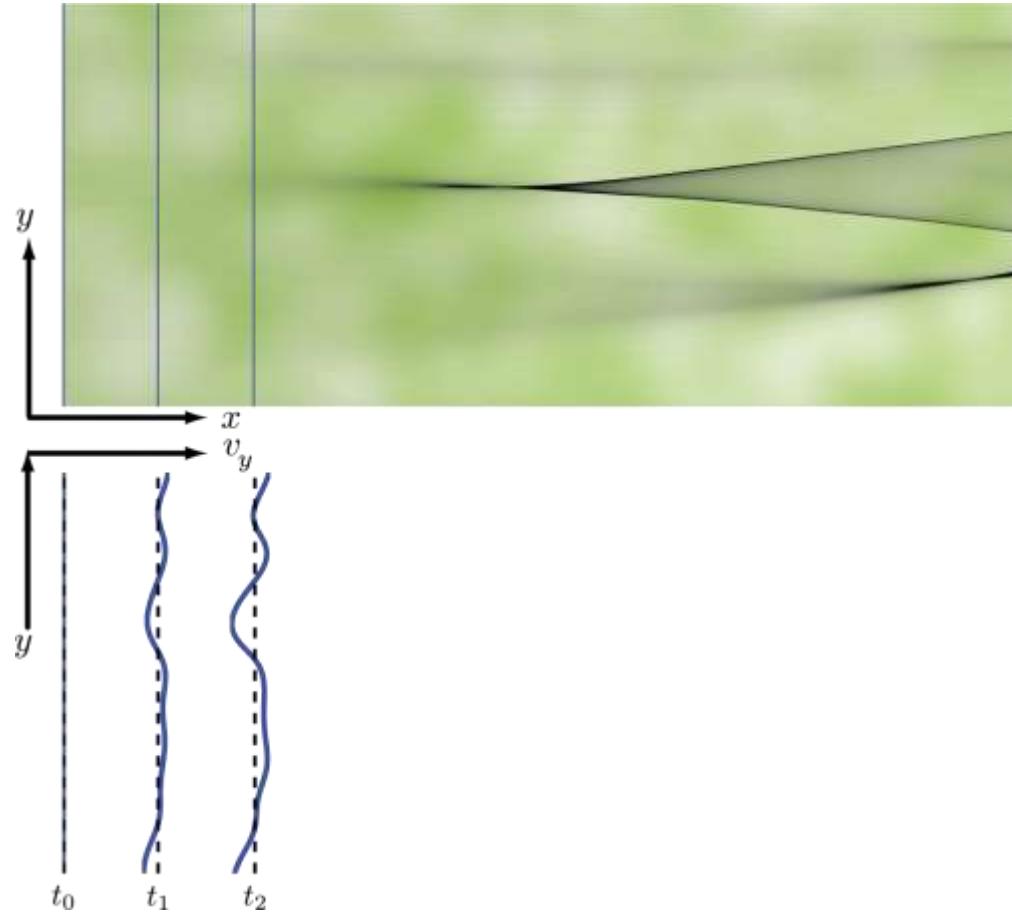
$x = t$

Formation of Random Caustics

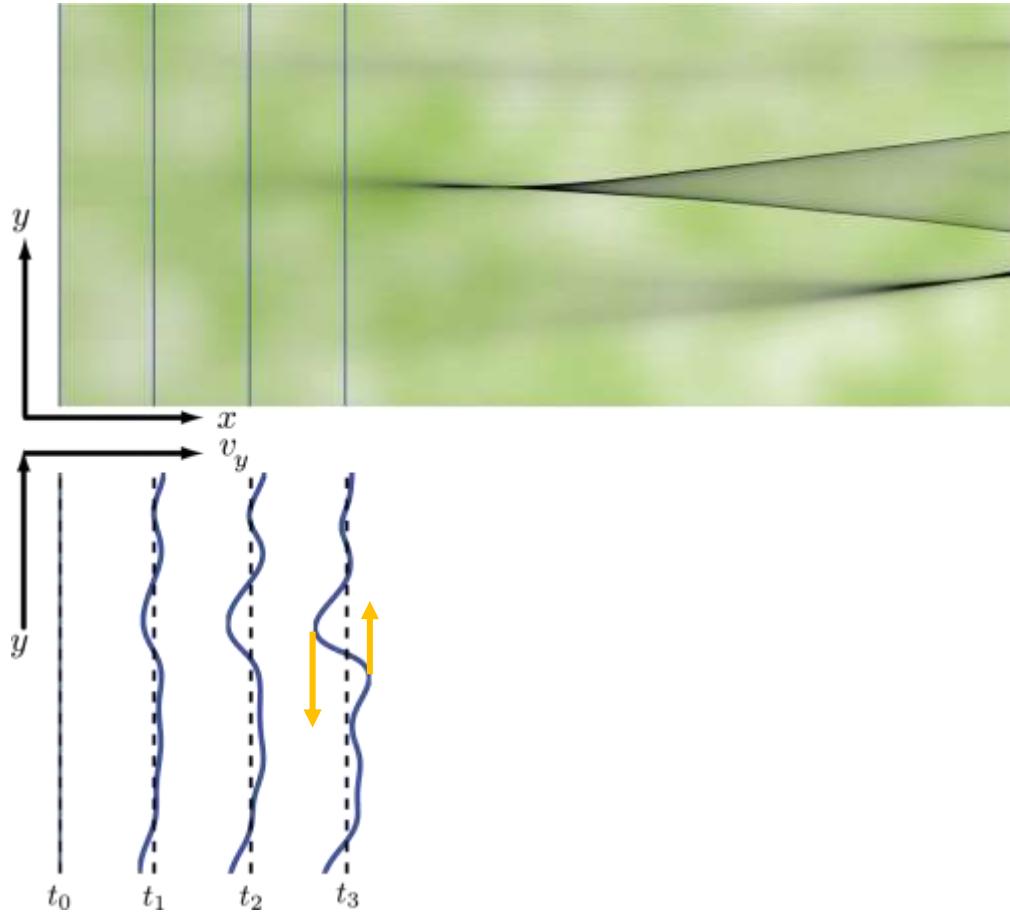


$$x = t$$

Formation of Random Caustics

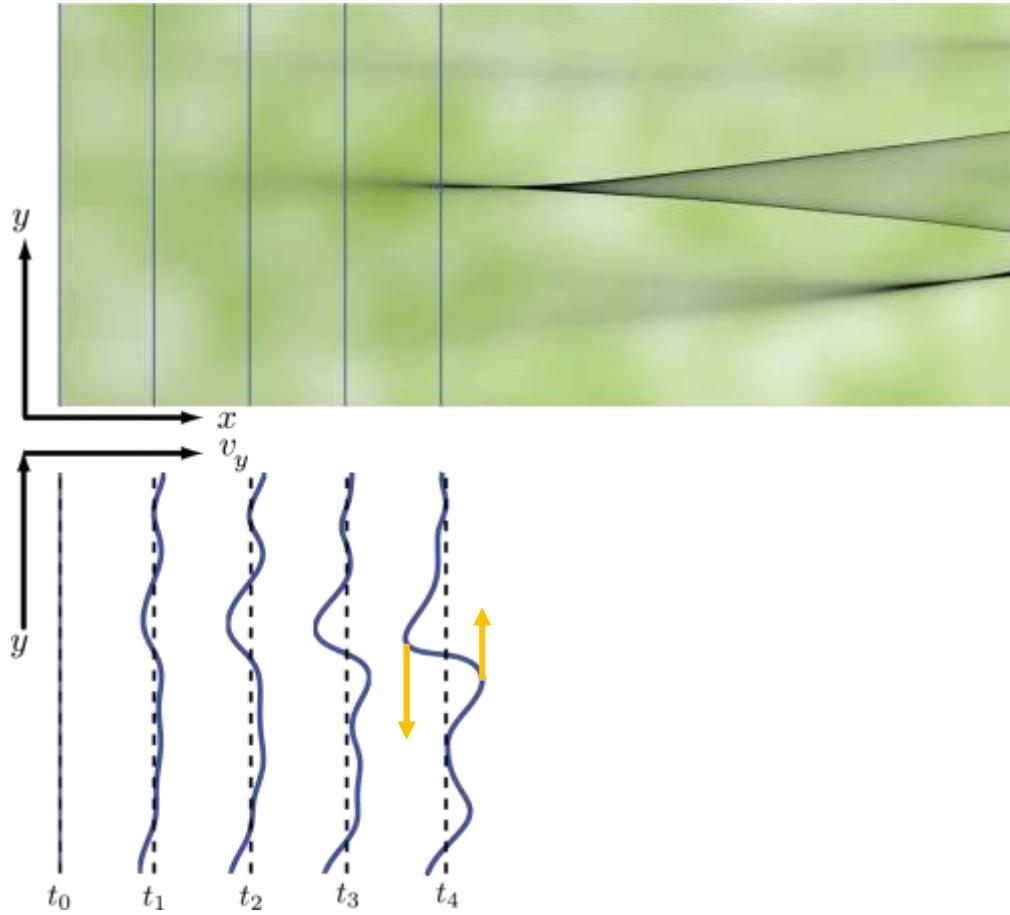


Formation of Random Caustics



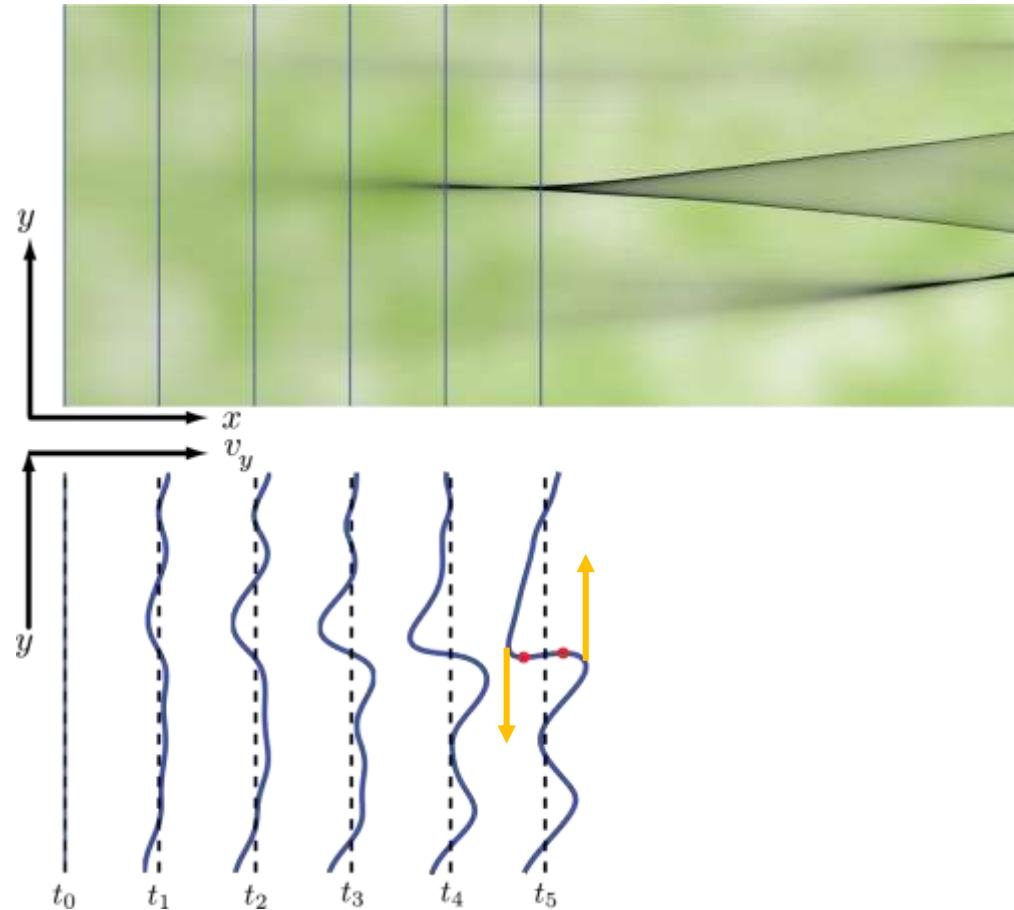
$$x = t$$

Formation of Random Caustics

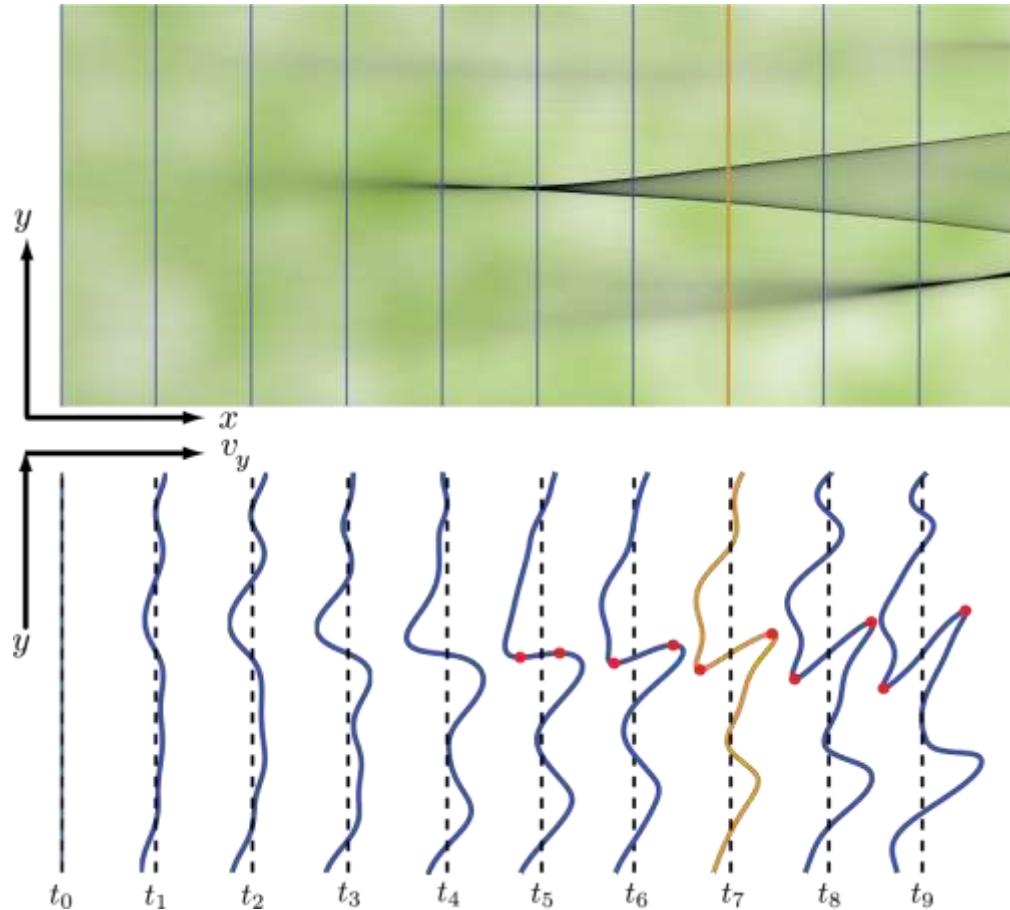


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Formation of Random Caustics

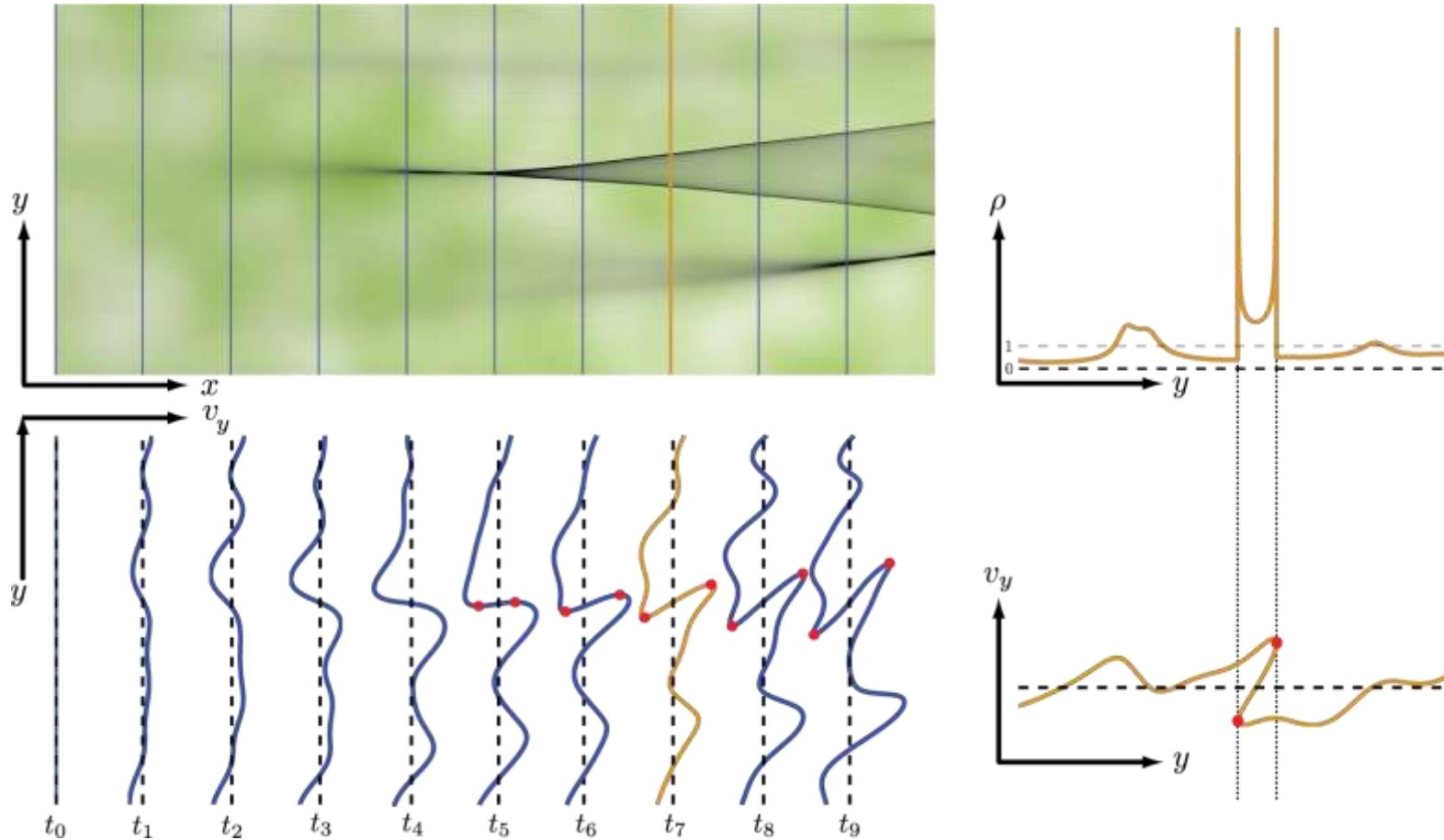


Formation of Random Caustics



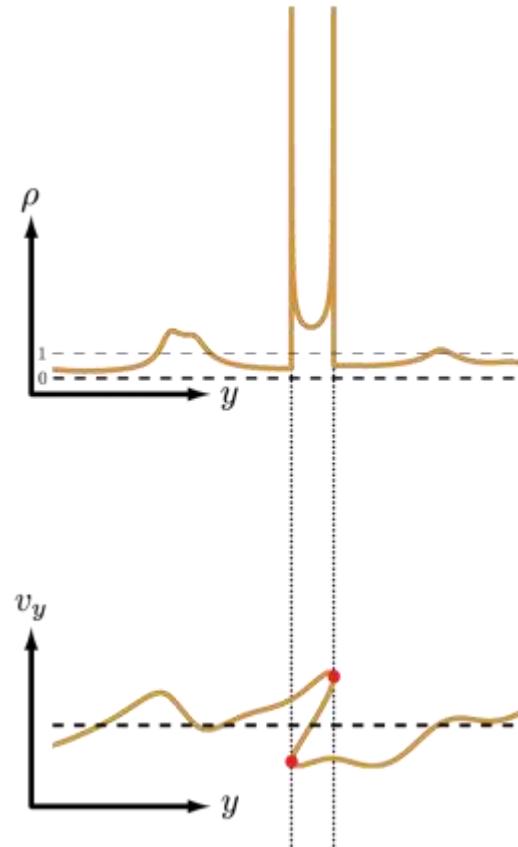
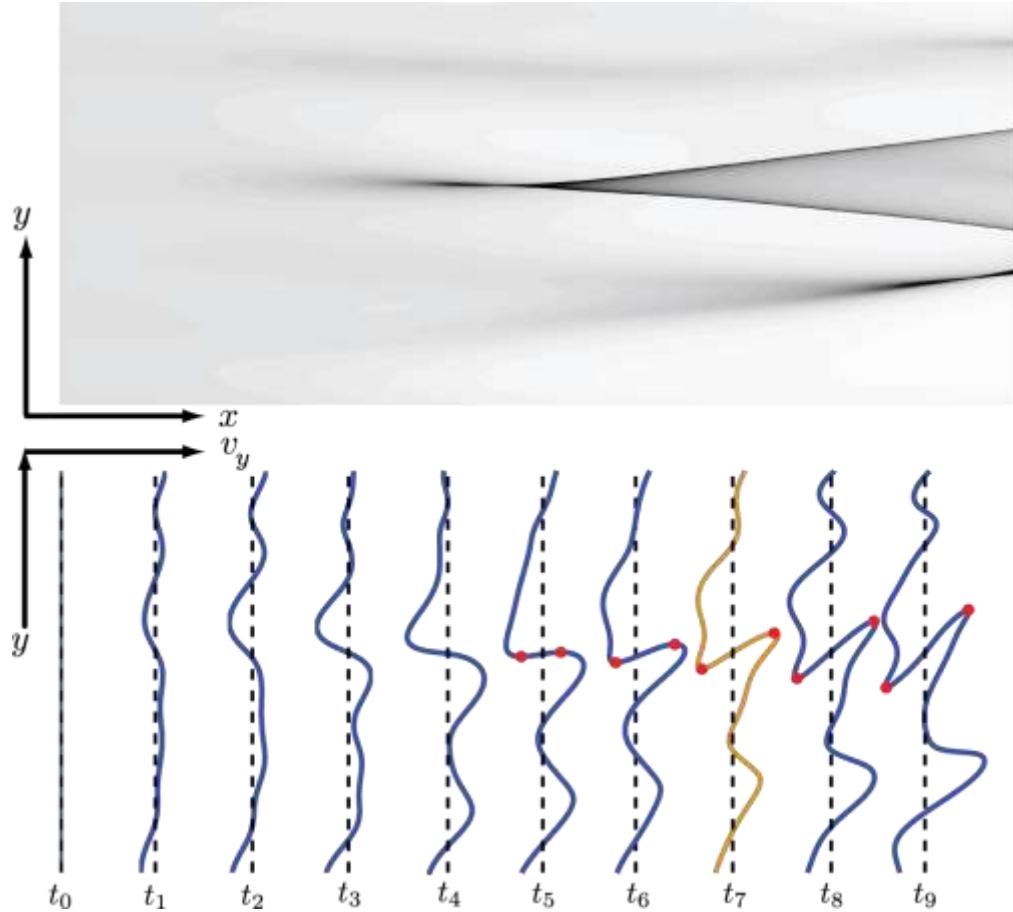
$$x = t$$

Formation of Random Caustics



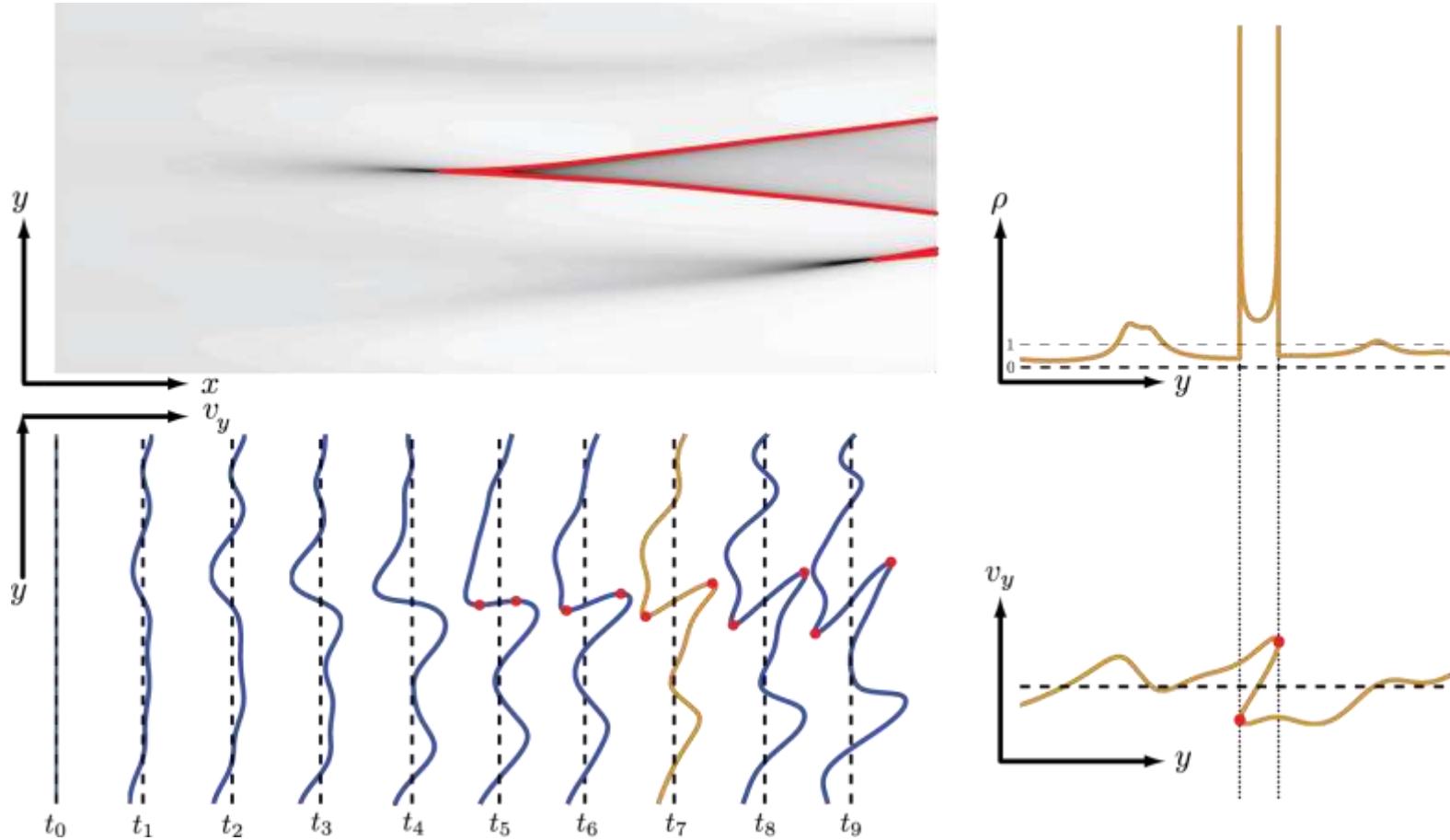
$x = t$

Formation of Random Caustics



$x = t$

Formation of Random Caustics



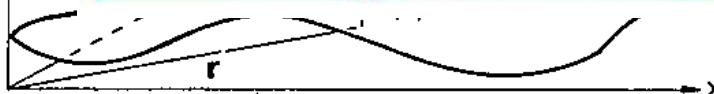
$x = t$

Light refraction/reflection by random surfaces or phase screens

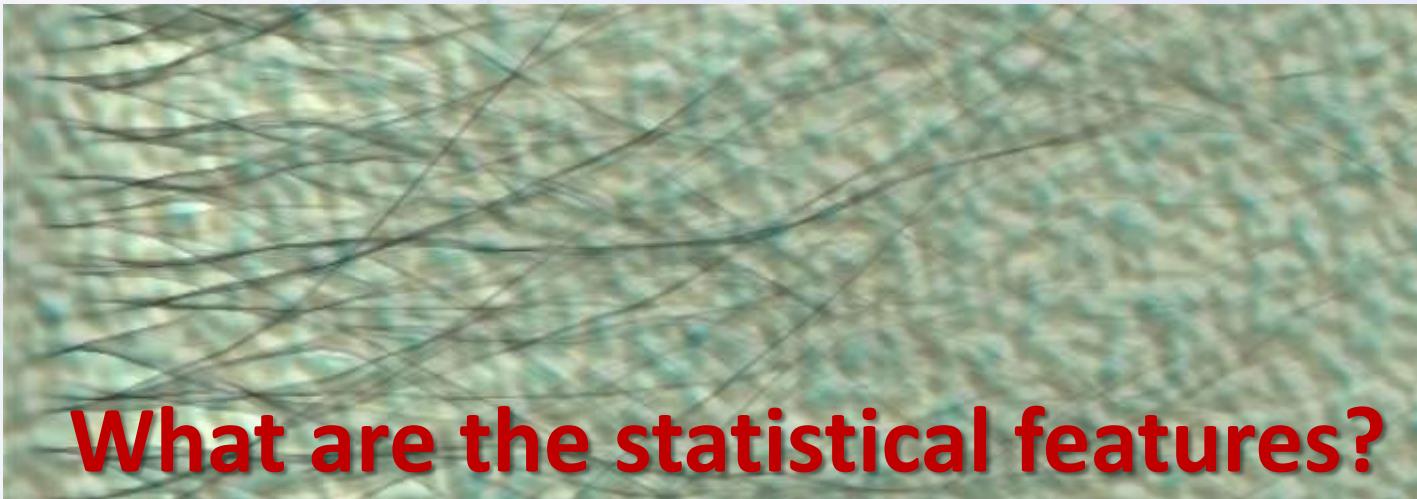
E. WOLF,

MORPH

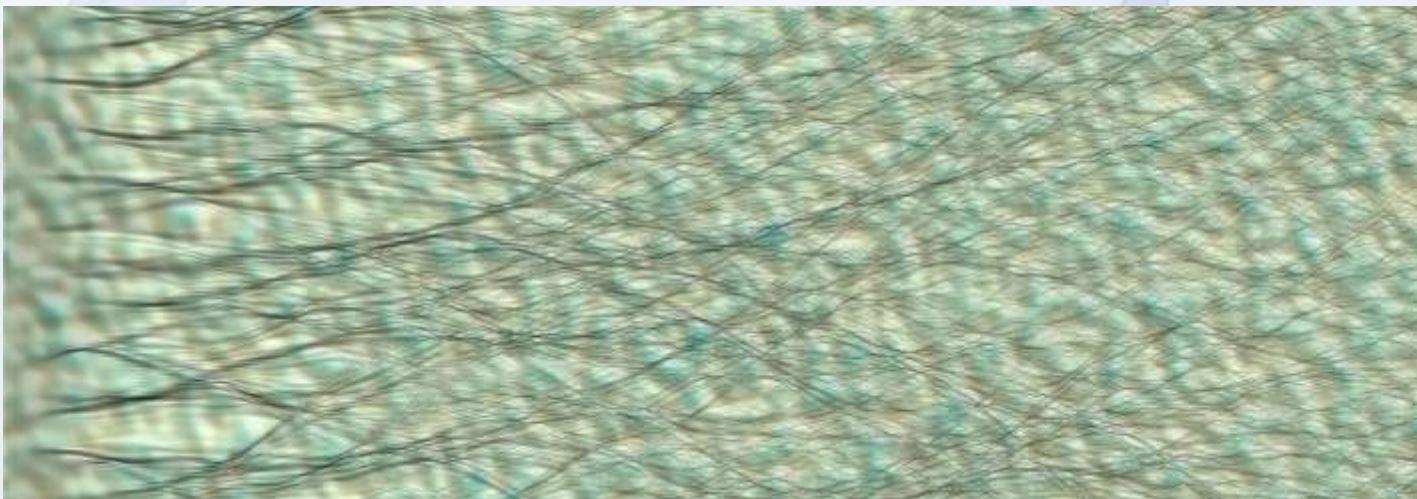
Z



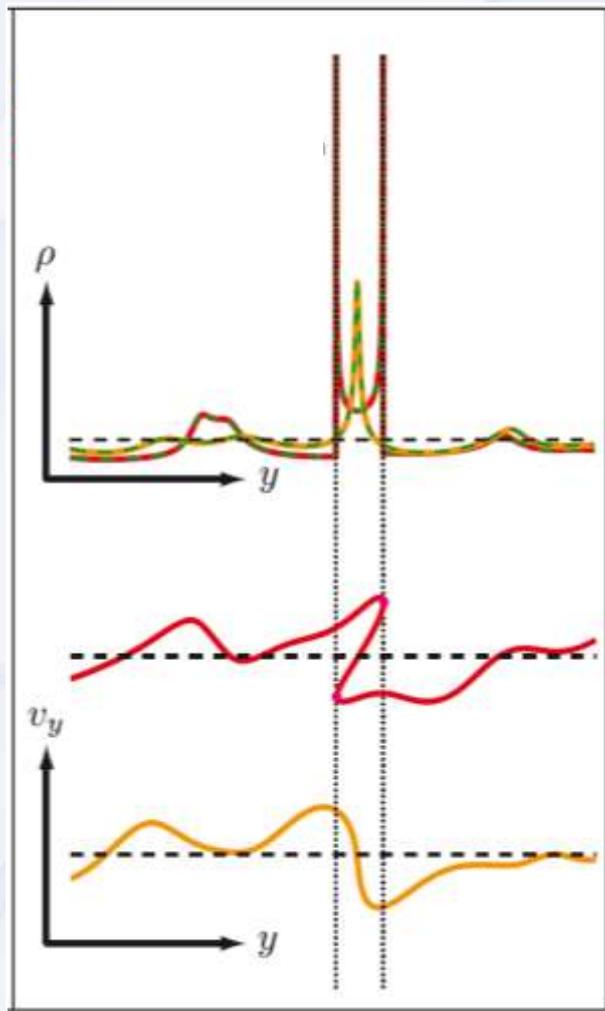
Rays and waves



What are the statistical features?



How to find caustics?



$$\rho \propto \left| \frac{\partial y}{\partial y_0} \right|^{-1} = \frac{\ell_0}{\ell} \sqrt{1 + u^2}$$

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y$$

$u \rightarrow \infty$ at caustic

Lagrangian caustic statistics

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y$$

Kulkarny & White '82, Klyatskin '93

$$\frac{d}{dt} u(t) + u^2(t) + \frac{\partial^2}{\partial y^2} V(t, y(t)) = 0$$

Curvature equation



Langevin Equation

Lagrangian caustic statistics

$$\dot{x} = f(t)$$

$$\langle f(t) \rangle = 0 \quad \langle f(t)f(t+\tau) \rangle = c(\tau)$$

$$\sigma^2 = \int_{-\infty}^{+\infty} d\tau c(\tau)$$



$$\langle x^2 \rangle = \sigma^2 t$$

$$\dot{x} = \sigma \xi(t)$$

$$\langle \xi(t) \rangle = 0 \quad \langle \xi(t)\xi(t+\tau) \rangle = \delta(\tau)$$

Lagrangian caustic statistics

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y \quad \frac{d}{dt} u(t) + u^2(t) + \frac{\partial^2}{\partial y^2} V(t, y(t)) = 0$$

Kulkarny & White '82, Klyatskin '93

$$\left\langle \frac{\partial^2}{\partial y^2} V(t, y) \frac{\partial^2}{\partial y'^2} V(t', y') \right\rangle = \frac{\partial^2}{\partial y^2} \frac{\partial^2}{\partial y'^2} \langle V(t, y) V(t', y') \rangle$$

$$= \frac{\partial^2}{\partial y^2} \frac{\partial^2}{\partial y'^2} c(t - t', y - y')$$

$$\sigma_2^2 = \frac{1}{2} \int_{-\infty}^{\infty} dx \frac{\partial^4 c(x, y)}{\partial y^4} \Big|_{y=0}$$

Diffusion coeffients

$$\sigma_k^2 = \frac{-1}{2}^k \int_{-\infty}^{\infty} dx \frac{\partial^{2k} c(x, y)}{\partial y^{2k}} \Big|_{y=0}$$

$$c(\mathbf{r}) = \langle V(\mathbf{r}')V(\mathbf{r}' + \mathbf{r}) \rangle = \epsilon^2 g |\mathbf{r}| / \ell_c$$

Lagrangian caustic statistics

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y$$

Kulkarny & White '82, Klyatskin '93

$$\frac{d}{dt} u(t) + u^2(t) + \frac{\partial^2}{\partial y^2} V(t, y(t)) = 0$$

Curvature equation



Langevin Equation

$$\frac{d}{dt} u(t) + u^2(t) + \sigma_2 \xi(t) = 0$$



Fokker Planck equation

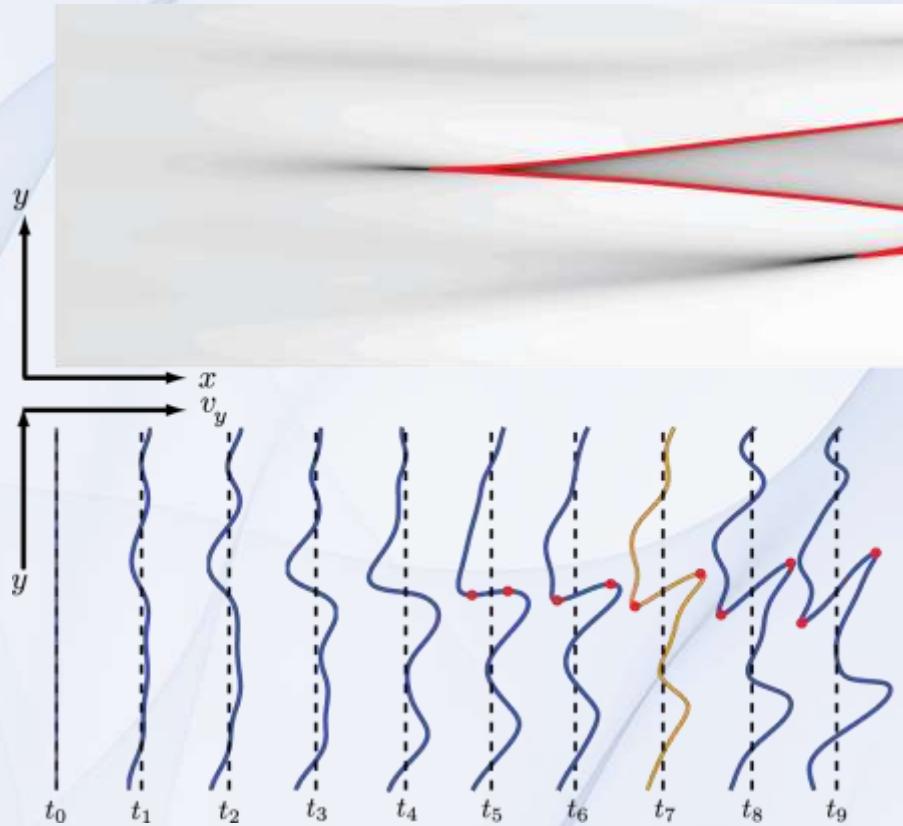
$$\frac{\partial}{\partial t} P(u, t) = \left[\frac{\partial}{\partial u} u^2 + \frac{\partial^2}{\partial u^2} \sigma_2^2 \right] P(u, t) .$$

First passage time calculations

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y$$

$$u_0 = 0 \rightarrow u(t_c) = -\infty \quad \Rightarrow \langle t_c(u_0) \rangle = 3.31 \sigma_2^{-2/3}$$

plane wave
initial condition



$$u = \left[\frac{\partial y}{\partial v_y} \right]^{-1}$$

First passage time calculations

$$u = \partial_y^2 S = \partial_y p_y = m \partial_y v_y$$

$$u_0 = 0 \rightarrow u(t_c) = -\infty \Rightarrow \langle t_c(u_0) \rangle = 3.31 \sigma_2^{-2/3}$$

plane wave

initial condition

$$u_0 = +\infty \rightarrow u(t_c) = -\infty \Rightarrow \langle t_c(u_0) \rangle = 4.97 \sigma_2^{-2/3}$$

point source

initial condition

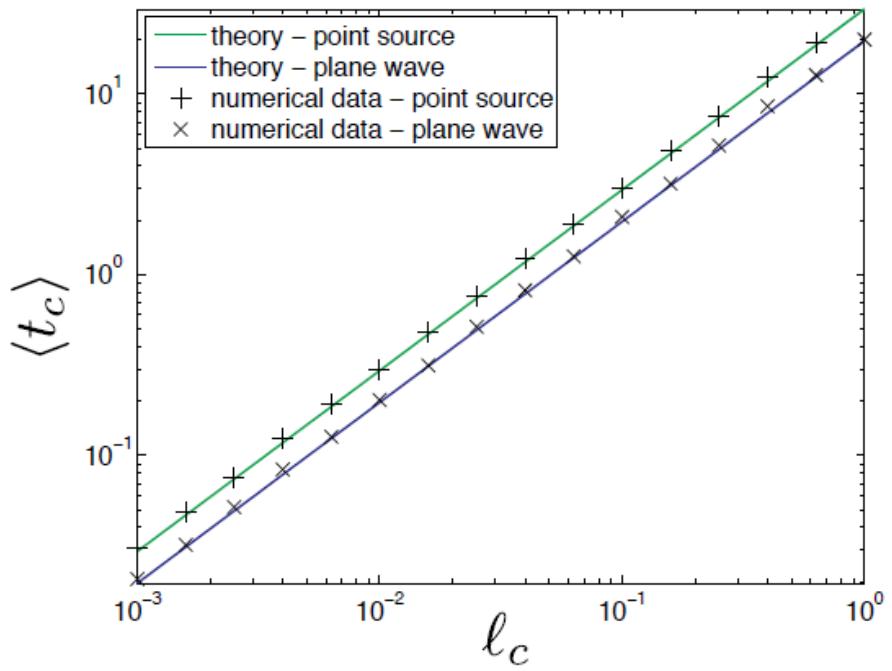
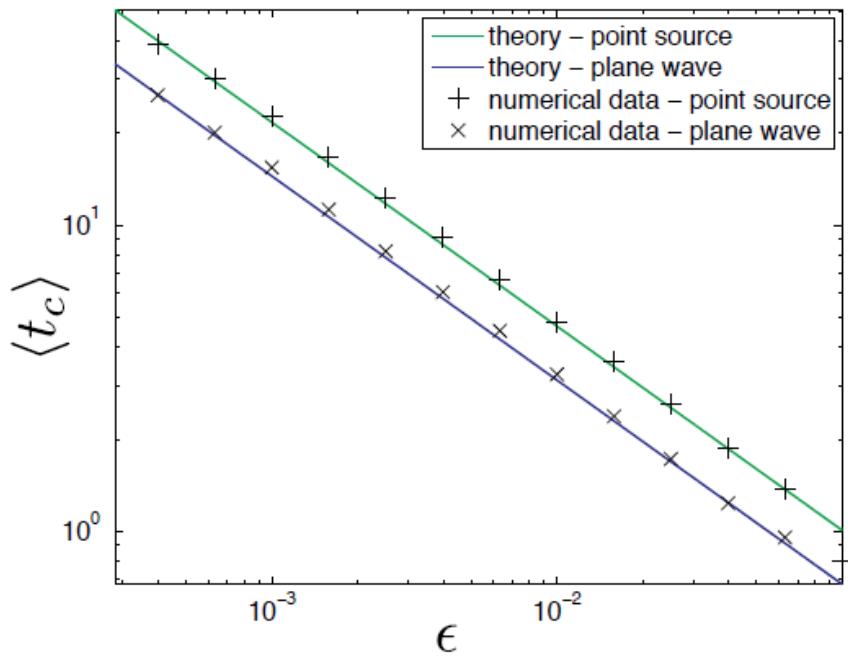
$$t_0 = \sigma_2^{-2/3} = \ell_c \epsilon^{-2/3}$$

Lagrangian caustic statistics

$$t_0 = \sigma_2^{-2/3} = \ell_c \epsilon^{-2/3}$$

$$\langle t_c(0) \rangle = 3.31 \sigma_2^{-2/3}$$

$$\langle t_c(\infty) \rangle = 4.97 \sigma_2^{-2/3}$$

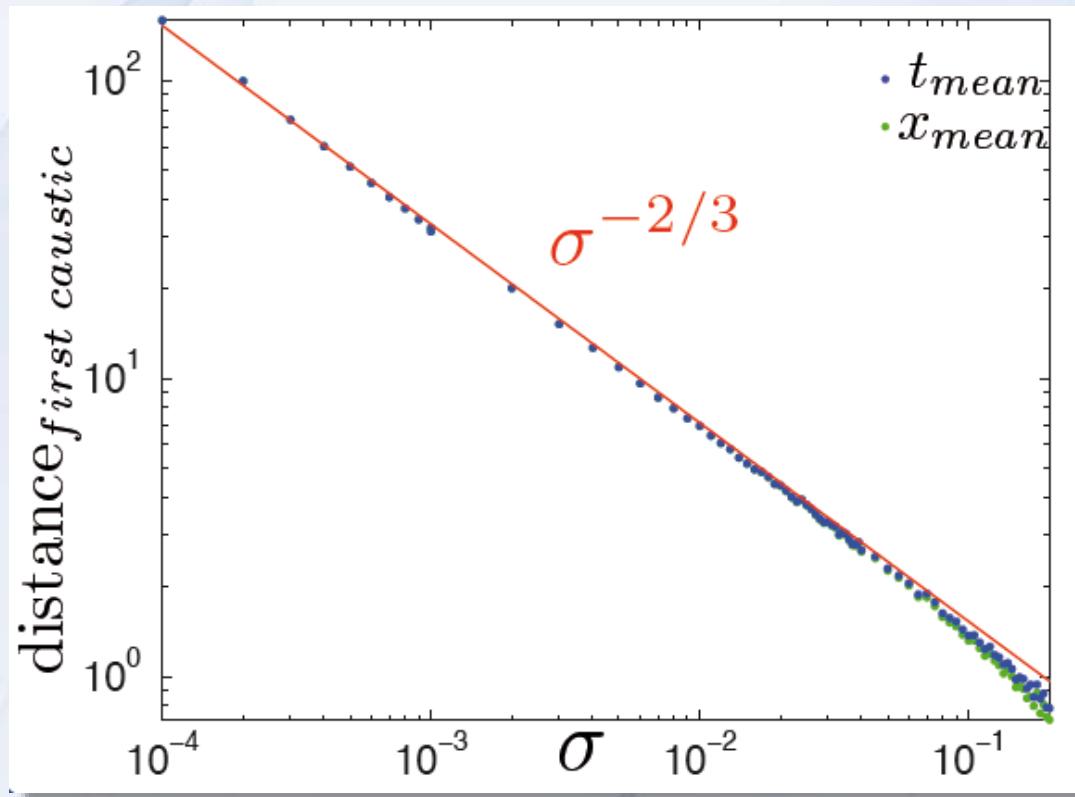


Lagrangian caustic statistics

$$t_0 = \sigma_2^{-2/3} = \ell_c \epsilon^{-2/3}$$

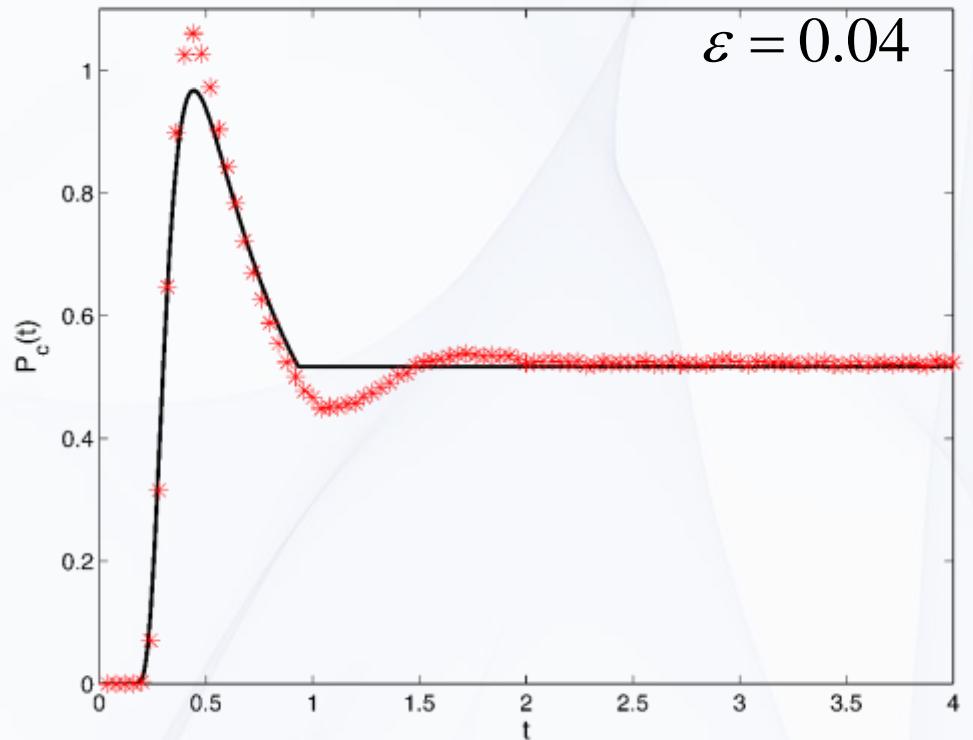
$$\langle t_c(0) \rangle = 3.31 \sigma_2^{-2/3}$$

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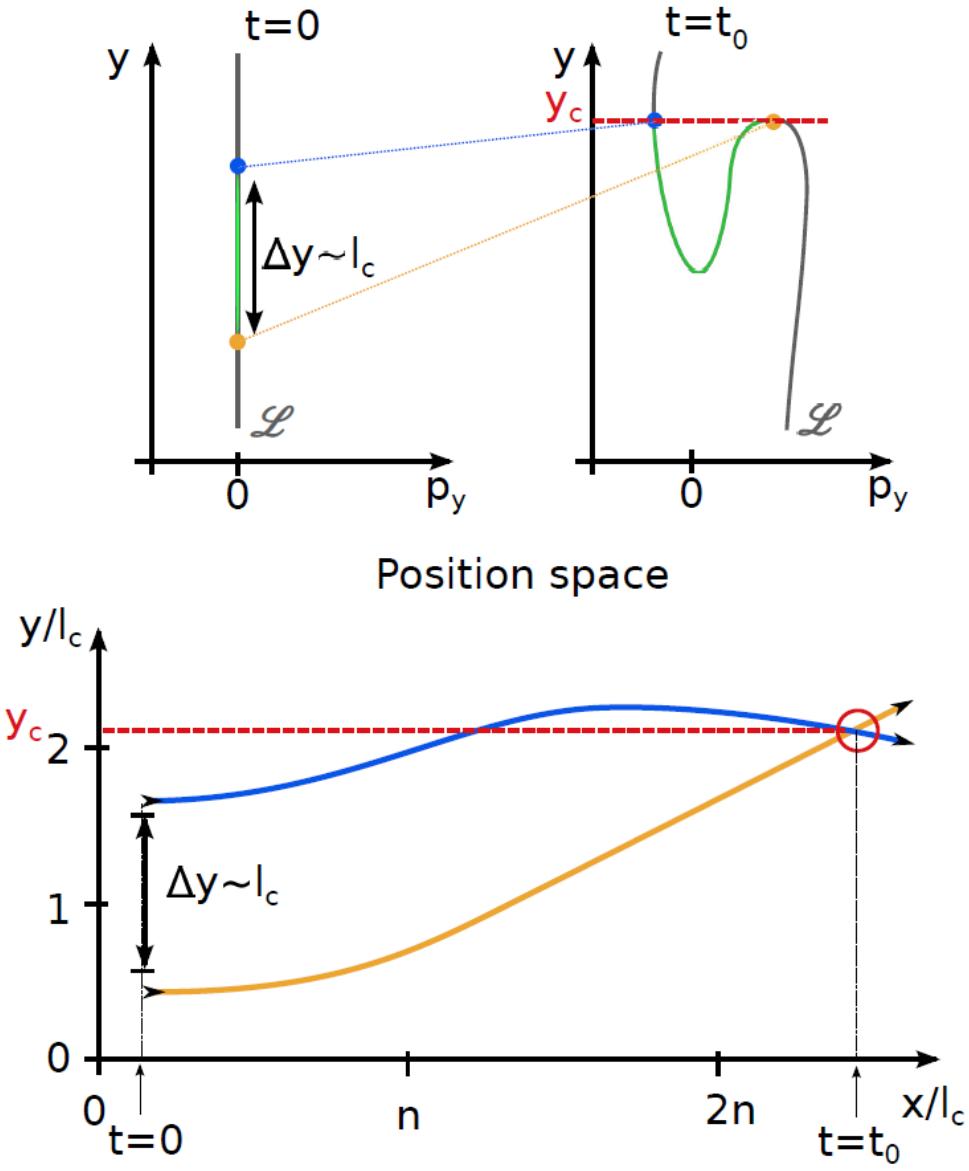


Lagrangian caustic statistics

$$P_c(t) = \begin{cases} \left(\alpha^2 (4\pi\sigma_2^2)^{-1/2} t^{-5/2} + C_F (2\sigma_2^2)^{1/3} \right) e^{-\lambda_1 (2\sigma_2^2)^{1/3} t - \alpha^4 / (12\sigma_2^2 t^3)} & \text{if } t \leq t_1 \\ 1 / \left(6.27 (2\sigma_2^2)^{-1/3} \right) & \text{if } t > t_1 \end{cases}$$



Scaling argument



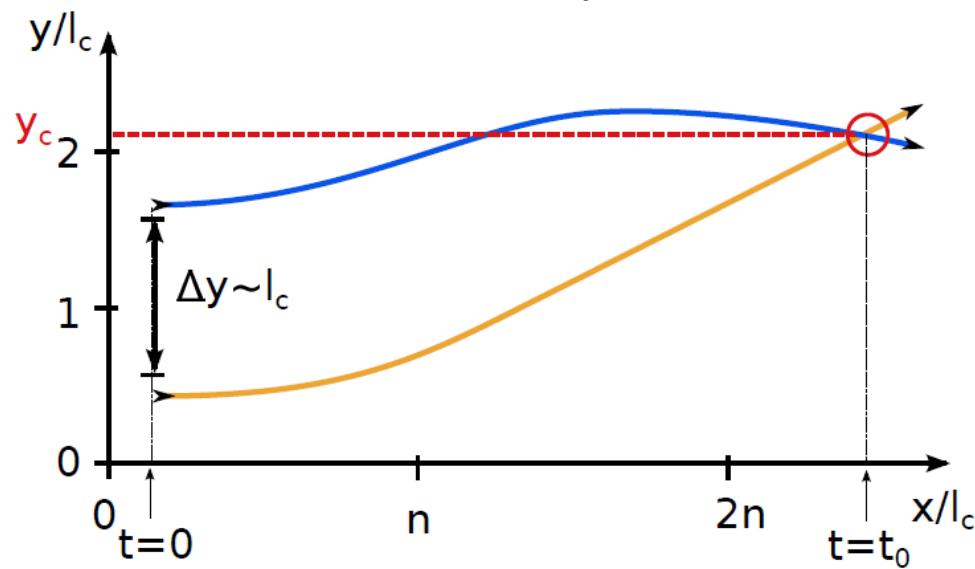
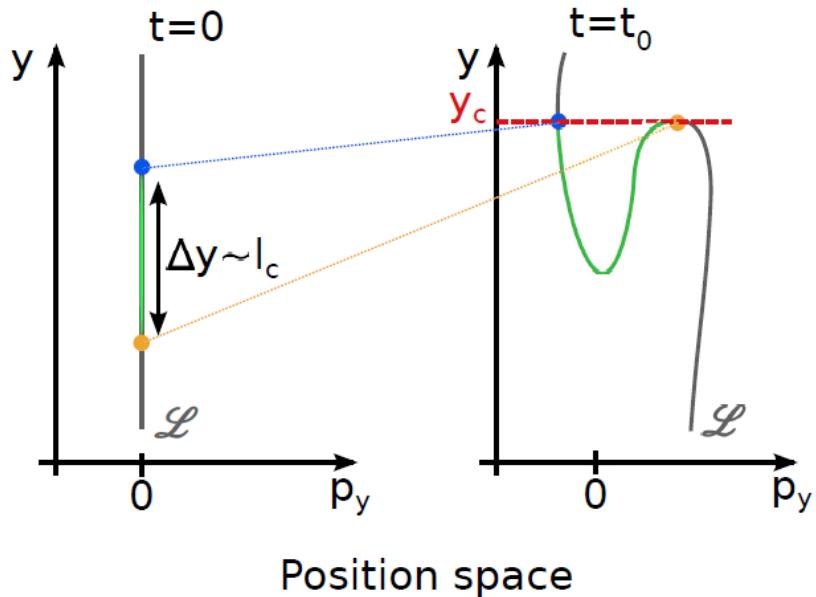
Kaplan, 2002

$$\ddot{y} = \sigma \xi(t)$$

$$\langle \dot{y}^2(t) \rangle = \sigma^2 t$$

$$\langle y^2(t) \rangle = \frac{\sigma^2}{3} t^3$$

Scaling argument



Kaplan, 2002

$$\langle y^2(t) \rangle = \frac{\sigma^2}{3} t^3$$

$$\langle y^2(t_c) \rangle \propto \ell_c^2$$

$$t_c^3 \propto \frac{\ell_c^2}{\sigma^2}$$

$$\sigma_1^2 \propto \frac{\epsilon^2}{\ell_c}$$

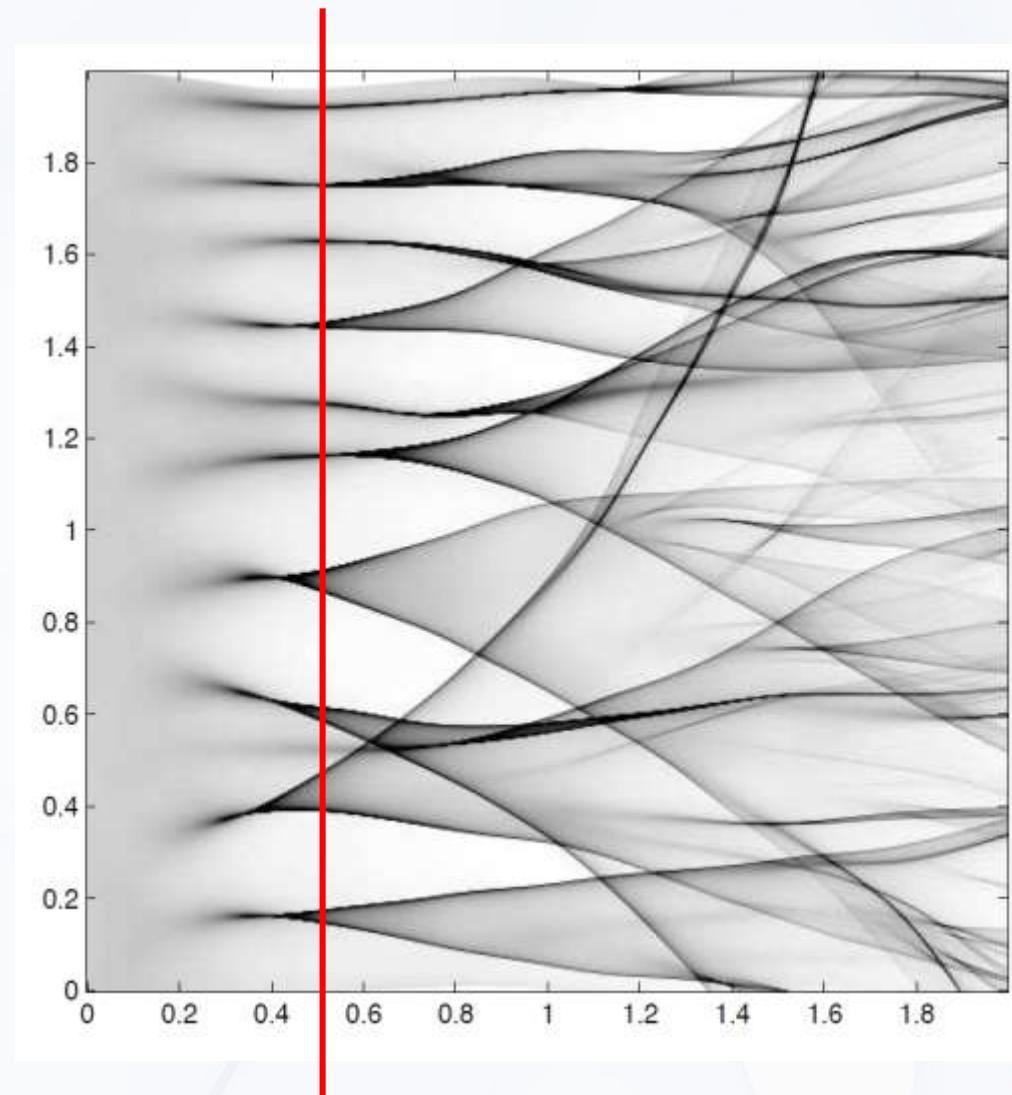
$$t_c \propto \frac{\ell_c}{\epsilon^{-2/3}}$$

Counting Caustics

$$N_c(x) = \frac{\# \text{Caustics}}{\text{Length}}$$

$$N_c(x) = \frac{1}{L} \left\langle \int_0^L \delta \left(\frac{\partial y_t}{\partial y_0} \right) dy_t \right\rangle$$

Berry & Upstill '80



Counting Caustics

$$\begin{aligned} N_c(t) &= \frac{1}{L} \left\langle \int_0^L \delta \left(\frac{\partial y_t}{\partial y_0} \right) dy_t \right\rangle = \frac{1}{L} \left\langle \int_0^L \delta(m) dy_t \right\rangle \\ &= \frac{1}{L} \left\langle \int_0^L \delta(m) |\partial_{y_0} m| dy_0 \right\rangle = \frac{1}{L} \left\langle \int_0^L \delta(m) |n| dy_0 \right\rangle \\ &= \langle \delta(m) |n| \rangle \end{aligned}$$

$$m(t) = \frac{\partial y_t}{\partial y_0} \qquad \qquad n(t) = \frac{\partial^2 y_t}{\partial y_0^2}$$

Counting Caustics

$$N_c(t) = \langle \delta(m)|n| \rangle = \left\langle \delta(t - t_c) \frac{1}{|\dot{m}|} |n| \right\rangle$$

$$\approx \langle \delta(t - t_c) \rangle \left\langle \left| \frac{n(t)}{\dot{m}(t)} \right| \right\rangle = P_c(t) \left\langle \left| \frac{n(t)}{\dot{m}(t)} \right| \right\rangle$$

$$P_c(t) = \langle \delta(t - t_c) \rangle$$

$$m(t) = \frac{\partial y_t}{\partial y_0}$$

$$n(t) = \frac{\partial^2 y_t}{\partial y_0^2}$$

Counting Caustics

Four coupled Lagenvin equations for
 $\vec{a} = (m, \dot{m}, n, \dot{n})$ yielding the Fokker-Planck-equation

$$\partial_t P(\vec{a}, t \mid \vec{a}', t') = \left[-a_2 \partial_{a_1} - a_4 \partial_{a_3} + \sigma_2^2 a_1^2 \partial_{a_2 a_2} + 2\sigma_2^2 a_1 a_3 \partial_{a_2 a_4}^2 + \partial_{a_4 a_4}^2 \sigma_3^2 a_1^4 + \sigma_2^2 a_3^2 \right] P(\vec{a}, t \mid \vec{a}', t')$$

Fokker-Planck-Equation governed by two parameters:

$$\sigma_2^2 = \frac{1}{2} \int_{-\infty}^{\infty} dx \frac{\partial^4 c(x, y)}{\partial y^4} \Big|_{y=0} \quad \sigma_3^2 = -\frac{1}{2} \int_{-\infty}^{\infty} dx \frac{\partial^6 c(x, y)}{\partial y^6} \Big|_{y=0}$$

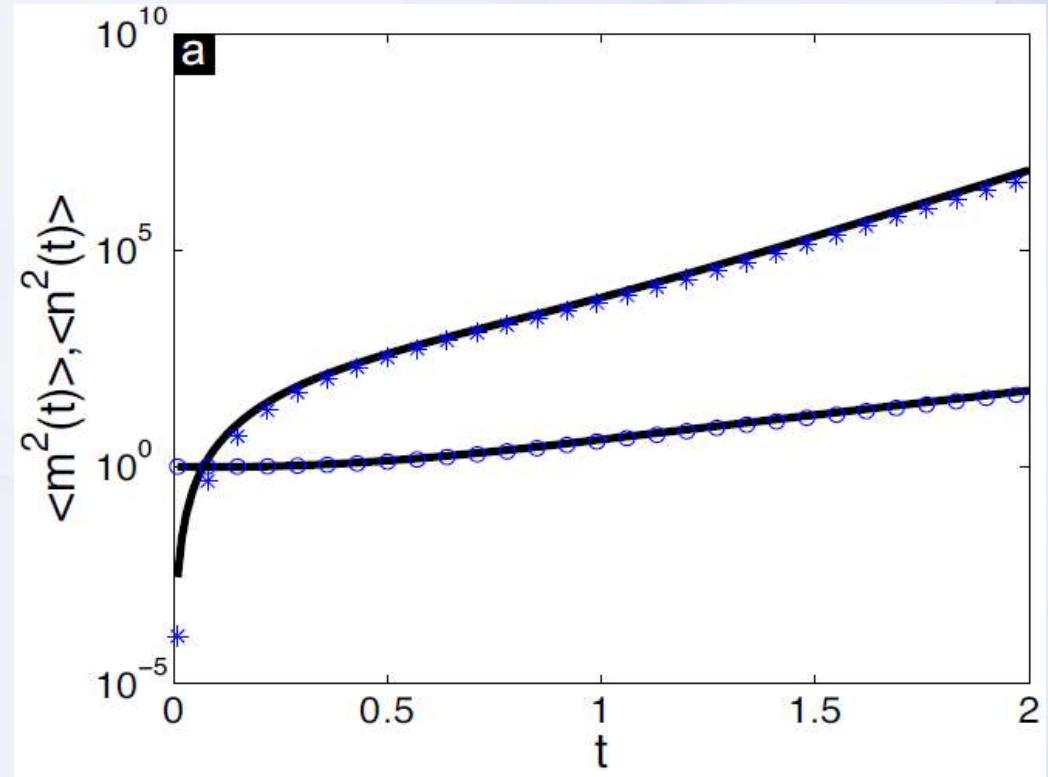
$$c(\mathbf{r}) = \langle V(\mathbf{r}')V(\mathbf{r}' + \mathbf{r}) \rangle = \epsilon^2 g |\mathbf{r}| / \ell_c$$

Counting caustics

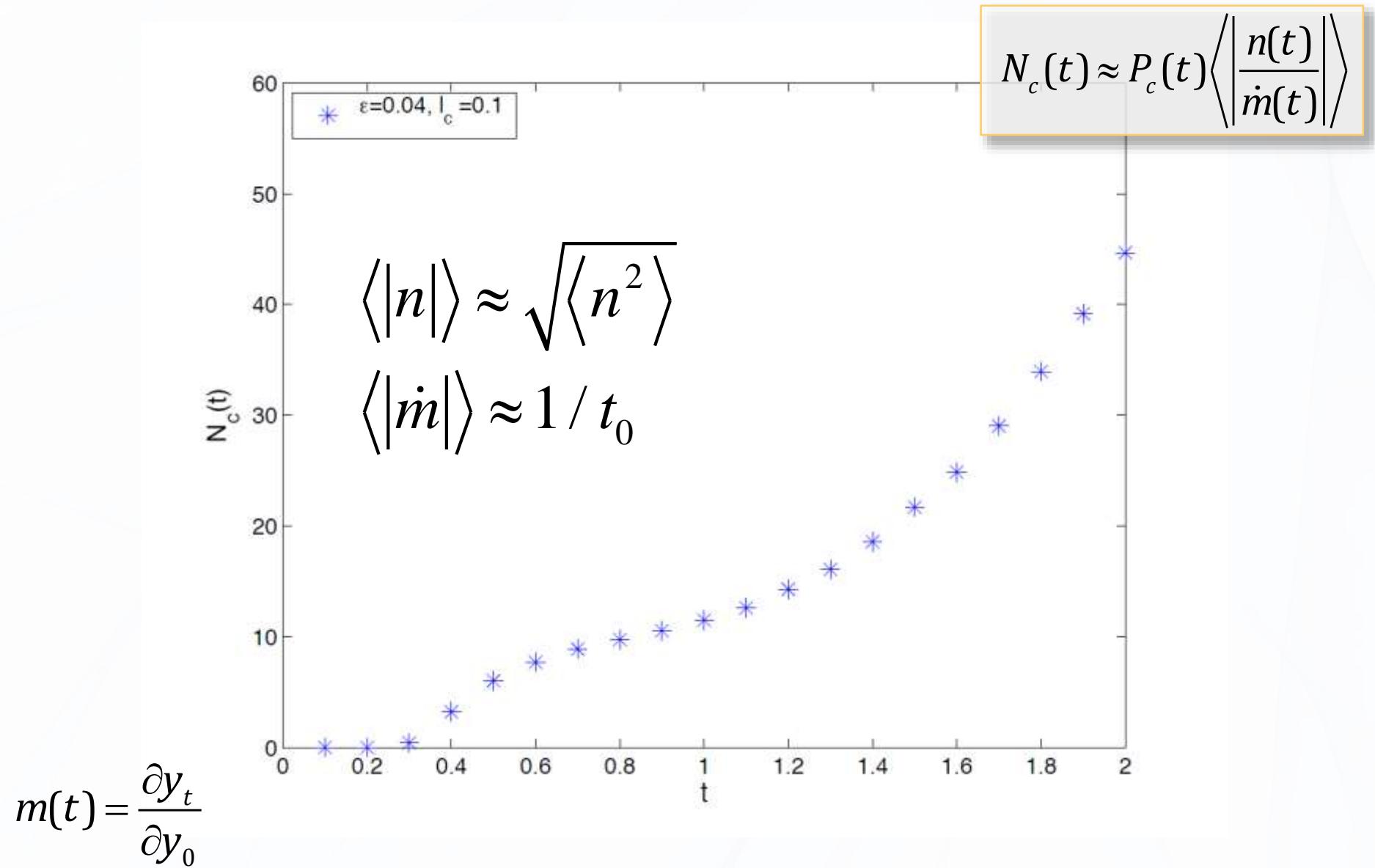
Fokker-Planck-equation leads to sets of ODEs for the momenta of a_j ,
e.g. 11 coupled equations for $\langle n^2(t) \rangle$

$$N_c(t) \approx P_c(t) \left\langle \left| \frac{n(t)}{\dot{m}(t)} \right| \right\rangle$$

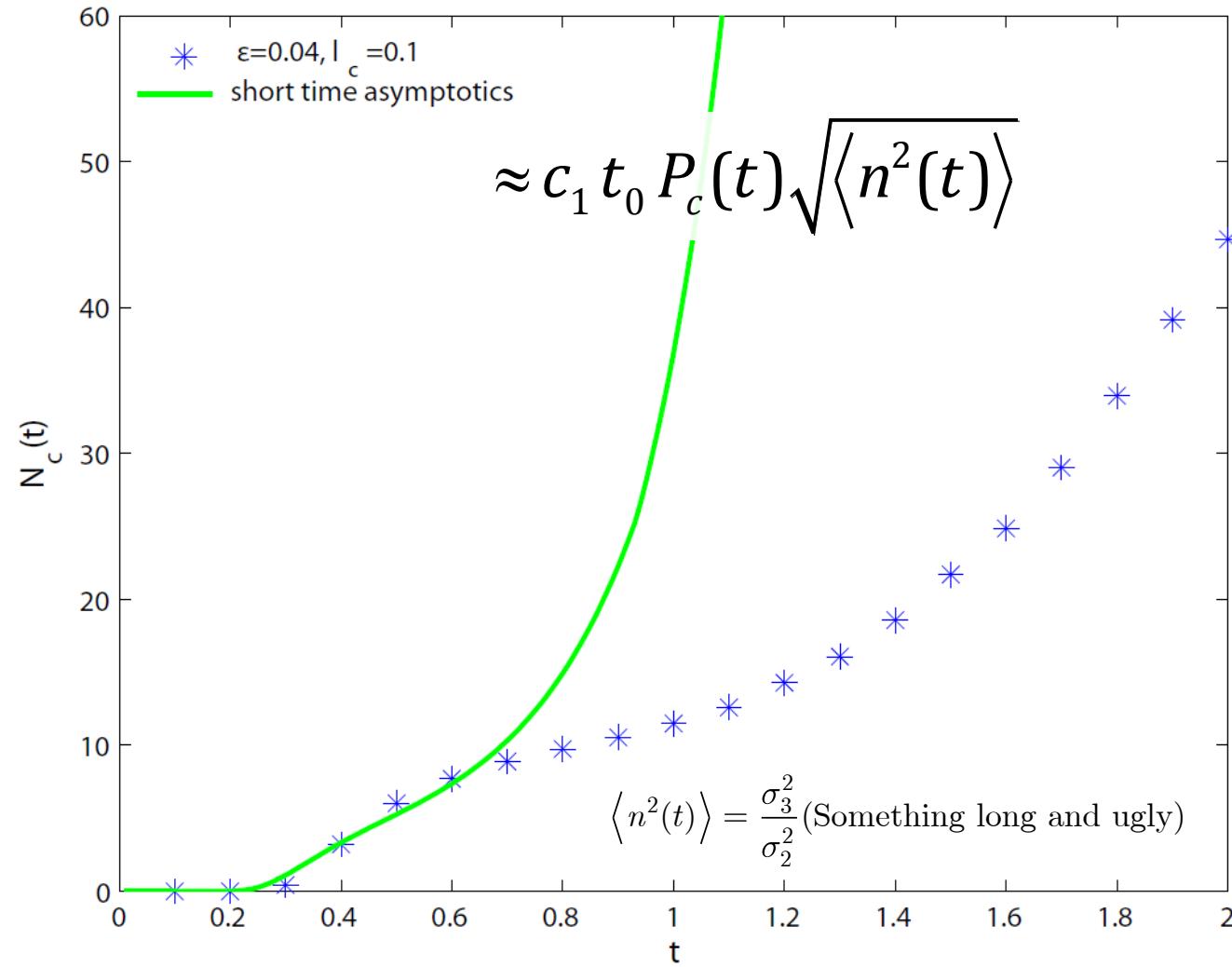
$$\langle n^2(t) \rangle = \frac{\sigma_3^2}{\sigma_2^2} (\text{Something long and ugly})$$



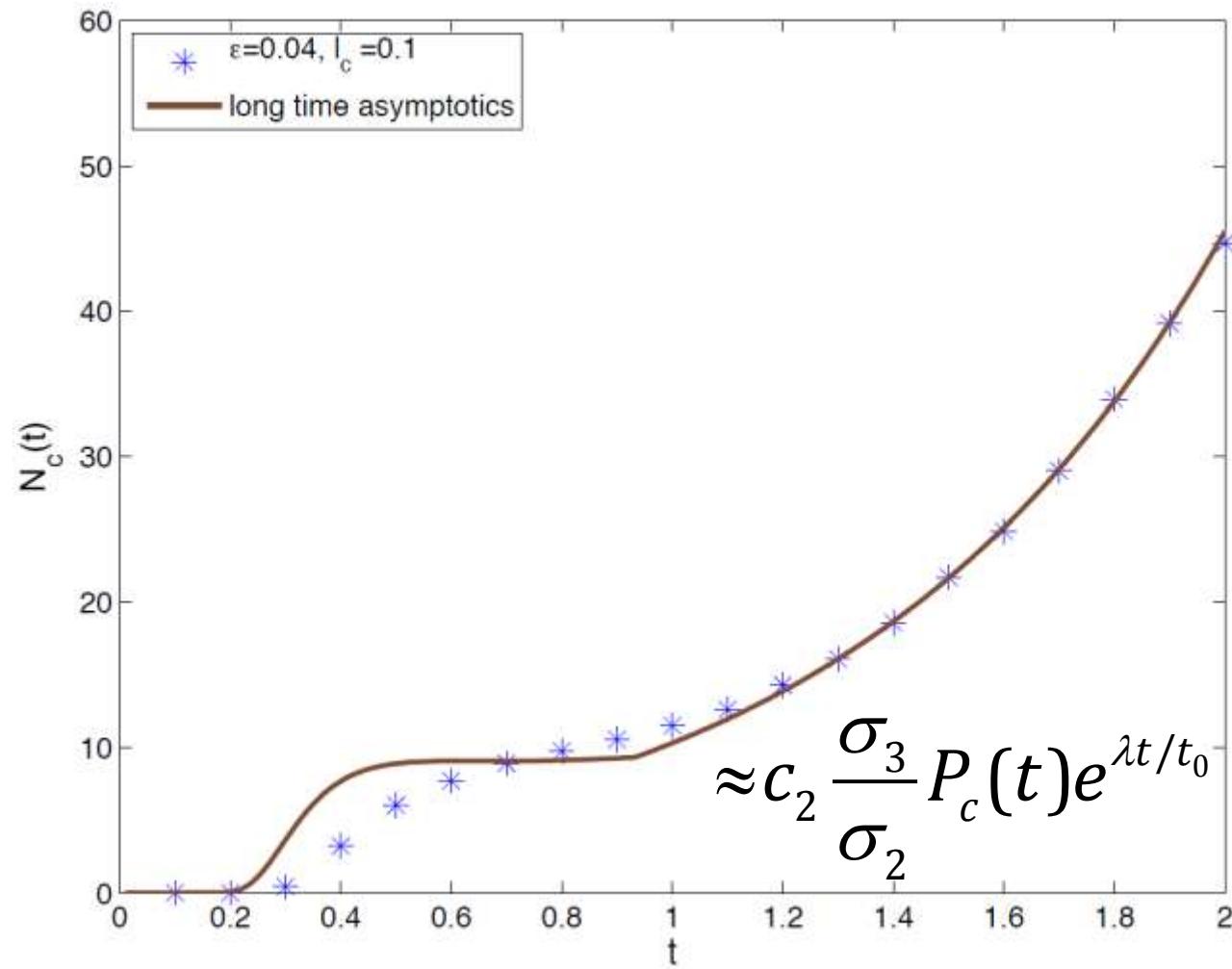
Counting caustics



Counting caustics

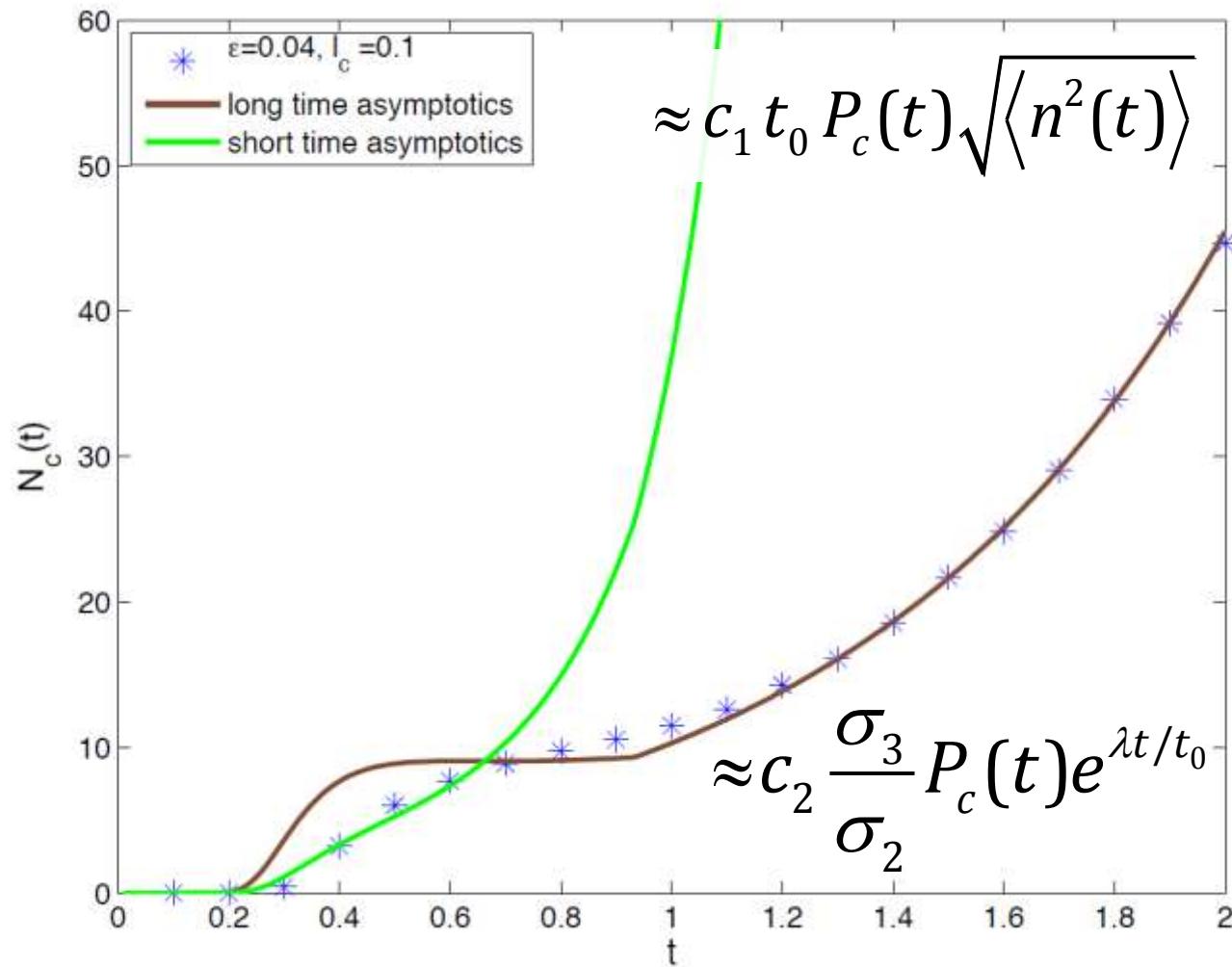


Counting caustics

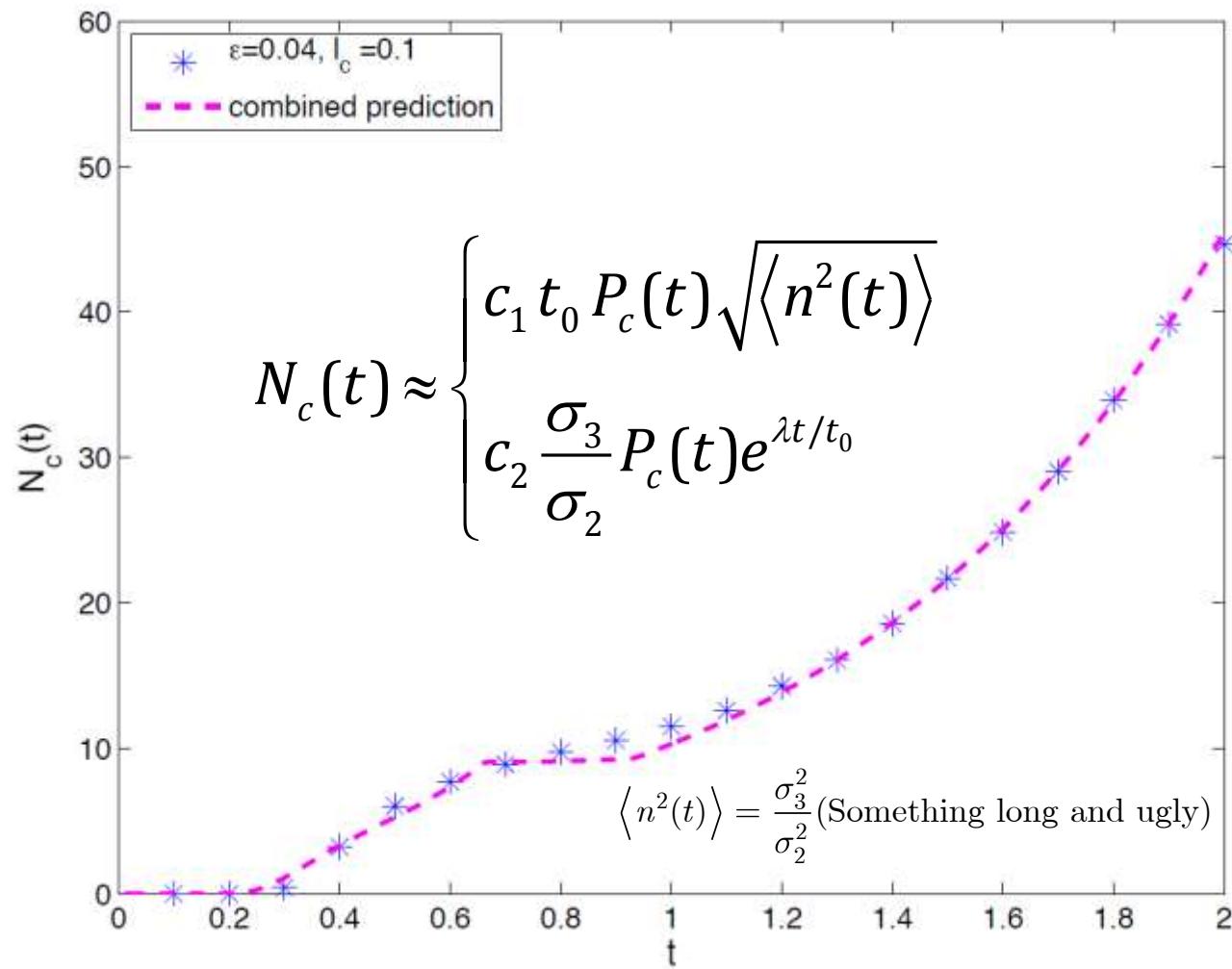


$\lambda \approx 2.87$: Kaplan 2002, Schomerus, Titov 2002

Counting caustics

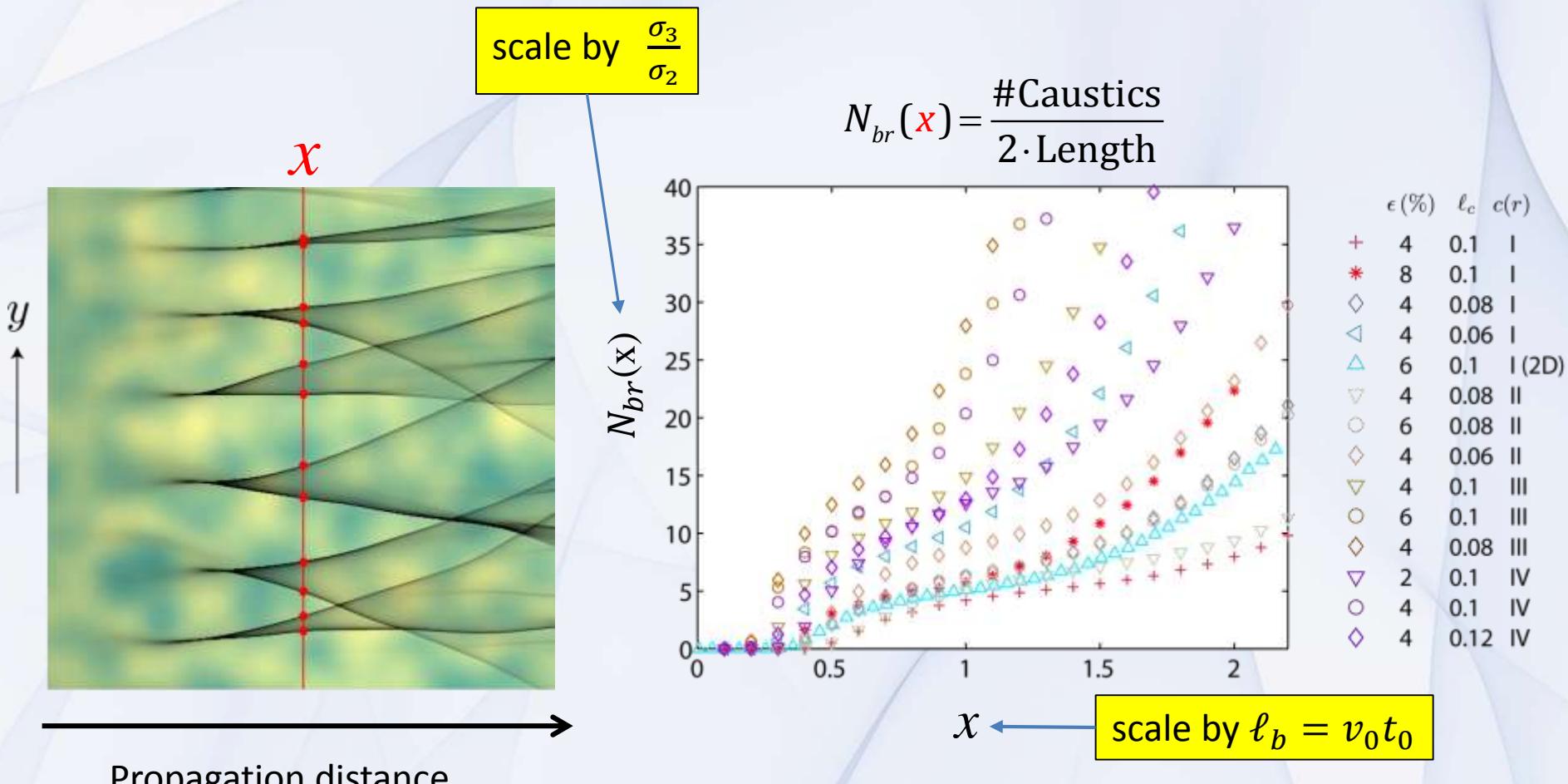


Counting caustics



Random caustics statistics

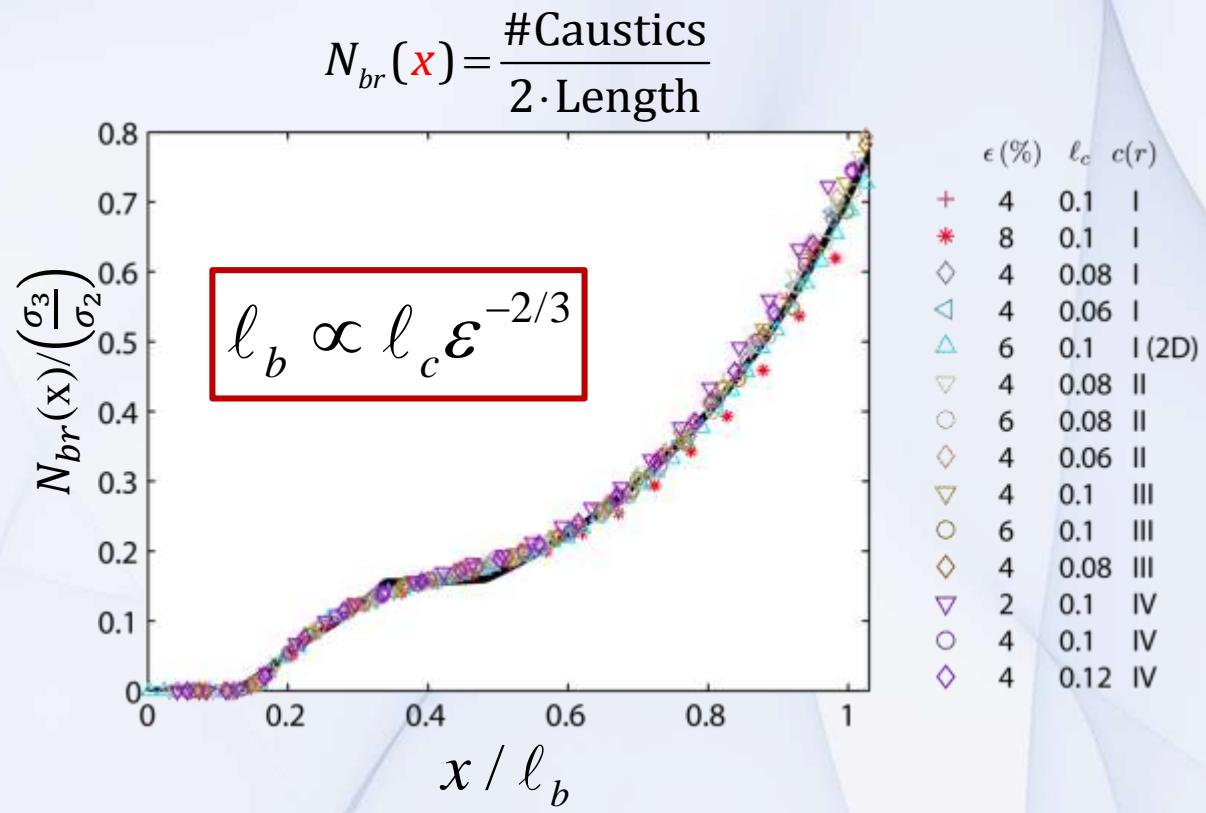
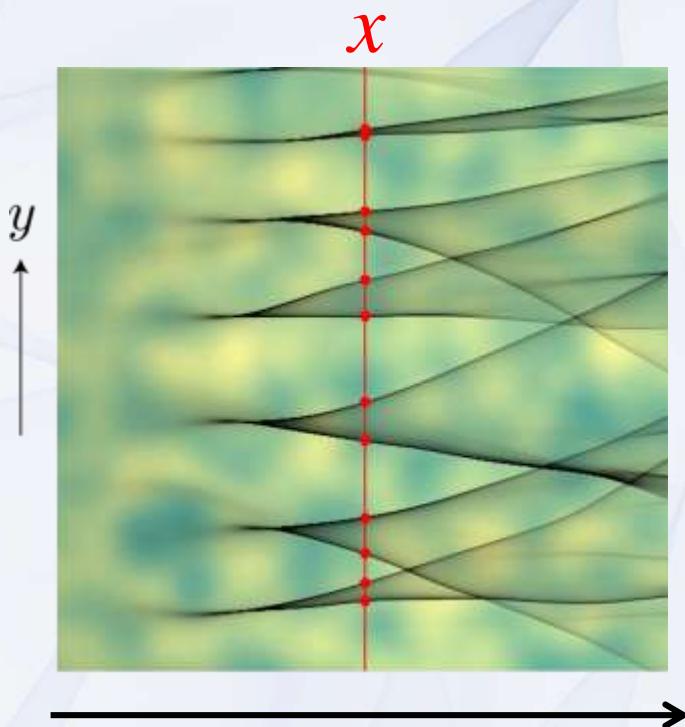
$\ell_c \gg \lambda$
 $(\lambda = \text{wavelength})$



J.J. Metzger, R. Fleischmann, and T. Geisel, Phys. Rev. Lett. **105** (2010).

Random caustics statistics

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Random caustics statistics

$\ell_c \gg \lambda$
 $(\lambda = \text{wavelength})$

branching length

$$\ell_b \propto \ell_c \epsilon^{-2/3}$$

\ll

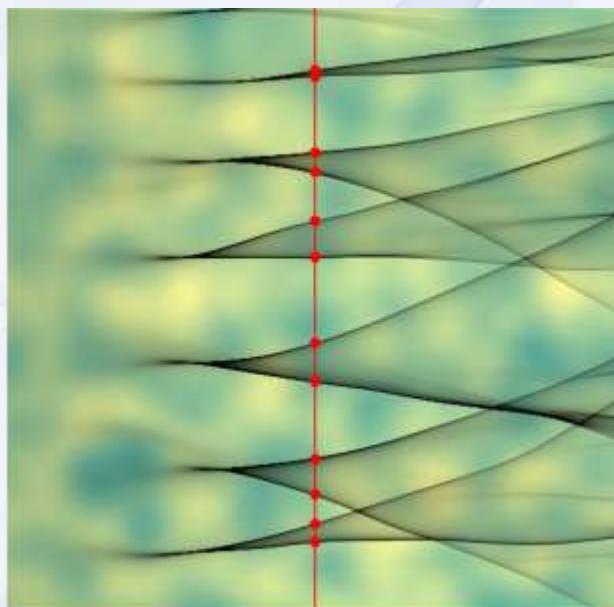
$$\ell_{\text{mfp}} \propto \ell_c \epsilon^{-2}$$

mean free path

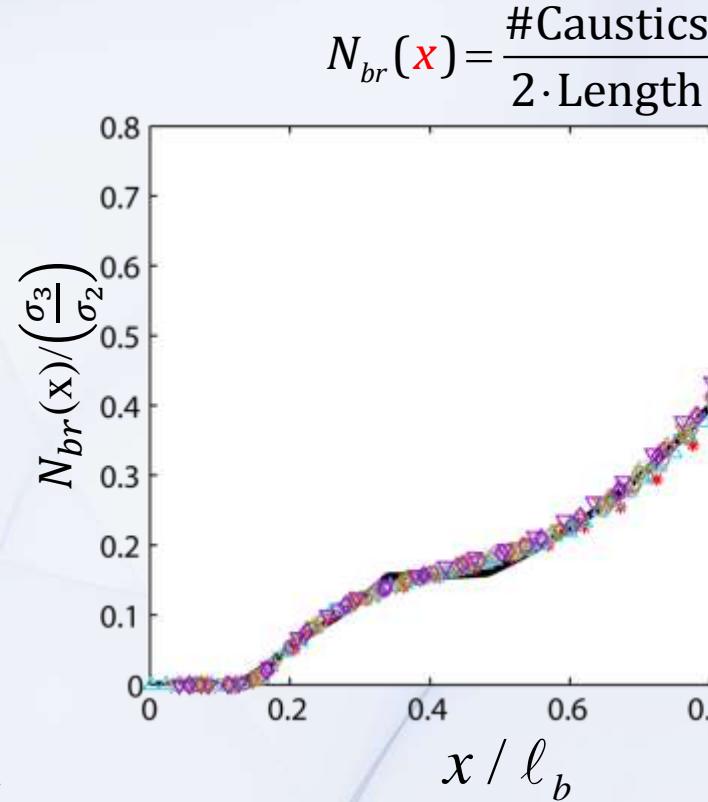
$\epsilon \ll 1$

x

y

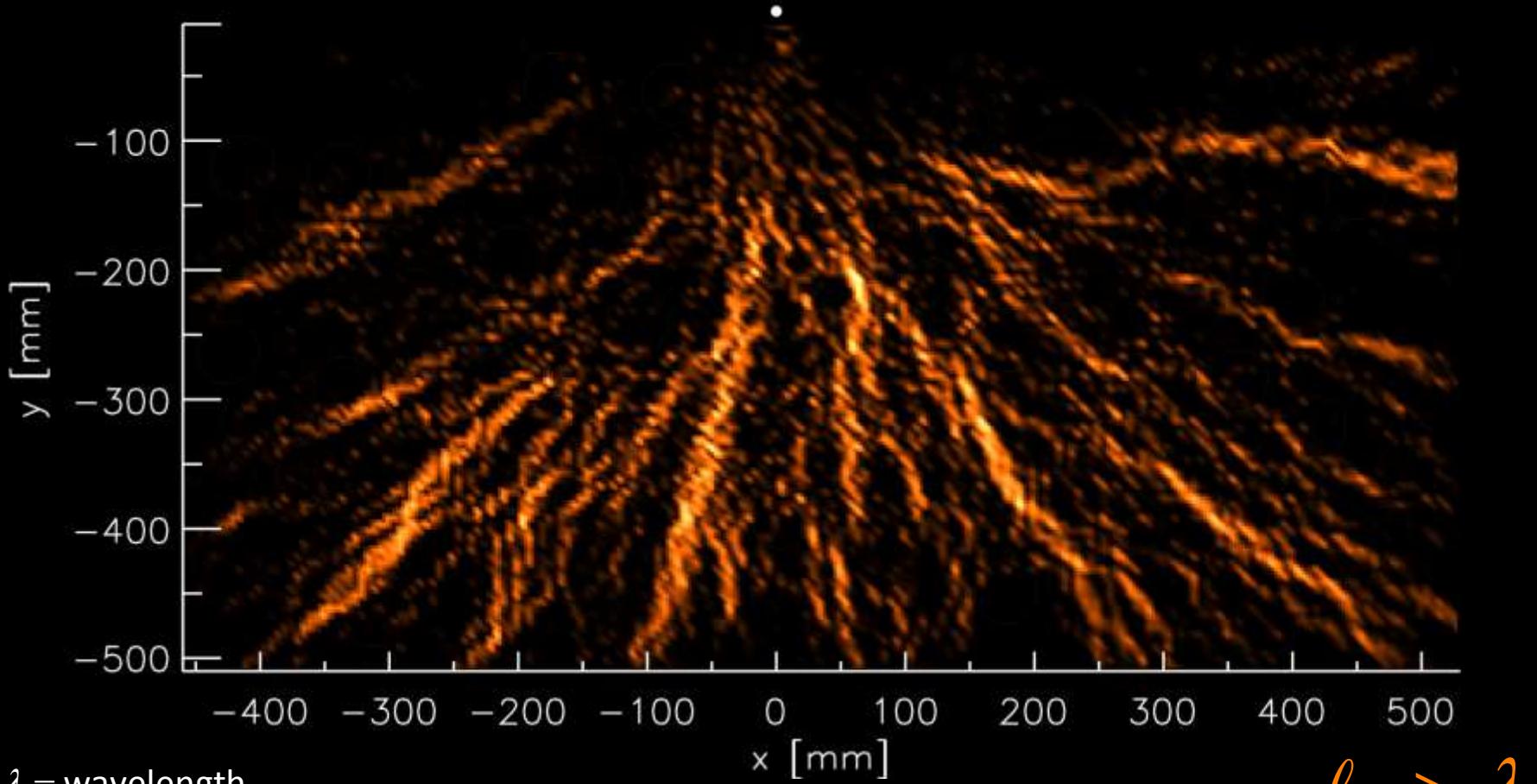


Propagation distance

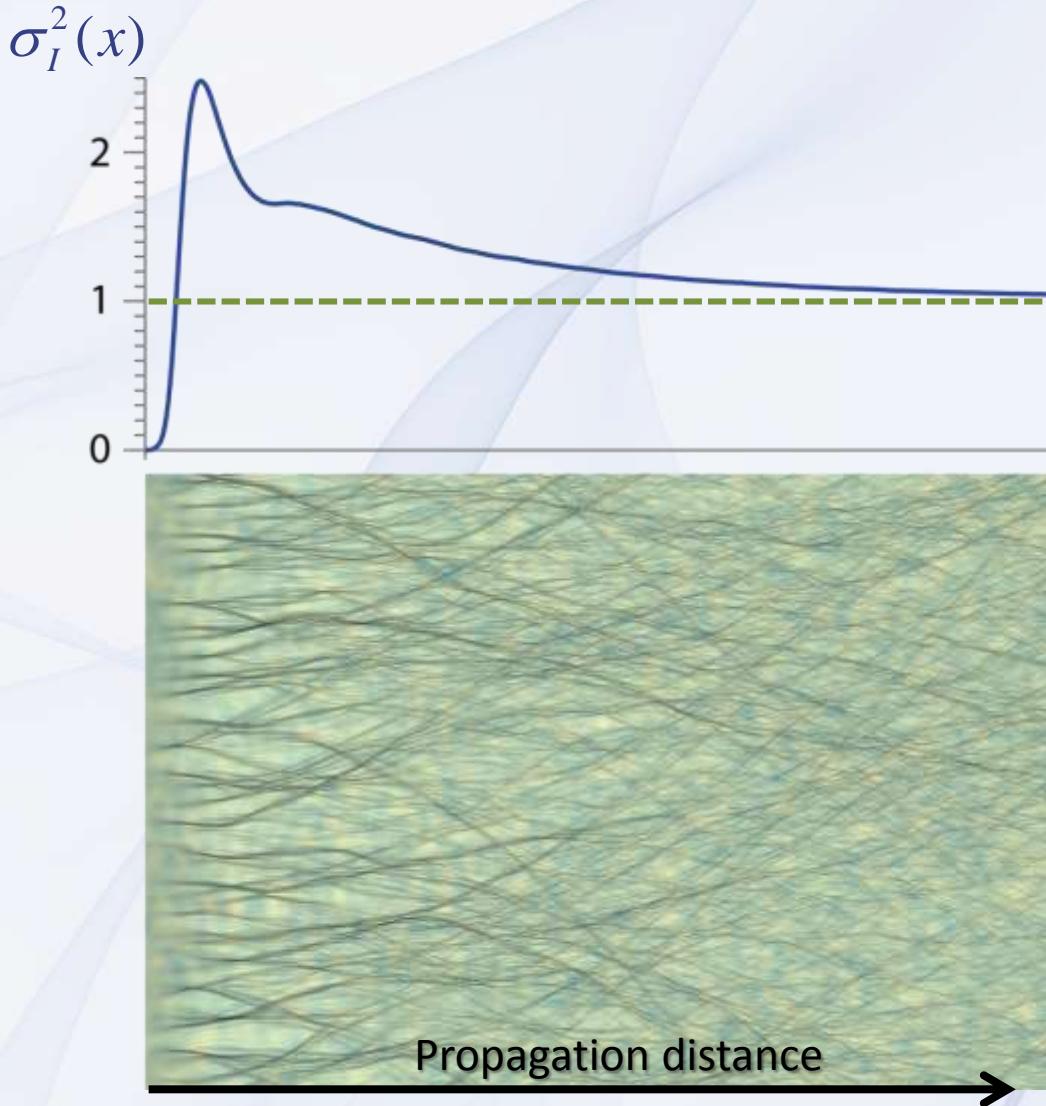


J.J. Metzger, R. Fleischmann, and T. Geisel, Phys. Rev. Lett. **105** (2010).

How can branching be characterized experimentally?



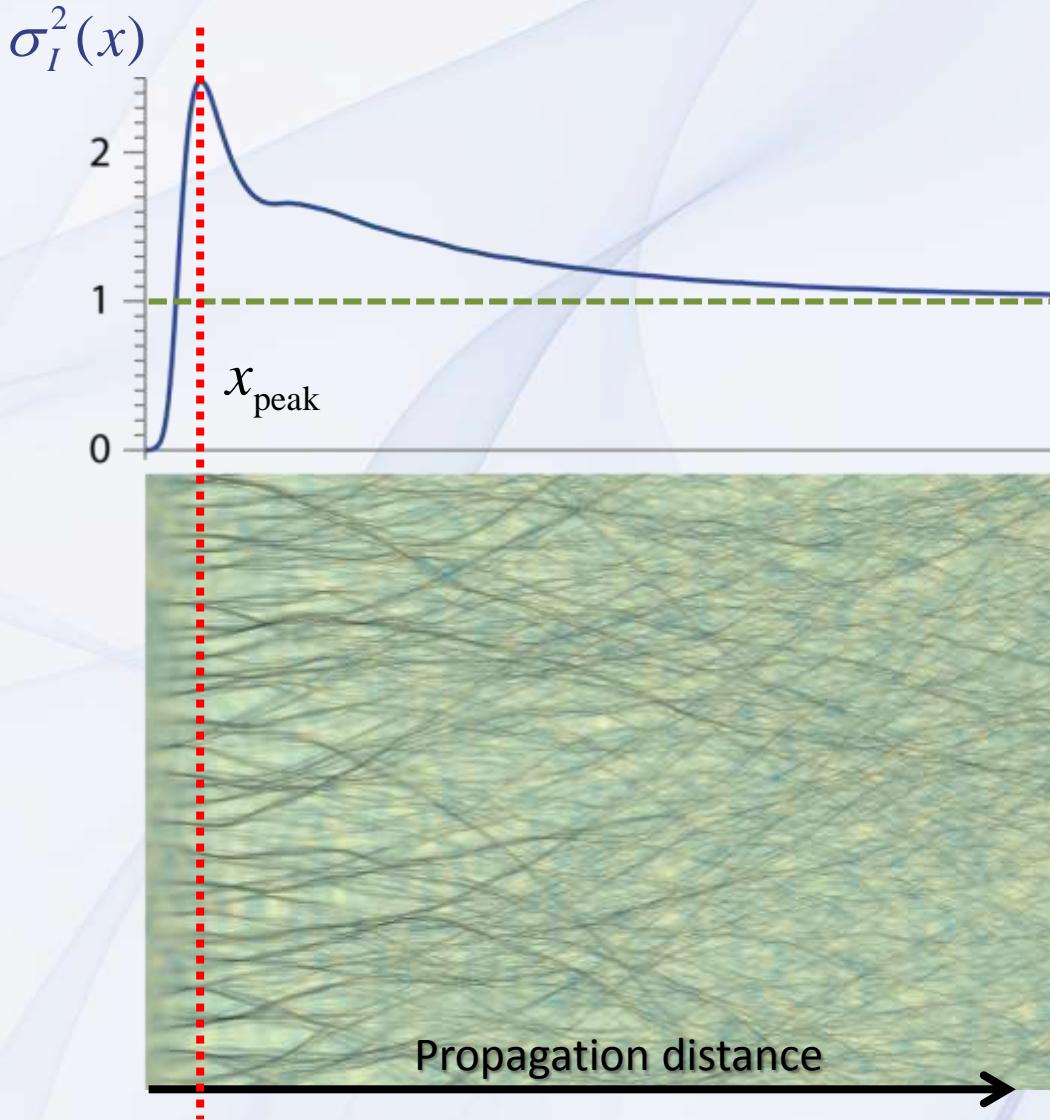
How can branching be characterized experimentally?



Scintillation index:

$$\sigma_I^2(x) = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}$$
$$= \frac{\text{variance of the intensity}}{(\text{mean intensity})^2}$$

How can branching be characterized experimentally?



Scintillation index:

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$$x_{\text{peak}} \propto \ell_b$$
$$\propto \ell_c \epsilon^{-2/3}$$

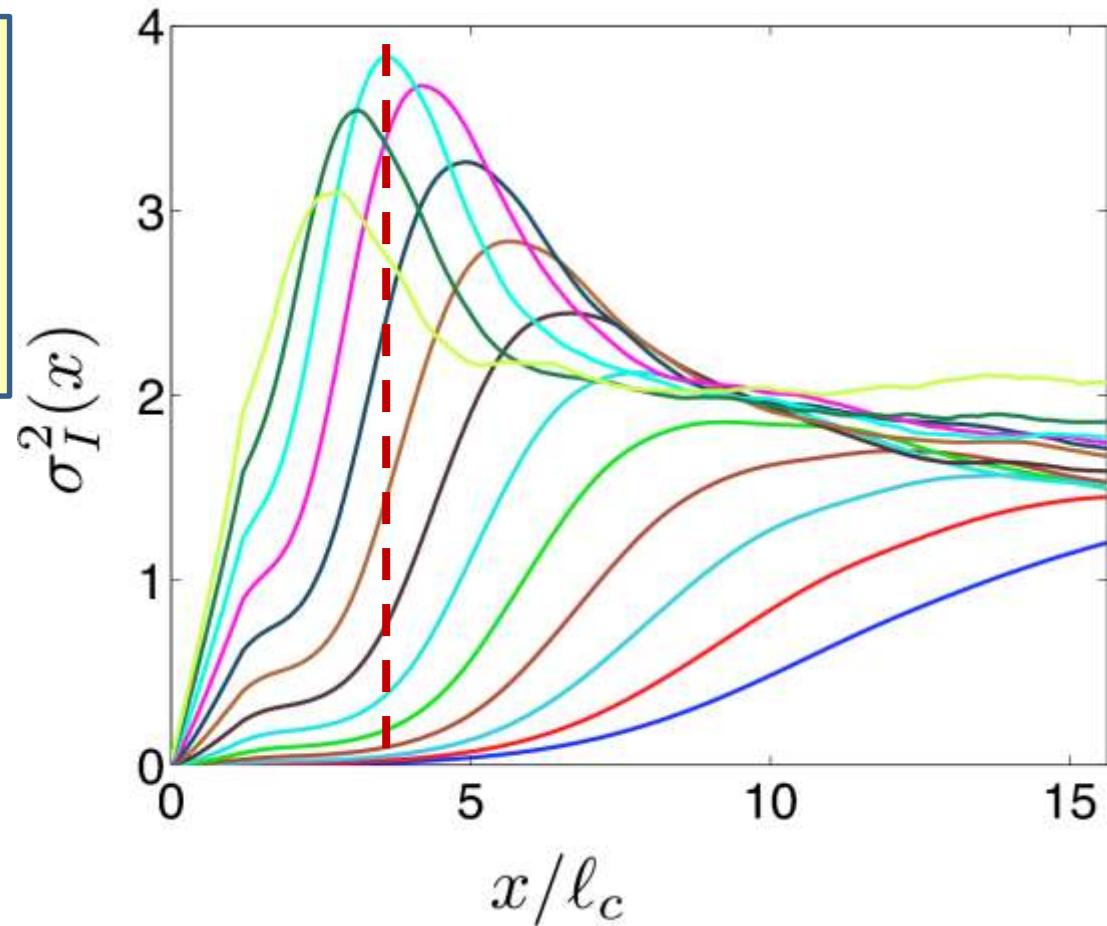
Scaling of the scintillation index

Scintillation index:

$$\sigma_I^2(x) = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1 = \frac{\text{variance of the intensity}}{(\text{mean intensity})^2}$$

scaling with
“branching length”

$$l_b \propto l_c \varepsilon^{-2/3}$$



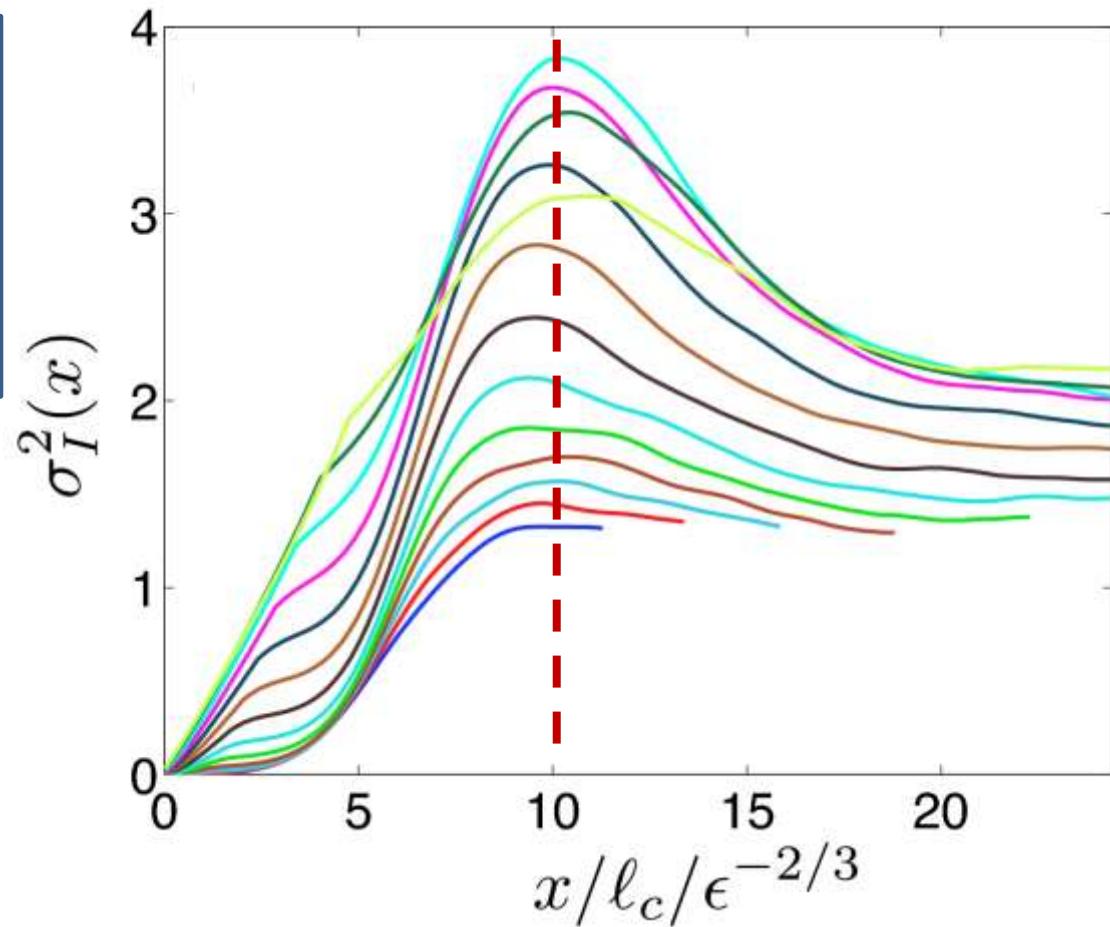
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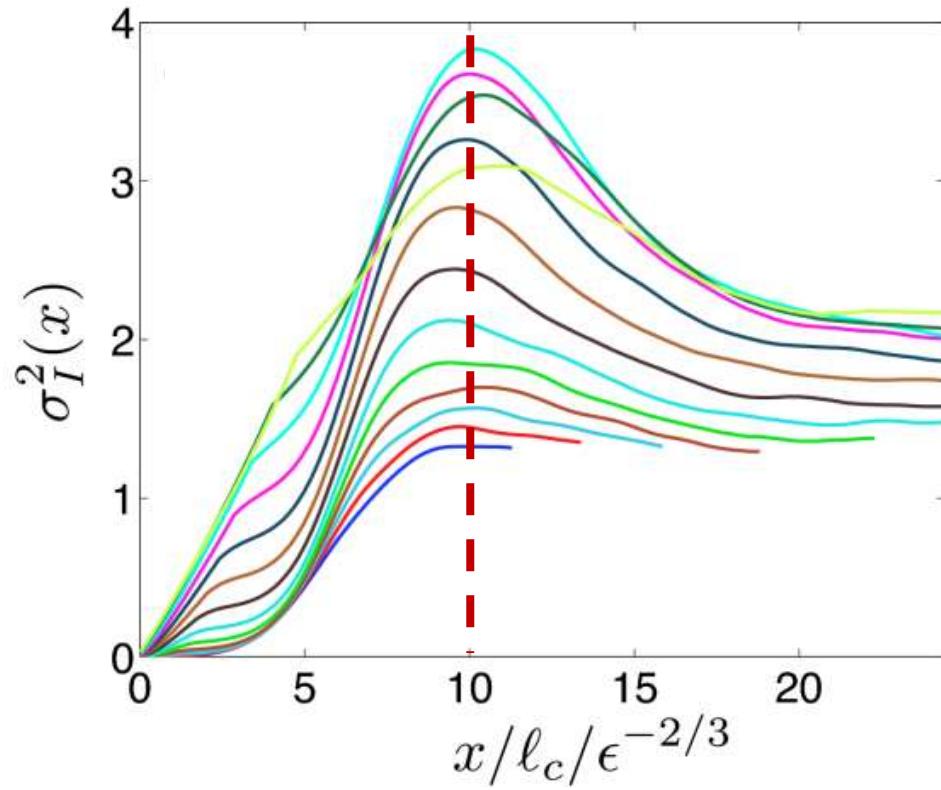
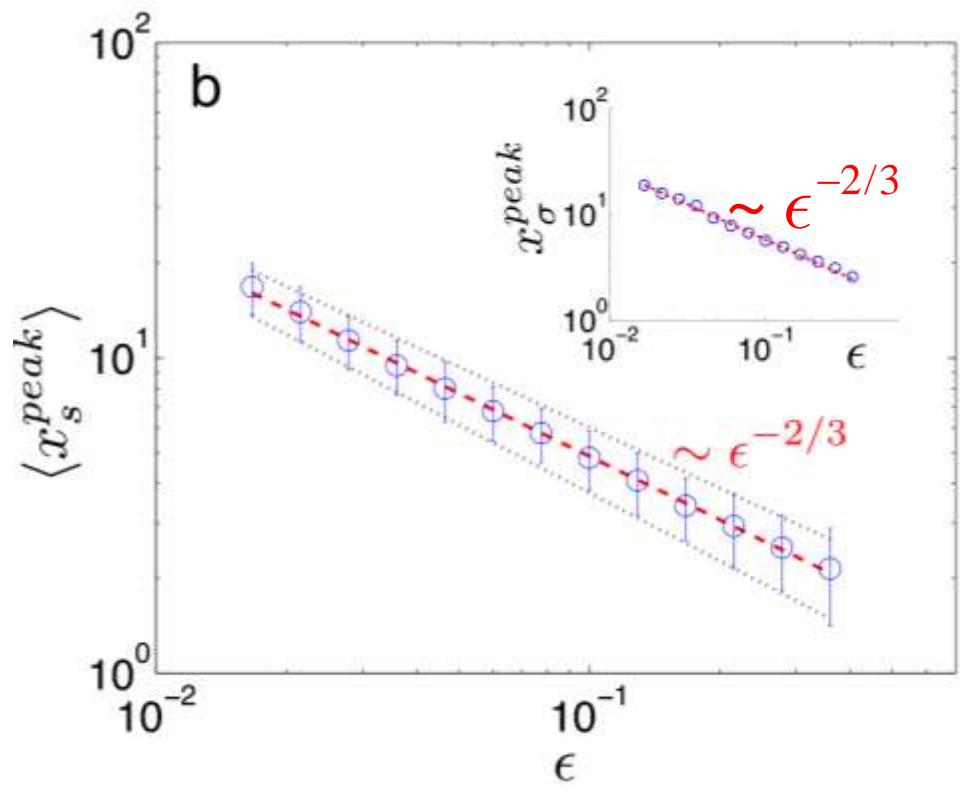
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Scaling of the scintillation index

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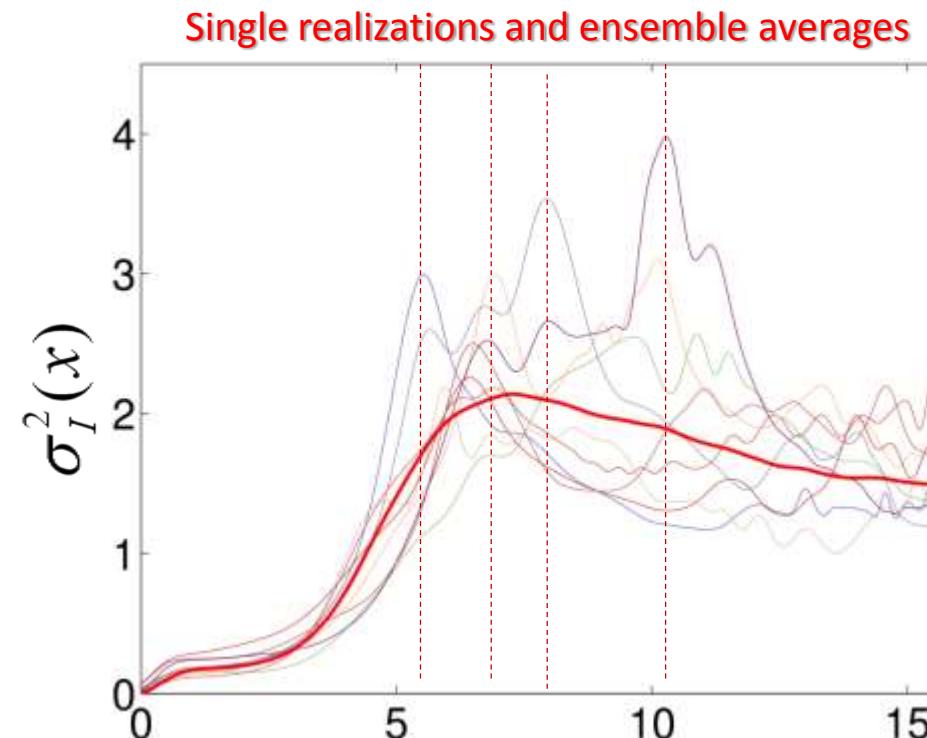
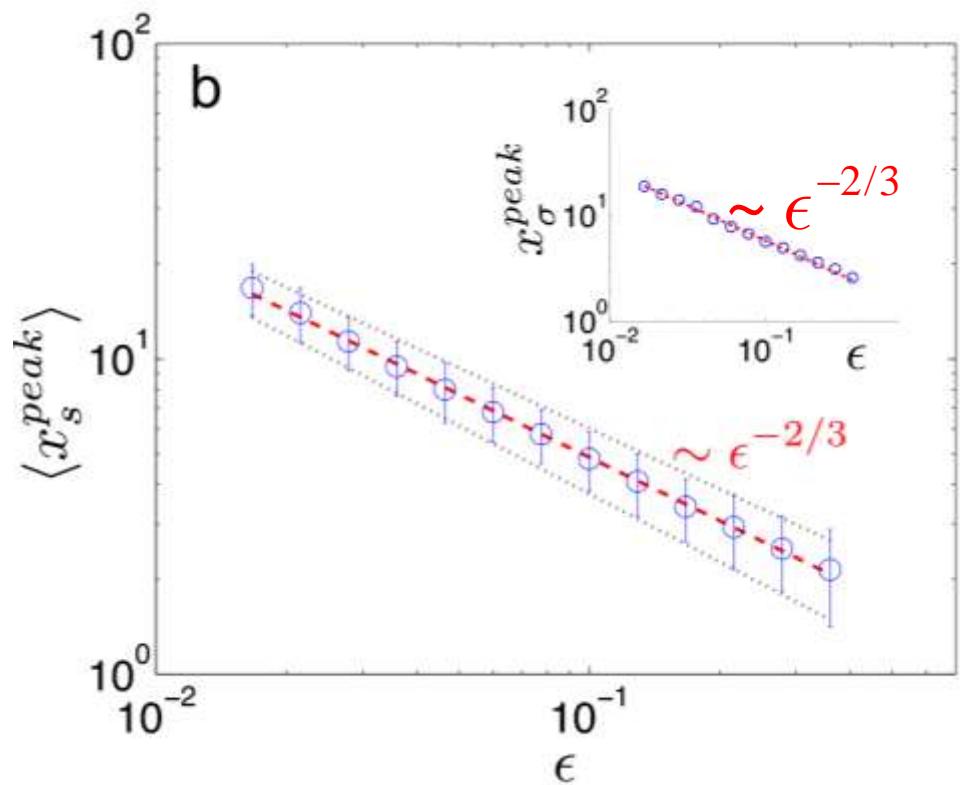


S. Barkhofen, J. J. Metzger, R. Fleischmann, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. Lett. **111**, 183902 (2013).

Scaling of the scintillation index

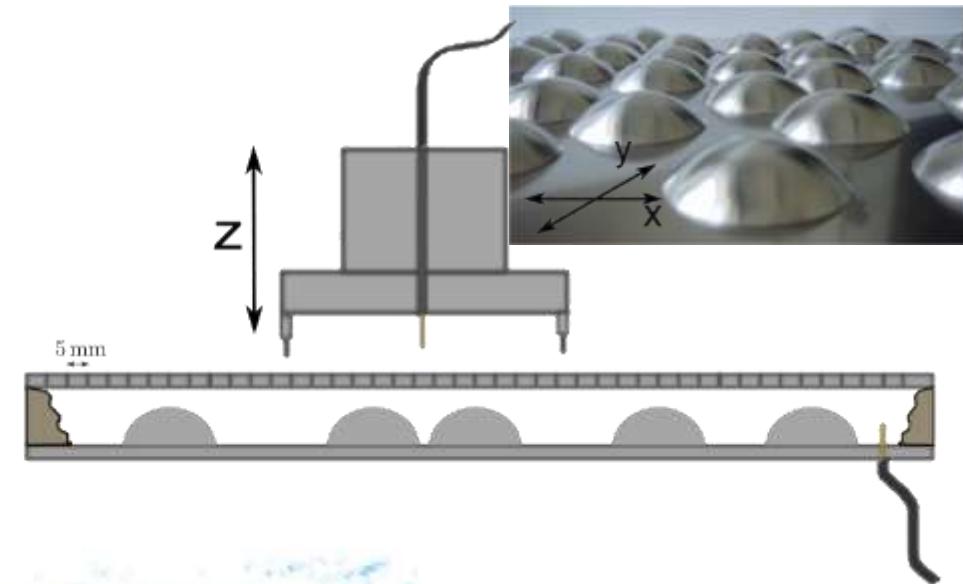
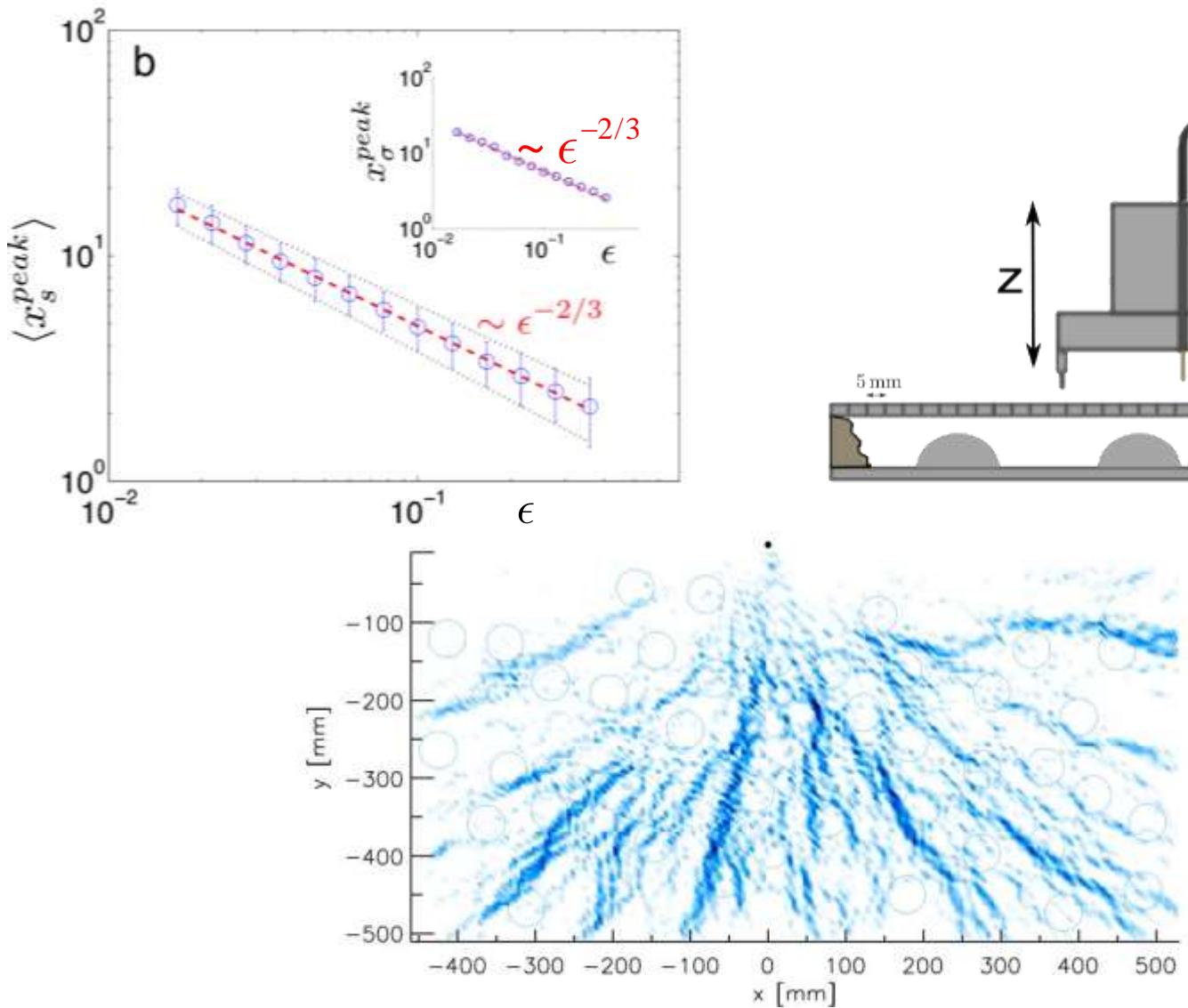
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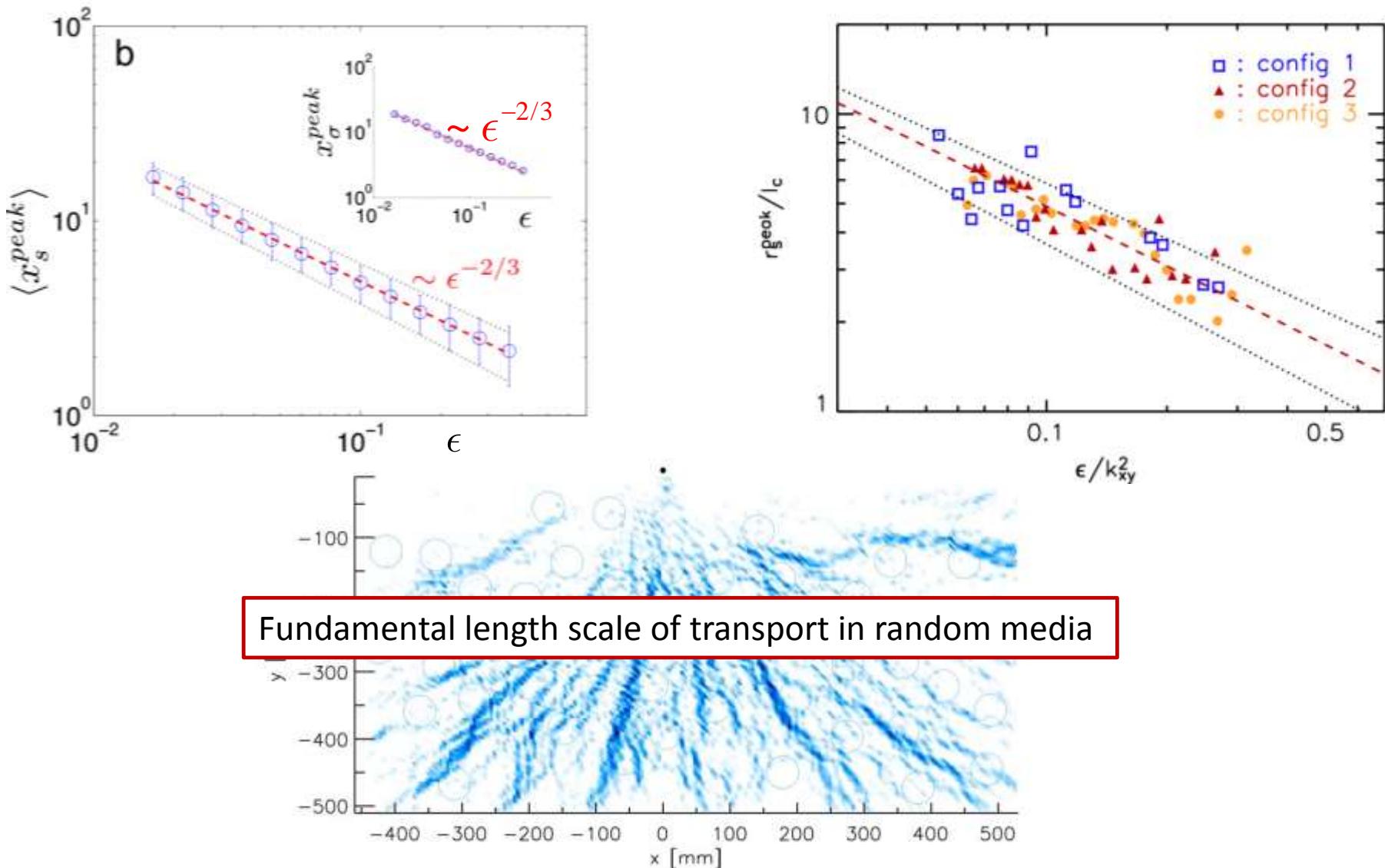
S. Barkhofen, J. J. Metzger, R. Fleischmann, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. Lett. **111**, 183902 (2013).

Observation in microwave experiments



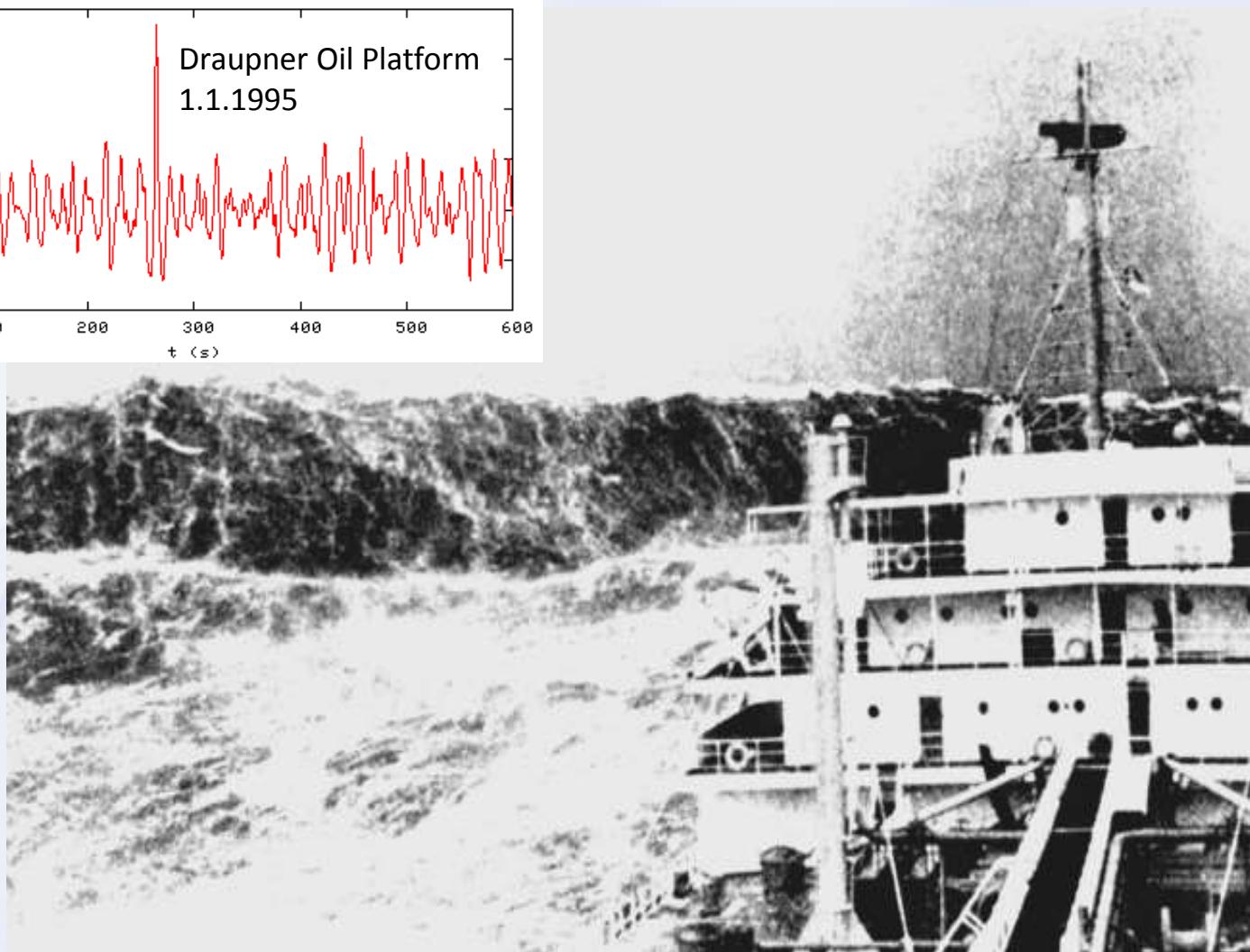
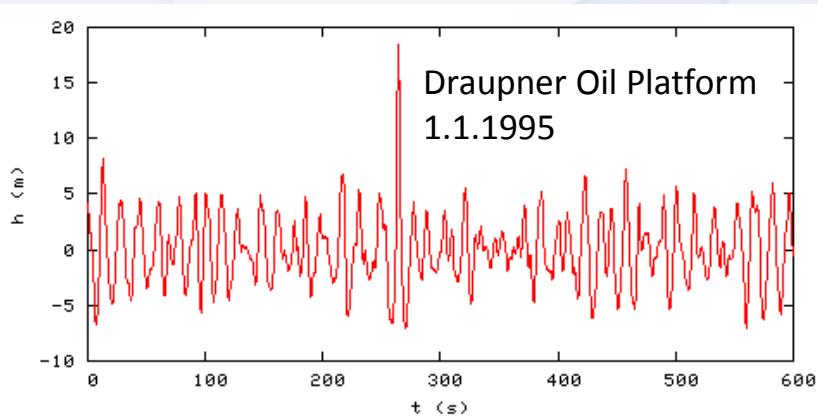
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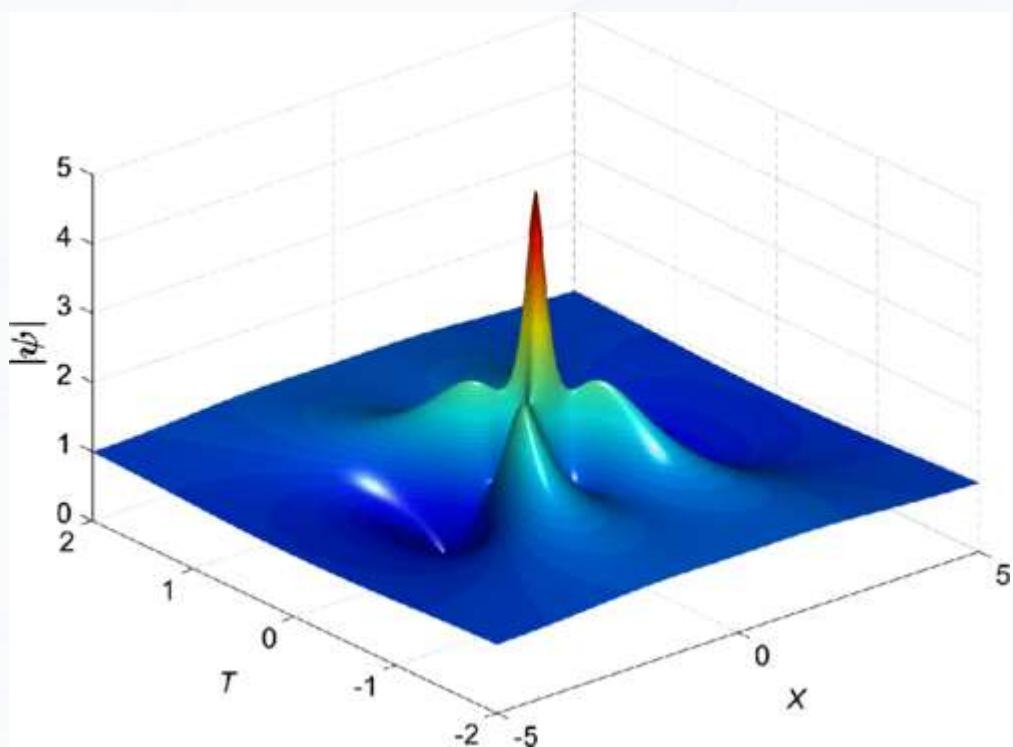
S. Barkhofen, J. J. Metzger, R. Fleischmann, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. Lett. **111**, 183902 (2013).

Statistics of wave heights/intensities: Rogue waves



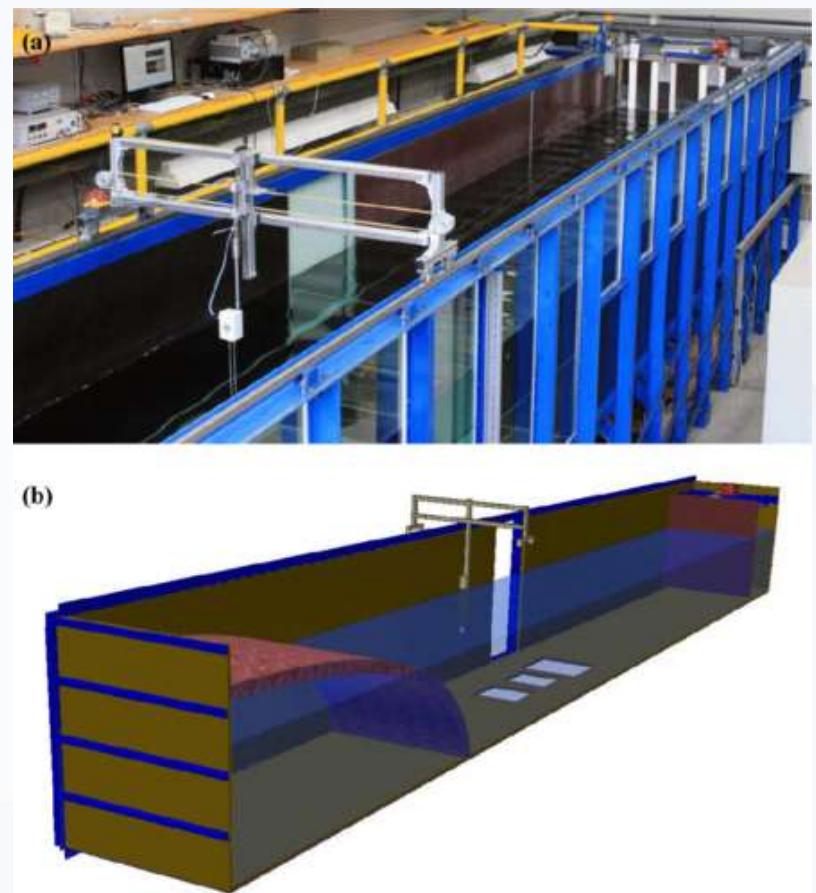
Heller europhys.news 2005

Nonlinear rogue waves



Breather Solutions

Solutions of the nonlinear Schrödinger equation
that are “localized” in space and time



Chabchoub et al. Phys. Rev. X 2012.
Chabchoub et al. Phys. Rev. Lett. 2011

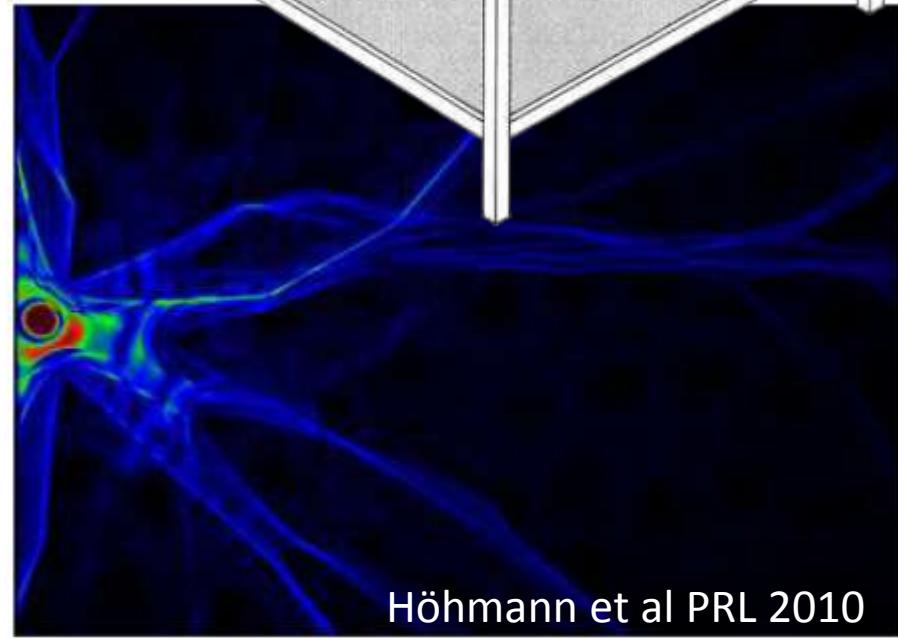
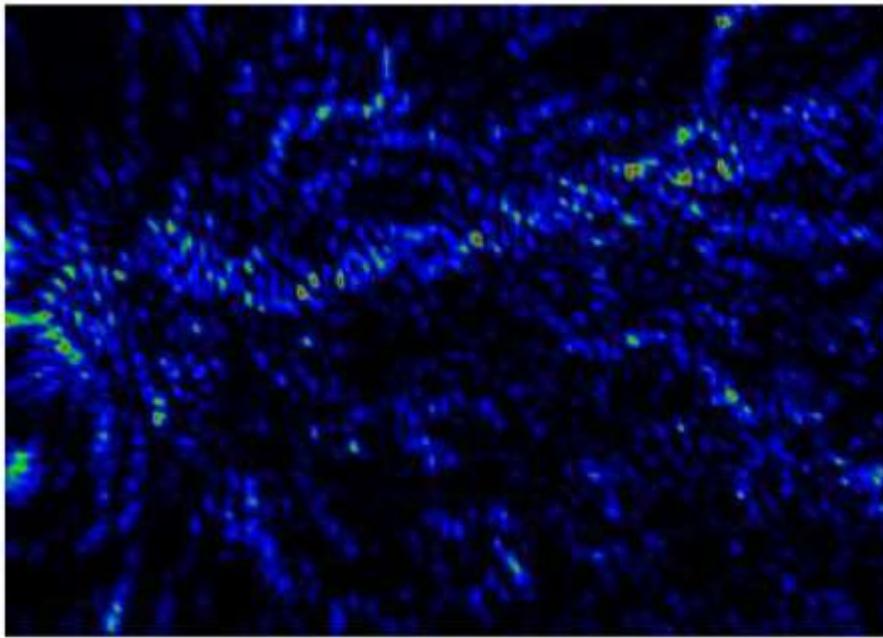
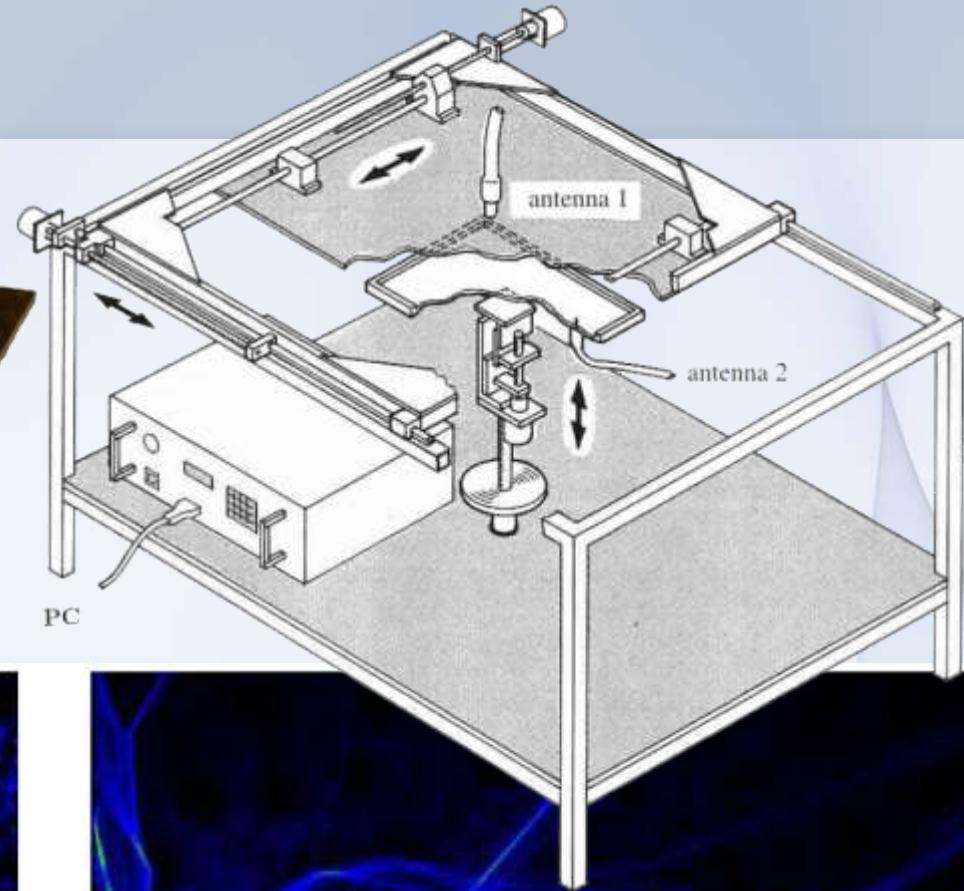
Nonlinear Rogue waves



http://journals.aps.org/prx/multimedia/10.1103/PhysRevX.2.011015/v1/e011015_vid1.movh264720x480

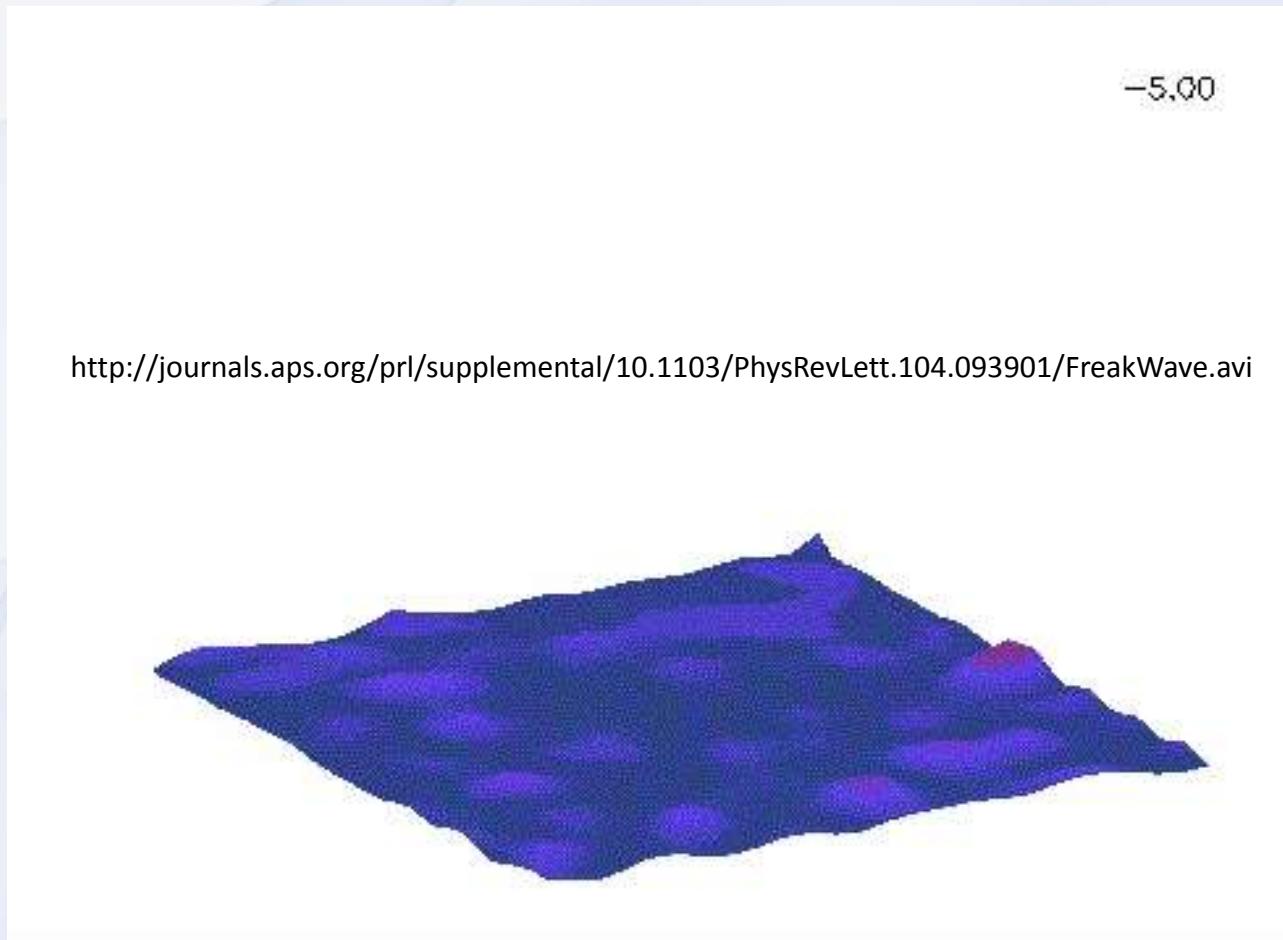
Chabchoub et al. Phys. Rev. X 2012

Linear rogue waves



Höhmann et al PRL 2010

Linear rogue wave



Höhmann et al PRL 2010

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Jakob Metzger



Theo Geisel



Henri Degueldre

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Brian LeRoy
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Stuttgart)

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Ulrich Kuhl
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